## IV. CONCLUSION

From (10) and (22), by using the constants of reference 5,

$$(ft)_{\text{core+sp}} \leq 1990/0.31 = 6420.$$

This calculation yields an upper limit only  $\sim 3$  times the single-particle ft value, despite the fact that it was based upon a quadrupole moment equal to about twice the observed one. The experimental ft value for Ge<sup>75</sup> is 25 times larger than this upper limit. It is evident from the nature of the calculation that the result for other like-core nuclei in this mass region will be similar. For lighter nuclei, the core matrix element  $\langle \alpha\{n\} | \beta\{n\} \rangle$ will be still larger, even though the measured ft values for unfavored transitions are not substantially lower.

Formula (8) indicates that, in fact, the matrix element equals 0 if  $J' \neq j'$ . A modification of the model in which at least one of the core states contains a large admixture of states with  $j' \neq J'$  could therefore yield an arbitrarily small matrix element. A regularity found by Kopfermann,<sup>8</sup> however, seems to indicate that, for several isotope pairs at least, the single particle appears in just one state: On graphs of quadrupole moment vs magnetic moment, the two experimental points for odd-A

<sup>8</sup> H. Kopfermann, Naturwiss. 38, 29 (1951).

isotopes and the theoretical point for the pure singleparticle model are approximately collinear. Another possibility of explaining the large ft values within the framework of the deformed core model would be by assuming that both the initial and final cores are deformed. The core functions would have to be substituted in formula (6). It does not appear, however, that this would change our result drastically. It should be mentioned, finally, that the results of calculations<sup>9</sup> based upon the collective model in the form given by Bohr and Mottelson, a form which assumes that both cores are deformed, are of the same magnitude as the one obtained here.

An alternate explanation of the unfavored ft values for allowed transitions in terms of the original shell model would postulate instead the predominance of different configurations in the initial and final states. The fact that all the allowed transitions between nuclei with double closed shells±one nucleon fall into the favored group is consistent with this explanation. It is not clear, however, how to explain the grouping of the allowed ft values which is observed. (There are very few with  $3.6 < \log ft < 4.5$ .)

<sup>9</sup>S. Suekane, Progr. Theoret. Phys. Japan 10, 480 (1953).

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# Energy Distributions of Fragments from Fission of U<sup>235</sup>, U<sup>238</sup>, and Pu<sup>239</sup> by Fast Neutrons\*

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The energies of fragments from fission of  $U^{235}$ ,  $U^{238}$ , and  $Pu^{239}$  by 14-Mev neutrons, from fission of  $U^{235}$  by 2.5-Mev neutrons, and from fission of  $U^{235}$  and  $Pu^{239}$  by thermal neutrons have been measured in a single Frisch grid ionization chamber. The energy distributions for fast neutrons are similar to those previously obtained for fission by thermal neutrons. The most probable energies of the light and heavy fragments for fission by 14-Mev neutrons do not change significantly from their values for slow-neutron induced fission. The valley between peaks is higher for fission induced by 14-Mev neutrons than for low-energy neutron-induced fission.

A double Frisch grid chamber has been used to measure simultaneously the energies of both fragments from fission of  $U^{235}$  by 14-Mev neutrons. The main change in the distribution of fission modes from that for thermal-neutron-induced fission is the increased probability for symmetrical fission.

## INTRODUCTION

S ince the first work of Frisch<sup>1</sup> many measurements of the energy distributions of fission fragments have been made, on a number of fissionable isotopes,

<sup>1</sup> Ŏ. R. Frisch, Nature 143, 276 (1939).

and at various neutron energies.<sup>2</sup> The purpose of the present work was to make measurements at a neutron energy of 14 Mev for comparison with measurements at other energies.

The experiments reported fall into two classes: (a) measurements of the energy of one of the fragments

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<sup>&</sup>lt;sup>2</sup> See, for example, (a) J. L. Fowler and L. Rosen, Phys. Rev. 72, 926 (1947); (b) D. C. Brunton and G. C. Hanna, Can. J. Research A28, 190 (1950) and Phys. Rev. 75, 990 (1949); (c) D. C. Brunton and W. B. Thompson, Can. J. Research A28, 498 (1950) and Phys. Rev. 76, 848 (1949); (d) J. Jungerman and S. C. Wright, Phys. Rev. 76, 1112 (1949).

from a fission event, obtained from the ionization produced in a single ionization or Frisch grid chamber; (b) simultaneous measurement of the energies of both fragments from a fission event in two chambers placed on opposite sides of the fission foil. These two classes will be referred to as single- and double-chamber experiments, respectively.

In the evaluation of fission fragment energies from the measured ionization, it has been customary to assume that w, the ratio of kinetic energy to ionization produced, is the same for fission fragments as for alpha particles. It is now known that this is not correct,<sup>3</sup> and also that the ratio varies for different fission fragments. Leachman<sup>4</sup> has estimated that  $w_L/w_H = 0.96$ , where L and H refer to the most probable light and heavy fragments. From a direct measurement of the velocities of fission fragments<sup>5</sup> it has been inferred that about 5.7 and 6.7 Mev (these corrections are called ionization defects) should be added to the energies of the most probable light and heavy fragments, respectively, to take account of energy not observed as ionization in argon. In the present work nitrogen was used as a chamber filling. No corrections for ionization defects have been made to the measured energies.<sup>6</sup>

#### APPARATUS AND EXPERIMENTAL METHODS

Fast neutrons were obtained from the Los Alamos Cockcroft-Walton accelerator. The reaction  $T(d,n)\alpha$ gave neutrons with a mean energy of 14.1 Mev and a spread, due to target thickness and finite solid angle subtended by the foil of fissionable material, which in the worst case was about  $\pm 0.5$  Mev. Neutrons of energy 2.52 Mev were obtained from the reaction D(d,n)He<sup>3</sup>; the energy spread was about  $\pm 0.3$  Mev. Thermal neutron measurements were made by placing both the chamber and a Po- $\alpha$ -Be neutron source in a graphite pile.

In the fast neutron measurements some of the neutrons seen by the fissionable material are low-energy neutrons arising from inelastic scattering and (n,2n)reactions in the chamber walls. An estimate has been made for the 14-Mev case, on the assumption of isotropic angular distribution of the scattered neutrons, and using the inelastic cross sections of Phillips et al.7 Recent measurement by Graves<sup>8</sup> have shown that the extra neutrons contributed by (n,2n) reactions roughly compensate for those lost to other reactions. It is calculated that in the single-chamber measurements about four percent of the neutron flux incident upon the foil is due to low-energy neutrons. For the double-

<sup>7</sup> Phillips, Davis, and Graves, Phys. Rev. 88, 600 (1952).

chamber measurements the figure is nine percent. Nearly all of these neutrons are below about 3 Mev.<sup>7,9</sup>

A measurement of fission rates with and without cadmium surrounding the single chamber showed that about  $2\pm 1$  percent of the fissions in the 14-Mev measurements on U<sup>235</sup> and Pu<sup>239</sup> were due to thermal neutrons. This figure should also apply to the doublechamber measurements.

The chamber used for the single-fragment measurements was similar to that used by Fowler and Rosen.<sup>2</sup> No collimator was used to limit the angular range over which fission fragments were observed. The foil-to-grid spacing was about 2 cm, and the chamber pressure of about two atmospheres of nitrogen was sufficiently high that all tracks of alpha particles and fission fragments were contained in this region. The calculated shielding efficiency of the grid<sup>10</sup> was 96 percent in the U<sup>235</sup> measurements, and 98 percent in those on  $U^{238}$  and  $Pu^{239}$ .

The double chamber consisted of two Frisch grid chambers back to back, with a common high voltage electrode and foil. The foil was mounted on Zapon and supported on a brass collimator 0.017 in. thick with about five thousand 0.032-in. diameter holes. The foil-to-grid spacing in this chamber was one inch. The grid shielding efficiency was more than 99 percent.

The chambers were filled with nitrogen which was purified continuously by allowing it to circulate through an oven containing calcium metal at 150°C. After purification, saturation for fission fragments was obtained at a chamber potential of about 8 kv. Measurements were made at 10 or 14 kv. It was verified that the shape of the fission fragment spectrum did not change with chamber potential above the saturation point. Attempts to use argon, with or without  $CO_2$ , failed because it was not possible to obtain voltage saturation for fission fragments. This may be due to the use of higher chamber pressures than were required by previous investigators.

Signals received by the collector plate of a chamber were amplified by a Model 101 preamplifier and amplifier and clipped to produce roughly rectangular pulses 3 to 4 microseconds long. Energy distributions were obtained with the use of a ten-channel pulse-height analyzer.<sup>11</sup> Energy calibrations were obtained from measurements of natural alpha particles whose energies are known<sup>12</sup> (U<sup>234</sup>: 4.76 Mev; U<sup>238</sup>: 4.18 Mev; Pu<sup>239</sup>: 5.16 Mev). Pulse heights due to fission fragments were compared with those from a generator of artificial pulses. Then alpha-particle pulse heights were compared with

 <sup>&</sup>lt;sup>8</sup> Knipp, Leachman, and Ling, Phys. Rev. 80, 478 (1950).
 <sup>4</sup> R. B. Leachman, Phys. Rev. 83, 17 (1951).
 <sup>5</sup> R. B. Leachman, Phys. Rev. 87, 444 (1952).

<sup>&</sup>lt;sup>6</sup> Recently ionization defects of various gases (including nitrogen) relative to argon have been measured by L. O. Herwig, thesis, Iowa State College, Ames, Iowa (unpublished)

<sup>&</sup>lt;sup>8</sup> E. R. Graves (to be published).

<sup>&</sup>lt;sup>9</sup> E. R. Graves and L. Rosen, Phys. Rev. **89**, 343 (1953); B. G. Whitmore and G. E. Dennis, Phys. Rev. **84**, 296 (1951); P. H. Stelson and C. Goodman, Phys. Rev. **82**, 69 (1951). <sup>10</sup> Bunemann, Cranshaw, and Harvey, Can. J. Research **A27**,

<sup>191 (1949).</sup> 

<sup>&</sup>lt;sup>11</sup> W. C. Elmore and M. Sands, *Electronics, Experimental Tech-niques* (McGraw-Hill Book Company, Inc., New York, 1949). <sup>12</sup> Way, Fano, Scott, and Thew, *Nuclear Data*, National Bureau of Standards Circular 499 (U. S. Government Printing Office, Washington, D. C., 1950).

artificial pulses of the same size, but passed through a 20-1 attenuator. Thus the energy measurement depends on the attenuation factor of the attenuator. This factor was obtained from measurements of the resistances of which the attenuator was composed. It is estimated that inaccuracies in the measurement of the ionization energies of the fission fragments do not exceed two percent.

For the double-chamber measurements, the amplified rectangular pulses due to the two fragments from a fission event were placed on the horizontal and vertical plates of an oscilloscope tube, whose intensifier was gated on for about one microsecond when the pulses were at their peak values. Thus two coincident fragments were represented on the oscilloscope tube by a well-defined bright spot whose horizontal and vertical distances from the position of the undeflected beam were proportional to the ionization energies of the two fragments.

The oscilloscope tube was photographed and the energies of the fragment pairs obtained from calibration marks placed on each frame by a pair of pulse generators. The energies of the fragments passing through the Zapon foil backing and the collimator is lower than the energy of fragments going into the uncollimated chamber. It was found that energy distributions from the two chambers could be made to agree when 2.5 Mev was added to the energy of the fragments in the collimated chamber. This correction was applied to the measured energies of this chamber.

The foils of fissionable material used in the single chamber measurements were prepared on 0.005-in. platinum sheet by painting on uranyl nitrate or plutonium nitrate with a brush.<sup>13</sup> The uranium foils were heated to convert the nitrate to oxide. Heating of plutonium foils showed apparent diffusion of the active material into the backing, and final measurements were made on unheated foils. The foil for the double chamber measurement consisted of a Zapon backing 12  $\mu$ g/cm<sup>2</sup> thick, on which was sprayed 10-20  $\mu g/cm^2$  of uranyl nitrate. Energy losses for fission fragments in the foil materials were calculated from their compositions and weights. The initial energy loss of fission fragments was taken as:  $dE/dx = kZ^{-\frac{1}{2}}$  with  $k=148 \text{ kev}/\mu\text{g}/\text{cm}^2$ . This equation can be derived from published measurements of stopping power and initial energy loss of fission fragments.<sup>14</sup> In the singlechamber measurements, the energy losses of fragments passing through the active material and normal to the plane of the foil are, for U<sup>235</sup>, 0.84 Mev; for U<sup>238</sup>, 1.2 Mev; and for Pu<sup>239</sup>, 0.01 Mev. The single-chamber energy distributions have not been corrected for foil

losses; however, the energies of the peaks of the distributions given in the text and in Table I are corrected. The energy loss of a fragment passing through the foil used for the double-chamber measurements is 0.7 Mev. Since the maximum angle of fragments is limited by the collimator to about  $60^{\circ}$  to the normal, a well-defined average energy loss exists in this case. The energies of the double-chamber distributions have been corrected for energy loss in the foil.

#### RESULTS

The energy distribution of fragments from fission of  $U^{235}$  by thermal neutrons was obtained as a check on the over-all operation of the equipment. The resulting spectrum is shown in Fig. 1A. The shape of the spectrum is in good agreement with previous measurements.<sup>2,15</sup> The peak energies, corrected for energy loss in the foil, are 91.1 and 58.8 Mev, about three percent lower than those obtained by Brunton and Hanna<sup>2</sup> or by Deutsch and Ramsey,<sup>15</sup> when the latter are corrected by  $\frac{1}{2}$  percent for energy loss in the foil

The spectrum of fragments obtained from fission of U<sup>235</sup> induced by 14-Mev neutrons is shown in Fig. 1B. The rise below 20 Mev is due to background from reactions in the chamber gas. The spectrum agrees with that of Friedland,<sup>16</sup> for fission of U<sup>235</sup> by 14-Mev neutrons, when account is taken of the poorer resolution occasioned by his greater foil thickness.

A spectrum for fragments from fission of U<sup>238</sup> by 2.5-Mev neutrons is shown in Fig. 1C. Because of the low neutron flux available at this energy, 5-Mev channel widths were used, and the statistical accuracy is poor. Another measurement differed somewhat from the spectrum shown, but the two agreed within their statistical errors, and the peak energies agreed to within 2 Mev. The spectrum is not in agreement with the measurement of Kanner and Barschall,<sup>17</sup> who used 2.8-Mev neutrons, or that of Jentschke,<sup>18</sup> who used Ra-Be neutrons. Their energies are about eight percent higher than the present results. The spectral shape of Kanner and Barschall differs considerably from that given here, while that of Jentschke is similar to that of Fig. 1C.

A spectrum for fission fragments from 14-Mev neutron bombardment of U<sup>238</sup> is shown in Fig. 1D. Another measurement agreed with these results within about 1 Mev on the average.

Spectra for fragments from fission of Pu<sup>239</sup> by thermal and 14-Mev neutrons are shown in Figs. 1E and 1F. The thermal measurement was made to permit comparison with previous work, and the agreement with spectra of Deutsch and Ramsey<sup>15</sup> and of Brunton and

<sup>&</sup>lt;sup>13</sup> The author is indebted to John Povelites and Jane Evans for

<sup>&</sup>lt;sup>16</sup> The author is indebted to John Poventes and Jane Evans for preparation of the foils. <sup>14</sup> F. Suzor, Compt. rend. **226**, 795 (1948): E. Segrè and C. Wiegand, Phys. Rev. **70**, 808 (1946); D. West, Can. J. Research **A26**, 115 (1948); N. O. Lassen, Phys. Rev. **68**, 230 (1945); R. Scherr and R. Peterson, Rev. Sci. Instr. **18**, 567 (1947); K. W. Allen and J. T. Dewan, Phys. Rev. **82**, 527 (1951).

 <sup>&</sup>lt;sup>15</sup> M. Deutsch and M. Ramsey, Atomic Energy Commission Report MDDC-945, 1946 (unpublished).
 <sup>16</sup> S. S. Friedland, Phys. Rev. 84, 75 (1951).
 <sup>17</sup> M. H. Kanner and H. H. Barschall, Phys. Rev. 57, 372

<sup>(1940).</sup> 

<sup>&</sup>lt;sup>18</sup> W. Jentschke, Z. Physik **120**, 165 (1943).



FIG. 1. Single-chamber energy distributions of fission fragments.

Thompson<sup>2</sup> is satisfactory. The valley between the peaks, which is about 30 percent higher in the present work than in the published spectra, indicates somewhat poorer resolution.

In all three cases the spectra for 14-Mev neutron bombardment are similar to those for low-energy neutrons. The peak energies change only by amounts of the order of 1 Mev. In view of the experimental errors this is not considered significant. In each case the valley between the peaks is higher at 14 Mev than at the lower neutron energies, and this may well be due to an increase in the probability of symmetrical fission. However, no quantitative data on this point can be deduced from the single-chamber measurements.

The observation that there is no significant change in fragment energies with change in energy of the incident neutron is in agreement with predictions of the fission theory of Fong.<sup>19</sup> In this theory, the main contribution to fragment kinetic energies is considered to arise from Coulomb repulsion.

In Table I are given data on the spectra obtained in this work, together with a selection from previously published spectra. The energies have been corrected for foil losses except where noted. In addition, the data of this paper for fast-neutron fission include a correction for center-of-mass motion. The purpose of this correction is to convert the measured energies (which are for fragments going into the forward hemisphere with respect to the incident neutron) to the energy averaged over all fragment directions. It is not known whether such corrections have been applied to the other fastneutron data.

The double-chamber data all refer to fission of U<sup>235</sup> by 14-Mev neutrons. In Fig. 2 are shown the energy distributions of fragments entering the collimated side of the chamber (C circuit) when the coincident fragments (T circuit) are restricted to energy bands 3 Mev wide, centered on the value given with each curve. All ordinates are normalized to the same number of fissions taking place in the foil. A comparison of these spectra with those obtained by Brunton and Hanna<sup>2</sup> for thermal neutron induced fission of U<sup>235</sup> shows that the main change in going to 14 Mev is the large increase in the probability of symmetrical fission, most of which appears as a tail on the low energy side of the light fragment peak. Figure 3 shows the variation of the most probable energy of the spectra of Fig. 2, as a function of the energy of the coincident fragments. In Fig. 4 are given the derived single fragment energy distributions which were obtained by summing over all energies of the coincident fragments.

<sup>&</sup>lt;sup>19</sup> P. Fong, A Theory of Nuclear Fission (unpublished).

	Nuclide	Neutron energy	Position of high- energy peak (Mev)	Position of low- energy peak , (Mev)	Width a maximun High- energy peak	at half- n, (Mev) Low- energy peak	Ratio of peak energies	Ratio of peak heights	Ratio of height of minimum to height of high- energy peak	Nominal foil thickness $\mu g/cm^2$	Reference
-	U <sup>235</sup>	Thermal <sup>a</sup> Thermal <sup>a</sup> Thermal 14 Mev 14 Mev <sup>e</sup> 14 Mev 14 Mev <sup>a</sup>	94.5 <sup>b</sup> 94.5 91.1 91° 89.9 91.1 89.5	$\begin{array}{c} 60.3^{\rm b} \\ 60.2 \\ 58.8 \\ 59^{\rm e} \\ 62.1 \\ 60.7 \\ 59.9 \end{array}$	$12 \\ 12 \\ 12.6 \\ 15^{d} \\ 14.7 \\ 16 \\ 14.3$	19     20     20.3     26d     24     25     23.2	$     \begin{array}{r}       1.56 \\       1.57 \\       1.55 \\       1.54 \\       1.45 \\       1.50 \\       1.49 \\     \end{array} $	$1.37 \\ 1.49 \\ 1.43 \\ 1.1 \\ 1.14 \\ 1.17 \\ 1.22$	$\begin{array}{c} 0.12\\ 0.13\\ 0.16\\ 0.57\\ 0.42\\ 0.39\\ 0.32\end{array}$	$<\!\!\!\!\begin{array}{c} <\!$	15 2(b) This paper 16 This paper This paper This paper
	U238	2.8 Mev Ra–Be 2.5 Mev <sup>e</sup> 2.5 Mev 14 Mev 14 Mev	98 97 90.0 89.0 89.8 89.6		29 <sup>d</sup> 14 13.5 12.8 17 <sup>d</sup> 17.8 <sup>d</sup>	19 <sup>d</sup> 22 19 26.5 27 <sup>d</sup> 28.8 <sup>d</sup>	$1.51 \\ 1.47 \\ 1.46 \\ 1.48 \\ 1.47 \\ 1.50$	0.89 1.43 1.24 1.49 1.16 1.15	$\begin{array}{c} 0.50 \\ 0.13 \\ 0.31 \\ 0.30 \\ 0.50 \\ 0.51 \end{array}$	120 40 50 50 50 50	17 18 This paper This paper This paper This paper
	Pu <sup>239</sup>	Thermal <sup>a</sup> Thermal <sup>a</sup> Thermal 14 Mev	93.5 <sup>b</sup> 94.6 92.8 91.0	65.3 <sup>b</sup> 65.2 64.5 64.7	$12.5 \\ 13.6 \\ 14.5 \\ 19.6^{\rm d}$	20.5 24 24 32 <sup>d</sup>	$1.43 \\ 1.45 \\ 1.44 \\ 1.41$	$   \begin{array}{r}     1.50 \\     1.47 \\     1.42 \\     1.23   \end{array} $	0.21 0.23 0.30 0.59	$<\!$	15 2(c) This paper This paper

TABLE I. Fission fragment energy distributions.

<sup>a</sup> Double-chamber data.
<sup>b</sup> Energies have been increased <sup>1</sup>/<sub>2</sub> percent to correct for energy loss in the foil.
<sup>c</sup> Uncorrected for energy loss in the foil.
<sup>d</sup> Widths estimated from extrapolated curves for the light and heavy peaks.
<sup>e</sup> This measurement has lower statistical accuracy than the other measurements.

Data obtained by summing either over fragments entering the collimated chamber (C channel) or the uncollimated chamber (T channel) are given and show good agreement with each other. To avoid confusion, the data for the dashed curves which define the energy distributions of the light and heavy fragments separately have not been plotted. The uncertainties of these curves are about the same as for the data shown.



FIG. 2. Energy distributions of fragments from fission of U<sup>235</sup> by 14-Mev neutrons, when the energy of the coincident fragments is fixed.

Figure 5 gives a contour diagram showing the relative probability for various fission modes as a function of the energies of the two fragments. Auxiliary coordinates give the ratio of energies of the fragments. as well as their masses, with the assumption that no neutrons are emitted from the fragments. Figure 6 gives the distribution in mass of the fragments, again with the assumption that no neutrons are emitted. In this figure a curve has also been plotted to show approximately the effect of including the ionization defect. The values given by Leachman for argon<sup>5</sup> have been used for the most probable light and heavy fragments, and it has been assumed that the ionization defect varies linearly with fragment energy. Some data on mass yields obtained radiochemically by Spence<sup>20</sup> have been shown in the same figure. His



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PR. W. Spence, Brookhaven National Laboratory Report BNL-C-9, 1949 (unpublished).

function of the energy of the coincident fragments.

fragment masses are masses after emission of secondary neutrons from the fragments, and thus a direct comparison of these data with the curves of this paper does not have much significance except in the region of symmetrical fission, where the latter is almost unaffected either by consideration of neutron emission from fragments, or by the ionization defect. The agreement here is satisfactory, and confirms the increase in the probability of symmetrical fission by a factor of 100 at 14 Mev as compared to that for thermalneutron-induced fission.

# DISCUSSION

The measured fission fragment energy distributions differ somewhat from the energy distributions of the fragments in the center-of-mass system of the fissioning nucleus, and before the emission of secondary neutrons. The effects to be considered are a spread in the energies of fragments which have the same initial energy, together with a shift in the mean energy of these fragments. The causes of these distortions are, (a)



FIG. 4. Energy distribution of the light and heavy fragments from fission of  $U^{235}$  by 14-Mev neutrons.

"inherent" effects, including motion of the center of mass, emission of secondary neutrons, and the ionization defect and ionization straggling; (b) statistical errors; and (c) instrumental errors, such as use of finite channel widths, energy loss in the foil, foil nonuniformity, noise and background, and errors in pulse-height measurement.

Calculations have been made of the change in fragment energy, based on a fission model in which fission occurs before the compound nucleus has lost any energy, either kinetic or internal. It is assumed that the fragments are emitted isotropically in the center-of-mass system.<sup>21</sup> Secondary neutrons are assumed to be emitted from the fragments with isotropic angular distribution in the frame of reference of the moving fragment and before the fragment has been slowed down.

On the basis of this model, it is found that the mean energy,  $\langle E_{\rm F} \rangle$ , of fragments in the laboratory system



FIG. 5. Contour diagram of fission modes for fission of  $U^{235}$  by 14-Mev neutrons.

and after the emission of  $\nu$  neutrons<sup>22</sup> is given very nearly by

$$\langle E_{\nu} \rangle = (E_1 + Q) (1 - \nu m_n/m) (M - m)/M + \Gamma E_1 Om (M - m) m_n ]^{\frac{1}{2}} M^{-\frac{3}{2}} (1 + E_1/2Q) \langle \cos \phi \rangle.$$
 (1)

Here  $E_1$  is the energy of the incident neutron, Q is that portion of the energy of fission which is released as kinetic energy of the primary fragments, and M, m, and  $m_n$  are masses of the compound nucleus, of the fragment before neutron emission, and of a neutron, respectively. In obtaining this expression averages have been taken over the angle of emission of the fragments, over the angles and energies at which secondary neutrons may be emitted, and over a distribution in the number of neutrons emitted, whose mean value is  $\nu$ . The second term in the equation involves an average over the angle  $\phi$  between the incident neutron and the fragment. This term is the



FIG. 6. Mass distribution of fragments from fission of U<sup>235</sup> by 14-Mev neutrons.

 $<sup>^{21}</sup>$  W. C. Dickinson and J. E. Brolley, Phys. Rev. 90, 388 (1953) have recently measured an anisotropy in the angular distribution of fragments from fission induced by 14-Mev neutrons.

so-called center-of-mass correction. In the singlechamber measurements the incident neutrons were normal to the foil and fragments were observed in the region  $0 < \phi < 90^{\circ}$ . The correction, which is nearly independent of fragment masses, is 0.65 and 1.54 Mev for the measurements at 2.5 and 14 Mev, respectively. For the double-chamber experiment neutrons entered parallel to the plane of the foil. Fragments were observed in each chamber at angles between  $30^{\circ}$  and  $150^{\circ}$  to the incident neutrons, and the correction vanishes.

The measured energies of a group of fragments which were initially monoenergetic are distributed around the mean value given by Eq. (1). The rms deviation of this distribution is made up of three parts:

$$\sigma_0 = 2 \left[ (E_1 Q m (M - m) m_n / M^3) (\langle \cos^2 \phi \rangle - \langle \cos \phi \rangle^2) \right]^{\frac{1}{2}} \times (1 + E_1 / 2Q), \quad (2)$$

due to motion of the center of mass;

$$\sigma_1 = 2 \left[ E_0 \langle E_n \rangle \nu m_n / 3m \right]^{\frac{1}{2}},\tag{3}$$

due to the emission of  $\nu$  neutrons of mean energy  $\langle En \rangle$ ; and

$$\sigma_2 = E_0 m_n \sigma_\nu / m, \qquad (4)$$

due to the distribution in  $\nu$ , with rms deviation  $\sigma_{\nu}$ .  $E_0$  is the energy of a fragment before neutron emission.

Estimates were made of the effect which small pulses due to natural alpha particles from the foil and due to the reactions in the chamber gas have on the spectrum. It was found that no observable distortion should result, except possibly in the case of gas reaction background. Experimental checks were performed on the effect of the gas background pulses by taking fission fragment spectra at various neutron flux strengths ranging from one-tenth to three times the flux used in the final measurements. No changes in spectral shape with neutron flux were found.

The over-all dispersion introduced in the fission fragment energies can only be estimated roughly, since some of the numbers required are not known. Even for the unreasonably large value  $\nu = 5$  the rms dispersions do not exceed 2.6 Mev for the heavy fragments and 3.4 Mev for the light fragments. The measured widths are about 25 and 16 Mev for the heavy and light fragment peaks. These are reduced to 24.2 and 13.8 Mev by removing the dispersions given above. Consideration of the effect of foil thickness further reduces these by about 0.3 Mev. The statistical errors introduce an uncertainty of about 0.5 Mev. Thus it appears that the distortions of the experimental

spectra from their true shapes, although appreciable, are not serious.

The rms dispersions given above, when converted to widths of distributions at half-maximum, become 6 and 8 Mev, respectively. These may be compared with an estimate<sup>4,5</sup> of 8 Mev for the instrumental spread in ionization chamber measurements. This estimate referred, however, to measurements made with thermal neutrons. Since one would expect a smaller energy dispersion for thermal neutron measurements than for those at 14 Mev, it may well be that there exist other sources of dispersion, such as ionization straggling, which have not been taken into account.

Estimates have been made of the corrections to be applied to the measured spectra to account for fission fragments produced by low-energy neutrons. It is found that, for fission of U235 by 14-Mev neutrons, the measured ratios of peak heights, 1.16 and 1.22, found in the single chamber and double chamber measurements, should be reduced to 1.15 and 1.20 and the measured ratios of the height of the valley to that of the high-energy peak, 0.40 and 0.32, should be increased to 0.41 and 0.34. These changes are of the order of magnitude of the statistical errors. The cause of the differences between the single-chamber and doublechamber measurements, which presumably should yield identical results, is not known. There may be a real change in energy distribution for fragments emitted at different angles to the incident neutron.

An analysis has been made of the effect of neutron emission by the fragments in modifying the measured mass distribution curve (Fig. 6) from the true distribution. To a good approximation the true curve is obtained by shifting the fragment masses by

$$\Delta m_H = -\Delta m_L = m_n (m_H \nu_H - m_L \nu_L) / M, \qquad (5)$$

where  $m_H$  and  $m_L$  refer to the masses of the heavy and light fragments before neutron emission.

The rms instrumental deviation in the mass distribution, as computed from the rms deviations in fragment energies given above, is 3.2 mass units. The experimentally observed distribution has a width at half-maximum of about 19 mass units, and this is reduced to about 17.5 units by removing the instrumental spread. Thus the observed distribution reproduces the main features of the true mass distribution. However, a fine structure peak, such as has been observed by Glendenin *et al.*<sup>23</sup> for fission of U<sup>235</sup> by thermal neutrons, would not be detected with the resolution obtained here.

<sup>23</sup> Glendenin, Steinberg, Inghram, and Hess, Phys. Rev. 84, 860 (1951).