deformation and the mass deformation as measured by the rotational energy.

Either of the following possibilities could increase the moment of inertia of the rotational state and thereby help to explain the existing discrepancy.

(1) Neutron structure different from proton structure. This could mean deformation of neutron structure greater than deformation of proton structure, for which there is no known reason, or neutron radius greater than proton radius, for which a qualitative argument has been advanced.[18] Such effects, together with possible effects of nonuniform density distribution, cannot reasonably account for all the factor four.

(2) Breakdown of the idea that the kinetic energy of collective motion is correctly described by an equivalent irrotational fluid flow. The observed moments of inertia lie, as they should, between the limiting moments for rigid-body rotation and for irrotational fluid flow. Independent evidence for a long mean path of nucleons within nuclear matter leads to the expectation that moments of inertia should lie close to the (small) moments of irrotational flow. The large observed moments, therefore, provide evidence against a complete independent particle picture.

It should be stressed that the very close agreement of theoretical and experimental energy ratios provides strong evidence for rotational states, (i.e., energy proportional to square of angular momentum), but does not check the unified (or collective) model in any fundamental way. Only the magnitudes of the energies test the independent particle assumption which underlies these models. The approximate proportionality of the moments of inertia to the square of the nuclear deformation favors the independent particle picture. The large values of the moments show that the picture is not wholly adequate.

Some helpful comments by Professor M. G. Mayer and Professor J. A. Wheeler caused this note to be written. The remarks here are based on earlier work with L. Wilets and D. L. Hill (reference 2).

[18] M. H. Johnson and E. Teller, Phys. Rev. 93, 357 (1954).

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Scattering of 1.32-Mev Neutrons by Protons*

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The total cross sections of polyethylene and graphite for scattering of neutrons of 1.315 ± 0.003 Mev have been measured and are $\sigma(CH_2)=(9.542\pm 0.035)\times 10^{-24}$ cm^2 and $\sigma(C)=(2.192\pm 0.020)\times 10^{-24}$ cm^2. From these the hydrogen cross section is $\sigma(H)=(3.675\pm 0.020)\times 10^{-24}$ cm^2. If the best current values of the binding energy of the deuteron (see reference 3) the coherent scattering length (see reference 4) and the epithermal cross section (see reference 5) are used, the singlet effective range in the shape-independent approximation is $(2.4\pm 0.3)\times 10^{-13}$ cm. This is to be compared with the proton-proton singlet effective range of $(2.7\pm 0.1)\times 10^{-13}$ cm.

I. INTRODUCTION

It seems probable that consideration of the singlet and triplet zero orbital angular momentum states suffices for an accurate description[1] of neutron-proton scattering from the bound state up to neutron bombarding energies of the order of 15 Mev. If so, four independent measured quantities may determine[2] the strength and effective range of both the singlet and triplet S-wave interactions, and six independent quantities may begin to differentiate in both cases between interactions of qualitatively different shapes. Three accurate independent quantities known at present are the binding energy of the deuteron[3] and the thermal neutron-proton singlet and triplet scattering amplitudes.[4,5] If applied to a description of the interactions by static potentials, these may be used to determine the strength and range of a triplet potential of a given shape and to determine the approximate product of the magnitude times the square of the range for a singlet potential of a given shape.

Other independent measurements which may be made are the total neutron-proton cross sections for fast neutrons of different energies. The region of neutron

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[1] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. 22, 77 (1950). That a tensor coupling of the triplet S state to a state of higher orbital angular momentum may be included effectively in the triplet S parameters is shown by J. M. Blatt and L. C. Biedenharn, Phys. Rev. 86, 399 (1952).

[2] George Snow, Phys. Rev. 87, 21 (1952).

[3] R. C. Mobley and R. A. Laubenstein, Phys. Rev. 80, 309 (1950).

[4] Burgy, Ringo, and Hughes, Phys. Rev. 84, 1160 (1950).

[5] E. Melkonian, Phys. Rev. 76, 1744 (1949).

energy of around 1 Mev is particularly suited for giving an effective singlet range independent of singlet and triplet shape, because the singlet contribution to the total cross section is comparatively large at energies less than a few Mev, and because the effect of potential shape is small. At around 5 Mev the triplet contribution to the total cross section is dominant, so an accurate cross section may be used to indicate a triplet shape if the singlet range is known accurately. At around 15 Mev the triplet phase-shift is nearly 90°, and therefore the triplet contribution to the total cross section is quite insensitive to triplet parameters, giving an opportunity to obtain the singlet shape if the singlet range is known accurately.

The total n-p cross sections have been measured over a wide range of neutron energies in many previous experiments, of which only the most recent[6–9] have been sufficiently accurate to give information about even the singlet range. In this experiment, aiming for a determination of the singlet range to the order of 0.1×10^{-13} cm, we needed to know the energy to 10 kev and the cross section to 0.2 percent. At transmissions of $T=0.4$ the latter permits an error in transmission of 0.2 percent, or at $T=0.6$ an error of 0.1 percent. In this range of accuracy in transmission measurements many precautions must be taken and many small corrections made. Both of these are discussed in Sec. II, and the corrections are displayed in Table I.

The final accuracy achieved in the singlet range, of the order of 0.3×10^{-13} cm, is not enough to select any but extreme singlet shapes when used in conjunction with even the most accurate higher energy experiments.

II. EXPERIMENT

The total neutron cross sections of polyethylene and graphite were determined from many transmission measurements. We have, however, a very accurate experiment only at a transmission of 0.4 and in a single geometry.

Neutrons were produced by the Li(p,n) reaction[10] using the Rockefeller electrostatic generator at M.I.T. Neutron energy was determined by observing the oxygen resonance at 1.315 Mev.[11] For each scatterer the counting rate of a small, propane-filled proportional counter, relative to that of a BF_3 "long-counter"[12] monitor, was determined many times with the scatterer alternately shielding the propane counter and removed. The setup is shown in plan drawing in Fig. 1.

The hydrogen cross section was computed on the assumption that the polyethylene was pure CH_2 by

[6] Lampi, Freier, and Williams, Phys. Rev. **80**, 853 (1950).
[7] Poss, Salant, Yuan, and Snow, Phys. Rev. **87**, 11 (1952).
[8] Hafner, Hornyak, Falk, Snow, and Coor, Phys. Rev. **89**, 204 (1952).
[9] Fields, Becker, and Adair, Phys. Rev. **89**, 908 (1953).
[10] Hanson, Taschek, and Williams, Revs. Modern Phys. **21**, 635 (1949).
[11] Freier, Fulk, Lampi, and Williams, Phys. Rev. **78**, 508 (1950).
[12] A. O. Hanson and J. L. McKibben, Phys. Rev. **72**, 673 (1947).

TABLE I. Scattering data, corrections, and cross sections.[a]

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	Properties of scatterers			Distance (inches)				Corrections to transmissions									
Date	Material		sample length (inches corrected to 23°C)	Source to scatterer	Scatterers to front of detector	Observed trans-missions	Adjustment of length and density to 23°C	Background in detector	Background in monitor	Low-energy group of neutrons	Rate dependence of detector efficiency	Rate dependence of monitor efficiency	Scattering into detector	Scattering into monitor	Corrected trans-mission	Cross section in 10^{-24} cm² with statistical error only	Standard deviation of errors in columns 4 and 8–15
10/9/51	CH_2	a	0.5411 ±0.0002	5.5 ±0.3	9.8 ±0.3	0.5982 ±0.0021	… ±0.0004	−0.0016 ±0.0004	…	+0.0007 ±0.0002	−0.0011 ±0.0006	…	−0.0007 ±0.0001	+0.0002 ±0.0001	0.5957	9.555 ±0.067	±0.028
12/14/51	CH_2	a	0.5411 ±0.0002	3.0 ±0.3	5.0 ±0.3	0.5997 ±0.0014	… ±0.0004	−0.0007 ±0.0002	…	+0.0008 ±0.0003	−0.0012 ±0.0006	…	−0.0023 ±0.0003	+0.0008 ±0.0002	0.5971	9.516 ±0.044	±0.028
12/14/51	CH_2	c	0.5478 ±0.0004	3.0 ±0.3	5.0 ±0.3	0.5960 ±0.0015	… ±0.0004	−0.0007 ±0.0002	…	+0.0008 ±0.0003	−0.0012 ±0.0006	…	−0.0023 ±0.0003	+0.0008 ±0.0002	0.5934	9.513 ±0.048	±0.029
1/15/52	CH_2	c	0.9802 ±0.0004	3.0 ±0.3	4.5 ±0.3	0.3958 ±0.0012	+0.0001 ±0.0003	−0.0009 ±0.0002	…	+0.0006 ±0.0002	−0.0012 ±0.0006	…	−0.0030 ±0.0004	+0.0001 ±0.0001	0.3915	9.556 ±0.031	±0.024
2/3/52	CH_2	c	0.9802 ±0.0004	3.0 ±0.3	5.0 ±0.3	0.3939 ±0.0009	+0.0005 ±0.0004	−0.0007 ±0.0002	−0.0002 ±0.0002	+0.0005 ±0.0002	…	+0.0004 ±0.0002	−0.0028 ±0.0004	+0.0001 ±0.0001	0.3917	9.551 ±0.023	±0.020
12/14/51	C	s	1.0004 ±0.0002	3.0 ±0.3	5.0 ±0.3	0.6284 ±0.0017	…	−0.0006 ±0.0002	…	+0.0008 ±0.0003	−0.0012 ±0.0006	…	−0.0012 ±0.0006	+0.0004 ±0.0002	0.6266	2.192 ±0.014	±0.008
12/14/51	C	1	0.9992 ±0.0002	3.0 ±0.3	5.0 ±0.3	0.6275 ±0.0017	…	−0.0006 ±0.0002	…	+0.0008 ±0.0003	−0.0012 ±0.0006	…	−0.0012 ±0.0006	+0.0004 ±0.0002	0.6257	2.192 ±0.014	±0.008

[a] All errors statistical or estimated standard deviations.
CH_2 average 9.542 (using both statistical and other errors) ±0.016 (statistical errors only)
C average 2.192 ±0.010 (statistical errors only)

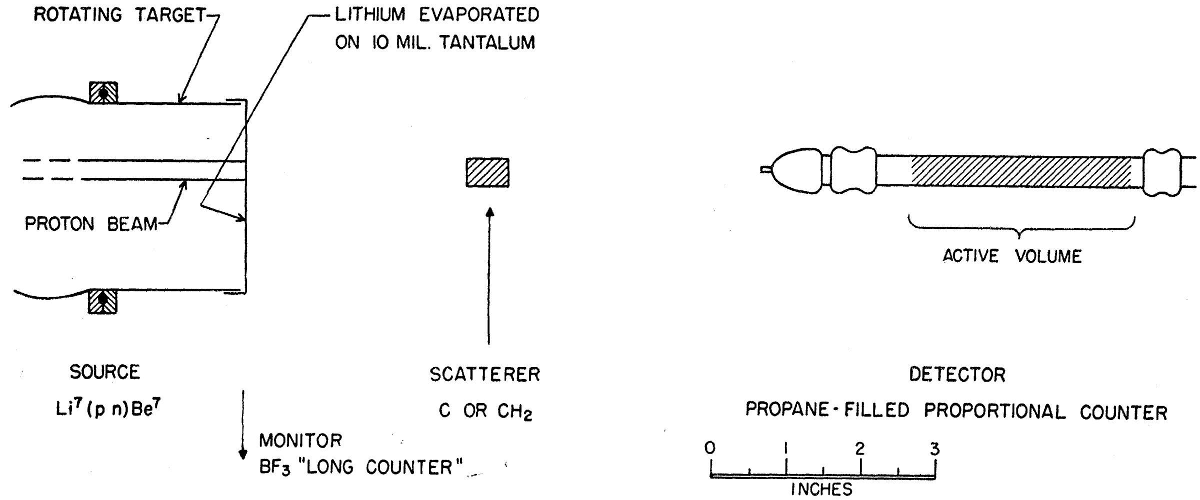

FIG. 1. Plan drawing of experiment.

subtracting the carbon cross section from that of polyethylene. In the final experiments the results were corrected for scattered neutrons entering the counters, the group of neutrons of lower energy from the Li(p,n) reaction, the change in efficiency of the counters as a function of counting rate, and background. A description of the salient experimental details follows.

A. Neutron Source

The electrostatic generator produced a proton beam of 10 microamperes and a fraction of a kilovolt spread in energy. This beam bombarded a thin film of lithium which had been evaporated on a rotating disk of 0.010-in. tantalum.

1. *Target Thickness*

The lithium target thickness—which chiefly determined the spread in energy of the resulting neutrons—was estimated roughly from the "geometrical resonance"[10] at threshold, both before and after using the target. In order to take into account more accurately the thickening and deterioration of the target, the neutrons produced in this manner were used to determine the transmission of an oxygen-containing scatterer as a function of proton beam energy. In the vicinity of 1.315 Mev the oxygen cross section for neutron scattering has a strong resonance. By locating this transmission minimum repeatedly throughout the experiment the average energy of the main group of neutrons from the Li(p,n) reaction was known at all times.

A discussion of the effect of the much smaller second group of neutrons will be given in Sec. 3 below. For the main group the very close approximation of the effective neutron energy to the energy of the center of the oxygen resonance is displayed schematically in Fig. 2. There $\epsilon(E)Y(E)$ is the counter efficiency times target yield for neutrons of each energy coming from the target. When weighted with the transmission T_{ox} of oxygen, this gives a counting rate for each machine energy proportional to $\int \epsilon(E)Y(E)T_{ox}(E)dE$. The ratio of this quantity to the unattenuated rate $\int \epsilon(E)Y(E)dE$ is minimized by changing the machine energy.

The machine energy corresponding to this minimum transmission is found by extrapolating from the two nearly straight parts of the observed transmission curve to their interaction as in Fig. 2. One thus sets the effective central energy $\bar{E}$ of the target, determined by the average of two approximately linear weightings with equal and opposite slopes, on the peak of the oxygen cross section, provided only that the resonance is approximately symmetric, and that the detector efficiency is (as observed) a linear function of energy over this small energy region, and that the relative yield from different parts of the target does not change rapidly with energy.

The central energy for polyethylene scattering is also very closely defined by a flat weighting, since the polyethylene cross section does not change by more than 10 percent over a 100-kev interval in this energy region.

The energy of a particular neutron resonance is thus chosen as the standard effective neutron energy for the experiment. It is believed that this method gives the effective energy to a few kev for any reasonable target yield profile and for targets up to 40-kev thickness. In practice targets of from 10 to 15-kev rise when fresh were used, thickening to 30 kev at most with use. The energy of the resonance was measured at Wisconsin[13] and checked by us as $E_n = 1.315 \pm 0.003$ Mev. If a future independent determination of the energy of this resonance gives a different value, the present data may be reassigned to that energy; a change of about 10 kev is required to change the singlet range by 0.1×10^{-13} cm.

After cleaning a used target only lightly with alcohol and steel wool, with 6 percent of the target activity remaining, the cross section was measured at the same

[13] R. K. Adair (private communication).

energy, and found (with insufficient statistics) to be greater than previously by 0.3 ± 0.4 percent. In this case of partial cleaning, the target may well have been thickened rather than thinned. After a thorough cleaning of another old target, only 0.15 percent of the activity remained and it gave the same transmission as the full target within the statistical error of 15 percent, reflecting a possible error of only 0.04 percent in cross section. In any case, unless there was target thickening of greater than about 50 kev, any effect from target nonuniformity would be included in the centering on the oxygen resonance.

2. *Scattering from Target and Nearby Material*

The 0.010-in. tantalum target backing was mounted on the end of a thin-walled steel cylinder, and there was an appreciable mass of scattering material several inches back of the target. The massive deflector magnet was several feet behind it. But because of the small scale of the experiment, and because of the sharp energy bias of the detector and the lower energy and yield of backward neutrons, the neutrons scattered back by this material were not expected to be observed. (All neutrons coming off at more than 85° had less than 10 percent counting efficiency relative to those at 0°, even if scattered elastically into the detector.) This was checked experimentally by background studies described below. An upper limit on forward scattering by the tantalum was set by observing that there was less than 0.1 percent difference in the number of neutrons counted with a $\frac{3}{8}$-in. shadow cylinder and with a $\frac{5}{8}$-in. cylinder.

3. *Monitoring of Source Strength*

The neutron intensity was monitored by a BF_3 "long counter" at 90° to the beam, with standard Model 100 pulse amplifier[14] and parallel lines of scaling circuits. Both monitor and detector scalers were turned off after a fixed number of monitor counts, usually of the order of 3×10^5 counts in a five-minute run.

B. Detector

Because the size of the beam spot on the target was not more than 4 millimeters in diameter under good operating conditions, the detector could be made quite small. It was placed as close to the target as compatible with sure shadowing of the whole detector and with keeping geometric corrections to less than 1 percent. The beam size and position were observed occasionally with a glass viewer. Some data were taken when there was poor control of the beam size, and gave lower cross sections because of incomplete shadowing.

1. *Counter Construction and Electronics*

The detector was a proportional counter $\frac{7}{16}$ in. in outside diameter with a 0.010-in. center wire. There were two sets of guard rings—0.035-in. hypodermic needles on the center wire and intermediate voltage rings on the end seals to aid in insulating the negative 12 kv on the outer electrode. The active volume was about 3 in. long. The filling gas was 5 atmospheres of cp propane, not purified in any way. This counter had an almost linear integral bias curve. The measured total efficiency was about 1.5 percent, but the discriminator bias was set sufficiently high to eliminate all but perhaps 20 percent of the pulses.

The pulses were amplified by a Model 100 preamplifier and amplifier. Two discriminators and scaling circuits in parallel followed the amplifier to provide a check on electronic malfunctions in these components. Scalers with discriminators, such as the Model 310, were used. The negative 12-kv power supply was a regulated-input half-wave rectifier and *RC* filter.

2. *Background in Detector*

Background studies were made using Lucite rods to attenuate the neutrons coming directly from the target to the counter. The true background was attributed to (1) scattering from the floor and walls of the room, essentially independent of small displacements of the counter; and (2) a small amount of reflection from the filter and preamplifier 30 in. behind the counter, depending on counter position and the amount of shielding afforded by a scatterer. It was also necessary to ascribe a transmission of 0.15 percent to the 12-in. Lucite cylinders used as attenuators, although this is a factor of

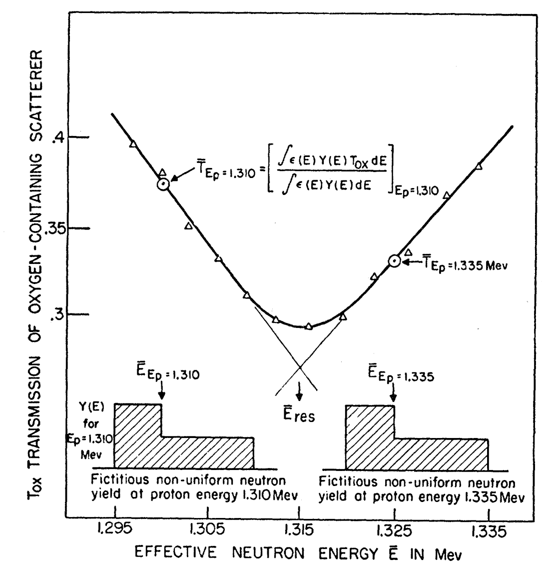

FIG. 2. Schematic drawing of method of determining effective neutron energy. The points are experimental transmissions with a target with approximately 14 kev "geometric" rise at threshold. The average transmission is shown, by circles, for a nonuniform target with the neutron yield shown, for two different values of the proton energy. The effective neutron energy of the target is $\bar{E}(E_p = 1.310)$ at the lower energy and $\bar{E}(E_p = 1.335)$ at the upper. These energies correspond closely to the effective energies for *n-p* scattering, as explained in the text.

[14] W. C. Elmore and M. Sands, *Electronics: Experimental Techniques* (McGraw-Hill Book Company, Inc., New York, 1949).

five higher than had been anticipated on a simple exponential attenuation basis. Cutting off $1\frac{1}{2}$ in. of the back end of a cylinder, while keeping the counter position fixed, increased the background by a larger amount than anticipated, also suggesting that at many mean free paths the neutrons were diffusing forward in appreciable numbers.

In most cases the desired geometry for transmission measurements had the counter too close to the source to permit placing a 12-in. cylinder between them. The background for these cases was extrapolated from the measurements with the counter farther back, approximately a factor of 2 in effective distance from the target. The validity of this extrapolation is based on detailed studies as a function of counter and shadow cylinder location and cylinder diameter and length.[15] Using the two true sources of background plus a small leakage through the shadow cylinder, the background could then be analyzed quite consistently (to 0.04 percent of the unattenuated rate) for various diameters of shadow cylinder and locations of cylinder and detector. The observed slight decrease of background by 0.12 ± 0.08 percent of the unattenuated rate on adding additional tantalum (0.18 in.) next to the target was also compatible with these studies.

The background correction to the cross section was usually about 0.3 percent and introduced an uncertainty estimated at considerably less than 0.1 percent.

3. *Correction for Binergy of Neutron Source*

The discriminator bias was set so that the relative sensitivity of the counter to neutrons of the lower-energy group from the Li(p,n) reaction was about 10 percent. The intensity of this group relative to that of the main group has been measured in several ways, giving always about 10 percent. Freier and Stratton[16] measured the relative intensity as 10 ± 3 percent at a proton energy of 3.49 Mev and we use this value for the proton energy of approximately 3.0 Mev in this experiment. In this experiment the cross section for polyethylene is 12 percent higher for the low-energy group than it is for the high-energy group. The corrections to the cross-section measurements for this effect were thus only from 0.1 to 0.3 percent.

4. *Rate Dependence*

The gas multiplication and the high discriminator setting made the counter extremely sensitive to changes in high voltage, amplification, and bias setting. A given fractional change in counting rate could be caused by one third the fractional change in amplification or bias, or by 1/15 the fractional change in high voltage. Precautions such as long warmup, heating the filaments of the amplifier tubes with a battery, avoiding air currents, and careful selection of components made it possible to reproduce transmission measurements to within statistical errors of less than 1 percent, each measurement taking 10 or 20 minutes. The counting rate usually stayed constant to the order of 1 percent for several hours.

A major source of error in the earlier stages of the experiment was the change of detector efficiency as a function of counting rate. Since the discriminators on the propane counter were biased very high, a slight change in gain of the electronics made a large fractional difference in counting rate—the price exacted for fast counting combined with good rejection of background and low-energy group. The first results we obtained gave quite high transmissions because of this effect, and while the effect was reduced considerably by working over the electronics, it was never completely understood. After testing with a sinusoidal signal generator, we felt that the possible shift of bias of the discriminators when counting at these rates would be sufficient to cause this effect. Self-biasing of the counter, by insulators charging up, and missing by scalers were also possible contributors. Instead of its expected resolving time of about 4 μsec, measured with a double pulser, the whole detector system had an "effective resolving time" of about 15 μsec, defined by the losses between two very high rates used to study it. In order to use this result in the real experiment at lower counting rates, the assumption must be made that there is a known dependence of lost counts on rate.

The following scheme to eliminate rate dependence of detector efficiency was finally used. The machine was run at low current when the scatterer was out, and at high current when the scatterer was in, in such a way as to keep the propane counter working at the same rate. The long counter monitor thus showed the variation of counting rate, and because of its flat bias curve, should not give such a large rate dependence of efficiency as the detector. But since the long counter might still have given trouble because it was run at a very high rate in order to get good monitor statistics, its efficiency was actually checked between the approximate rates used by bringing up and removing two equivalent radium-beryllium sources in sequence, finding the very slight nonadditivity of their counting rates. (This method was also used in a more complex way between transmissions $\frac{2}{3}$ and 1.) Thus the relative efficiency of the long counter and its whole electronic equipment was found at the two rates used, and the experiment did not depend on any assumption about efficiency of counters or electronics as a function of counting rate.

The result on the one high-accuracy run (0.22 percent statistics) taken by this constant detector count method agrees well within statistics with the other high-accuracy run on the same scatterer (0.33 percent statistics) using the resolving time derived from ratios of detector to monitor counts at two different very high detector rates,

[15] C. L. Storrs, thesis, Massachusetts Institute of Technology, February, 1952 (unpublished).

[16] G. Freier and T. F. Stratton, Phys. Rev. **79**, 721 (1950).

and assuming a conventional quadratic dependence of lost counts on rate.

C. Scatterers

1. *Composition of Scatterers*

Most of the CH_2 scatterers for this experiment were cut from the same stock used by Lampi, Freier, and Williams,[6] and some from the stock used by Hafner *et al.*[8] The carbon scatterers were cut from pile graphite sent from Brookhaven. The chemical analysis of the particular scatterers used revealed no significant differences between them. The hydrogen to carbon ratio was measured as two to one by numbers, to well within the maximum error of 0.1 percent, by Dr. R. A. Paulson of the Bureau of Standards. The scatterers were less than 0.1 percent oxygen by weight and had negligible ash. At this energy the high cross section of oxygen makes the oxygen impurity especially unimportant.

2. *Dimensions of Scatterers*

From the standpoint of statistics alone[17] a transmission of $T\cong 0.2$ would be desirable for determining cross sections with the low background present in this experiment. But at low transmissions the corrections to the cross section for neutrons multiply scattered but still counted becomes large, and possible errors in interpretation of background are magnified. A reasonable compromise between best use of time and background corrections seemed to be to work at $0.4\leq T\leq 0.6$.

Five polyethylene cylinders were used with diameters of $\frac{3}{8}$ in. and $\frac{5}{8}$ in., and transmissions of 0.4 and 0.6, there being two of the smallest cylinders. Most of these were machined on a lathe, but one of the smallest was faced with a grinder.

The dimensions of the cylinders were determined by comparing them with size blocks by means of a dial indicator gauge. It was estimated by inspection with a microscope that the scatterer with the ground ends had irregularities of not more than 0.0003 in. The length of this scatterer was measured with a size block on top of it to distribute the force of the dial indicator gauge, with the dial gauge pressing directly on the plastic, and with an 0.11-in. size block edgewise beneath it. The three results indicated that the plastic was deformed about 0.0005 in. when the force of the dial gauge was concentrated in a small area. The true length of the ground scatterer was taken to be the one observed with size block on top of the plastic and this small correction for deformation was applied to measurements of the other CH_2 scatterers.

3. *Density of Scatterers*

A major source of uncertainty in this whole experiment was caused by the interchangeable use of two previously intercalibrated thermometers, one of which was found to be off by the order of 4°C at the end of the experiment. Despite accurate density and length measurements there is thus an uncertainty in the temperature at which the experiment was carried out, displayed in the errors in the adjustment of length and density in Table I.

4. *Single and Multiple Scattering-in Corrections*

A lower limit for the scattering correction is obtained by computing the relative number of neutrons from the target which were scattered into the detector by each point in the scatterer, taking into consideration attenuation both before and after scattering, and integrating. The scatterer is thus considered to be a secondary source of neutrons, and its total strength per neutron incident on it from the target is given by this integration to be $T\ln(1/T)$, where T is the transmission. This gives quite accurately the effect of singly scattered neutrons.

A high upper limit for the secondary source intensity in the forward direction, including multiple scattering if there is a reasonable angular distribution of scattering, is to assume that the secondary source is comprised by the $1-T^{-N\sigma X}$ neutrons which are scattered per incident neutron, and has the angular distribution of singly scattered neutrons. Since the detector discriminated strongly against neutrons which had lost energy by multiple scattering, the lower limit is a much better first approximation.

The scattering correction depends upon the angular distributions of the scattered neutrons. For hydrogen the scattering is approximately isotropic in the center-of-mass system at this energy. For carbon the ratio of scattering at 0° to that at 90° was measured as $\sigma_C(0°)/\sigma_C(90°)=2.0\pm 0.5$ by increasing the mass of a graphite scatterer by a factor of 10, without changing its length, by slipping a coaxial shell of graphite around it. The monitor was at 90°, and the graphite was in this case so close to the target that the scattering into the monitor just cancelled the scattering into the detector. Actually, the monitor was at 40° when the graphite cross sections were taken, so a smooth angular distribution[18] had to be assumed to make the scattering-in correction for carbon for those runs in which the monitor was at 90°. (As pointed out in reference 8, if the monitor were at the same angle throughout, the angular distribution from carbon would not enter into single scattering at all.) The assumptions were $\sigma_C(0°)/\sigma_C(45°)\cong 1.5$, and $\sigma_C(90°)\cong(1/4\pi)\sigma_{C\ \mathrm{total}}$. A factor of two in the assumed anisotropy would make a difference of only 0.0001 in the CH_2 transmission.

The very small effect on cross section of the scattering by the air displaced by the polyethylene was compensated to less than 0.02 percent by the longer average path length in the scatterer for neutrons going through

[17] M. E. Rose and M. M. Shapiro, Phys. Rev. 74, 1853 (1948).

[18] Feld, Feshbach, Goldberger, Goldstein, and Weisskopf, Atomic Energy Commission Report AEC NYO-636, 1951 (unpublished).

the scatterer at a slight angle to the axis. The energy of the neutrons at these small angles is not significantly lower than in the exact forward direction.

An estimate was made of the secondary source strength due to doubly scattered neutrons, taking into account the fact that the detector would not count neutrons which were scattered from hydrogen by more than 30° and then scattered back in by hydrogen. The inclusion of these doubly scattered neutrons raises the hydrogen cross section by not more than 0.07 percent. More double scattering than computed could raise the hydrogen cross section further by only about 0.2 percent before reaching the upper limit of multiple scattering discussed above. Since in the final measurements the scattering correction was about 0.8 percent, the uncertainty in the result from this source was thus shown experimentally to be not more than about 0.3 percent. It is our belief that the actual uncertainty in cross section due to scattering-in correction, using the correction estimated theoretically, is not more than 0.1 percent.

Some of the formulas used in this experiment are tabulated in the Appendix.

III. RESULTS

The best determinations of polyethylene and graphite transmissions are shown with corrections in Table I. In early runs a series of measurements was made with scatterers of various lengths and diameters with scattering corrections from 0.3 percent to 3 percent of the cross sections. Within the standard deviation of about 0.5 percent, these measurements all give the same cross section around 9.52×10^{-24} cm^2, when corrected for rate effects as well as possible in retrospect. Only one of the earlier runs is included, because only it had a very small rate correction, and because it was taken in a geometry for which background was observed directly and for which scattering into the monitor (then at 40°) was known to be small. We do not feel that we understand the other runs well enough to use their data, the average of which lies within the final error we will quote for the best runs.

Some data taken on scatterer a around the 12/14/51 run showed rather large fluctuations in the direction of higher transmission, and it is possible that the transmissions tabulated for a and e on that data are high because of incomplete shadowing of the target. The accuracy of the previous measurements cannot rule out a possible undiscovered systematically higher transmission for shorter scatterers, but in the absence of any positive evidence we average these runs in as if their higher transmissions arose from statistics alone.

The corrections are as discussed in Sec. II. Using the statistical standard deviation as the sole source of error, the resultant n-p cross section at this energy is found to be $(3.675\pm0.008)\times10^{-24}$ cm^2. This error will be raised by uncertainties in the corrections, and, to a much lesser extent, by the uncertainty in the energy by assigning the cross section to the definite energy $T_n=1.315$ Mev. A reasonable final estimate is $\sigma_{np}=(3.675\pm0.16)\times10^{-24}$ cm^2.

IV. DISCUSSION

A. Relativistic Corrections

We may put the above value of the total cross section, together with the binding energy of the deuteron and the thermal scattering amplitudes, into a consistent shape-independent approximation by using, for the k of the reduced mass $\mu=m_pm_n/(m_p+m_n)$, the momentum p of either particle in the center-of-momentum system. Then $\hbar^2k^2/2\mu$ is given by $p^2/2\mu=\frac{1}{2}T_n\{1-[(m_n-m_p)/2m_n]\}$, where T_n is the neutron bombarding energy $E_n-m_nc^2$. If m_n and m_p are taken as equal, this expression is the same as that used by Snow[5] to the approximation used, i.e., neglect of β^4 compared with unity. We hope, by using the value of the relativistic momentum in the center-of-momentum system for $\hbar k$ wherever it appears in the shape-independent approximation, to separate out kinematic relativistic corrections from those effects possibly arising from a velocity-dependent force field.

B. Comparison with Proton-Proton Effective Range

The best value of the p-p singlet S-wave effective range comes at present from a shape-independent fit to data[19] in the 2- to 4-Mev region. This effective range is an average taken at a somewhat higher energy than the combination of present data and other n-p parameters represents, and so shape-dependence may affect the local slope of $k\cot\delta$ in a given energy region. There is the possibility of a change compatible with the data of not more than 0.1×10^{-13} cm in the p-p effective range, depending on what shape-dependent parameter is chosen.

In addition to this uncertainty in interpreting the p-p data, there are two small differences to be expected between the n-p and p-p singlet S-wave systems. The first is that if the observed π-mesons give rise to nuclear forces in even approximately the manner which led to Yukawa's original suggestion, the p-p intrinsic range may possibly be longer than the n-p intrinsic range. If neutral mesons are exchanged in the p-p case, and charged mesons, among others, in the n-p case, the lighter mass of the neutral meson might give a longer p-p range. For example, in a symmetric scalar theory the fractional difference would be $\frac{2}{3}(m^{\pm}-m_0)/m^{\pm}$, so the n-p range would be 2.59×10^{-13} cm for a p-p range of 2.65×10^{-13} cm.

A second difference comes from the influence of the electromagnetic interactions on the effective range. For example,[20] turning off the Coulomb interaction but keeping the same nuclear well parameters changes the

[19] Worthington, Findley, and McGruer, Phys. Rev. **87**, 223 (1952).

[20] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 108 (1950).

effective range by only 0.2 percent for a Yukawa well, but by 5.6 percent, i.e., from 2.64 for p-p to 2.79×10^{-13} for n-p, for a square well. The effect of a repulsive core should be to minimize this difference.

V. CONCLUSION

Using the value $\sigma=(3.675\pm0.016)\times10^{-24}$ cm² at $T_n=1.315$ Mev, we get $r_s=(2.36\pm0.20)\times10^{-13}$ cm in the shape-independent approximation. It should be pointed out that the quoted errors in the measurements of the other quantities that enter into the present evaluation of the singlet range, particularly the coherent thermal scattering amplitude, give a minimum uncertainty of almost 0.2×10^{-13} cm in r_s, so that several different experiments are now at about the same limit of accuracy as far as their application to the singlet n-p range. A reasonable figure for our result is therefore $(2.4\pm0.3)\times10^{-13}$ cm. This value[21] touches the p-p singlet range discussed in IV–B and is compatible with charge independence of singlet S-wave effective ranges.

No attempt at a comparison of singlet ranges obtained from other n-p scattering experiments will be made here, except to remark that a simple interpretation of a succession of experiments of comparable accuracy is that they average out to give a singlet n-p range that is of the order of 0.2×10^{-13} cm less than the p-p range.

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APPENDIX

Formulas Used in Experiment

1. Correction to transmission for background in detector in typical geometry:

$$\Delta T=(1-T)[(2.7\pm0.3)+(1.5\pm0.3)]\times10^{-3},$$

where the first term is from reflection from the walls and floors of the room, and the second term is from unshadowed and partially shadowed parts of the detector preamplifier, etc.

2. Correction to transmission for single scattering into detector and monitor:

$$T_{\text{observed}}=T_{\text{true}}\left[1+\frac{4\pi\sigma(0^\circ)}{\sigma_{\text{tot}}}\frac{\omega_{sq}\omega_{ds}}{\omega_{dq}}\ln\frac{1}{T}-\frac{I(0^\circ)}{I(\theta)}\frac{4\pi\sigma(\theta)}{\sigma_{\text{tot}}}\omega_{sq}(1-T)\right].$$

Here $4\pi\sigma(0^\circ)/\sigma_{\text{tot}}$ is the ratio of scattering at zero degrees to the average scattering for all other angles. $I(0)/I(\theta)$ is the ratio of source intensity in the 0° and θ directions. ω_{sq} is the solid angle subtended by scatterer at source, ω_{ds} by detector at scatterer, and ω_{dq} by detector at source.

3. Estimate of upper limit on double scattering: detector assumed to have a linear response as a function of energy, starting from zero at E_c.

$$\frac{N_{\text{doubly scattered}}}{N_{\text{singly scattered}}}\cong\bar{\epsilon}\sin^2\theta_c\left(\frac{1-T}{T\ln(1/T)}-1\right).$$

Here $\bar{\epsilon}$ is average efficiency, taken to be $\frac{1}{2}$, for those doubly scattered neutrons which are counted. θ_c is the scattering angle for which the energy of the doubly scattered neutron drops to E_c. Since $\sin^2\theta_c$ is about 0.3, for $T=0.4$ the double scattering gives not more than 0.07 percent correction to the transmission.

4. Calibration of rate dependence of monitor efficiency ϵ with two natural sources:

(a) Between rates R and $2R$: bring up source A to a distance such that the monitor counts at Rate $[A]\cong R$. Then bring up B so that $[A+B]\cong 2R$. Remove A, giving $[B]\cong R$. Thus, $\epsilon_{2R}/\epsilon_R=[A+B]/([A]+[B])$. Note that the locations of A and B need not be measured or reproduced.

(b) Between rates $2R$ and $3R$: proceed as above for first three steps. Then, without moving B, move A up to a closer position than before, such that $[A'+B]\cong 3R$. Then remove B, giving $[A']\cong 2R$. Thus,

$$\frac{\epsilon_{3R}}{\epsilon_{2R}}=\frac{[A']+[B]\{[A+B]/([A]+[B])\}-[A'+B]}{[A']+[B]\{[A+B]/([A]+[B])\}}.$$

Again no distances need to be measured or locations reproduced.

[21] In progress reports $r_s=2.2\pm0.3$ was computed tentatively from almost the same cross section. This error arose in part from use of $T_n=1.320$ Mev, the neutron energy computed without relativistic corrections to the reaction energy measurement.