Nuclear Deformation and the Moment of Inertia of Nuclear Rotational States

KENNETH W. FORD
*Department of Physics, Indiana University, Bloomington, Indiana**
(Received May 19, 1954)

Evidence concerning nuclear deformation from isotope shifts and from rotational states in even-even nuclei is compared for nuclei near neutron number 82. It is concluded that the moment of inertia of the rotational states is 4 ± 1 times greater than the theory predicts, if the nuclear radius is $1.20\times10^{-13}A^{\frac{1}{3}}$ cm, and if the interpretation of isotope shifts in terms of nuclear deformation is correct.

THE recently published measurements by Arroe[1] of the (very anomalous) isotope shifts in cerium call attention to the significant role of the nuclear deformation in this phenomenon[2,3] and suggest that a more careful evaluation of the magnitude of the deformation be carried out for cerium and neighboring elements. A number of isotope shifts have been measured[3] relative to isotopes with 82 neutrons, among which only cerium includes neutron numbers both greater and less than 82.

A knowledge of nuclear deformation is important both for the detailed analysis of experiments which measure the distribution of charge in the nucleus[4] and for the interpretation of many collective phenomena[5,6] in the nucleus. We are concerned in this note in particular with the quantitative relationship between the nuclear deformation and the moment of inertia of nuclear rotational states—a relationship which has been shown previously to be qualitatively but not quantitatively in accord with theory.[6,7]

ISOTOPE SHIFTS

Nuclear deformations calculated from isotope shift data[1,3] are shown by circles in Fig. 1. In order to obtain these deformations, it was necessary to make several assumptions.

(1) Nuclei containing 82 neutrons have a negligible deformation. This assumption is based on the large energies (1.38 to 1.6 Mev) of the first excited states of ${}_{54}Xe^{136}$, ${}_{56}Ba^{138}$, ${}_{58}Ce^{140}$, and ${}_{60}Nd^{142}$, which are in good agreement with the weak coupling limit of the unified model of the nucleus.[6]

(1a) It is a corollary that the zero point oscillation of the nuclear surface is ignored. This is justified if the mean square amplitude of oscillation is nearly independent of the equilibrium shape of the nucleus.

(2) In the absence of deformation, the isotope shift is 0.40 times the predicted magnitude for incompressible uniform density spherical nuclei of radius $1.40\times10^{-13}A^{\frac{1}{3}}$ cm. This assumption is based on the trend of isotope shifts from $N=50$ to 126, and particularly on the fact that the shift due to deformation should be zero at $N\cong68$ and at $N=126$. A previous fit[2] used 0.50 as the "zero deformation ratio". Near $N=82$, however, the trend of shifts favors 0.4, with a probable error of about 0.05. (The value 0.4 is consistent with a nuclear electric radius of 1.1 to $1.2\times10^{-13}A^{\frac{1}{3}}$ cm and a nuclear compressibility in accord with theoretical estimates.[8,9])

(3) The nuclear electric radius is taken to be $1.20\times10^{-13}A^{\frac{1}{3}}$ cm.[10]

(4) The addition of two neutrons to a nucleus increases the volume occupied by the protons by an amount proportional to $2/A$ (but not necessarily equal to $2/A$). This is the vital assumption, for it implies that all deviations from constancy of the ratio of the observed isotope shifts to those predicted[11] by a model of a constant density spherical nucleus are due to deformation. Shell structure effects, on this assumption, manifest themselves in nuclear deformations but not in nuclear proton density. Deformations were computed from the data of references 1 and 3 with Eq. (21) of reference 2.

OTHER ELECTROMAGNETIC EVIDENCE

Nuclear deformations deduced from quadrupole moments[6,7] are shown by crosses in Fig. 1. Except at $N=90$, they are very much smaller than deformations based on isotope shifts. It seems likely that this anomaly stems from the changing sign of the quadrupole moment and the mixing of states of prolate and oblate shapes; Q is negative from $N=74$ to 77, should be positive just below $N=82$, should be negative just beyond $N=82$, and is positive at $N=88$ and 90. The importance of the quadrupole moment in the present discussion is that it corroborates the isotope shift evidence in suggesting a maximum deformation of about 25 percent just beyond $N=92$. Deformations from this source were calculated from Eq. (30b) of

* Supported in part by a joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
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reference 7, but with the numerical coefficient increased from 67.5 to 100 to account for the smaller electric radius of the nucleus now assumed.

The $E2$ transition rate from first excited to ground state in even-even nuclei[6] measures the nuclear deformation. Two values of β^2 found from this source[12,13] are shown by squares on Fig. 1 at $N=90$ and 94. They confirm the other electromagnetic sources of evidence as to the maximum deformations encountered in nuclei. The deformations were calculated from Eqs. (VII.1) and (VII.17) of reference 6. Total conversion coefficients of 0.95 for $_{62}Sm^{152}$ and 4.5 for $_{66}Dy^{160}$ were used.

MOMENTS OF INERTIA

The existence of rotational states in nuclei, especially in the rare earth region and in the region near uranium, seems to be well established.[6,14] If one accepts the deformations given by isotope shifts as approximately correct, however, the moments of inertia of these states are experimentally much greater than the theoretical expression,[15]

$$I=3B\beta^2,$$

where $B=(15/16\pi)$ times the moment of inertia of a rigid spherical nucleus. To illustrate this point we plot in Fig. 1 (as triangles) the quantity

$$\tfrac{1}{4}\beta_E{}^2=\tfrac{1}{4}(I/3B),$$

found from the energies of first excited states[16,17] of even-even nuclei near $N=82$ (excluding those with energies greater than 1 Mev, for which the rotational assumption should not be valid). These values are seen to be in fair agreement with deformations based on isotope shifts and transition rates. (For all nuclei for which more than one datum exists, the agreement is within the error of the electrically determined deformation).

Between 88 and 90 neutrons one observes a sharp decrease in energy of rotational states, a sharp increase in quadrupole moments, and very large isotope shifts, all corresponding to a marked increase in nuclear deformation. This sharp change, as seen nowhere else in the periodic table, is not understood. It is probably not due to the filling of the first two $i_{13/2}$ states because this should tend to produce negative quadrupole moments, while the observed moments are positive. It may be correlated, however, with the stabilization of the cylindrically symmetric form of deformation. Below 88 neutrons, the quadrupole moments are too small to be consistent with deformations of cylindrical symmetry.

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CONCLUSION

It is concluded that the moments of inertia of nuclear rotational states near neutron number 82 are 4 ± 1 times greater than given by the theory (for uniform density nuclei with $R=1.20\times10^{-13}A^{\frac{1}{3}}$ cm). The rotational interpretation of many nuclear levels has met with striking success[6] in explaining energy level ratios and transition rates. Nevertheless, the large discrepacny between theory and experiment as to the absolute values of the energies remains an important puzzle. All electromagnetic sources of evidence agree about the order of magnitude of the *charge deformation* (predicting a maximum deformation at $N\cong96$ of about 25 percent). The discrepancy exists, therefore, between the charge

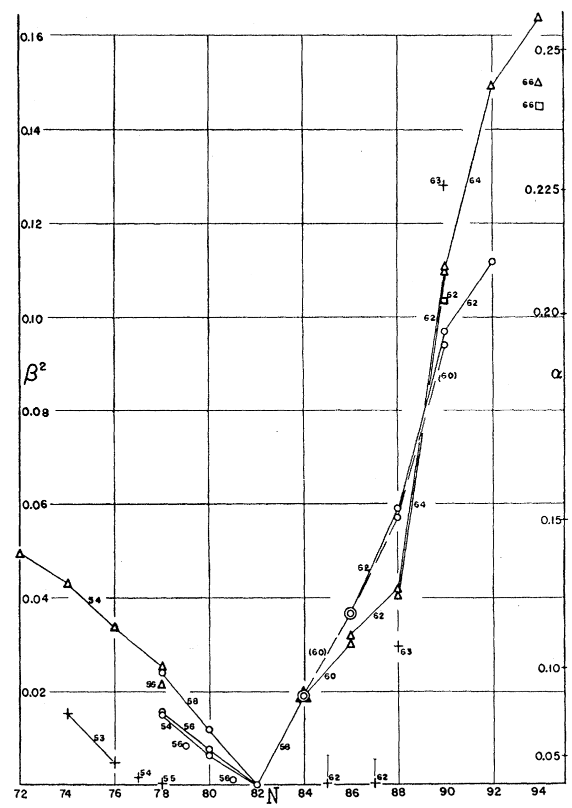

FIG. 1. Nuclear deformations *vs* neutron number. Left scale: square of deformation parameter β. Right scale (nonlinear): fractional deformation $\alpha(=|\Delta R|/R$ along nuclear symmetry axis). $\alpha=0.631\beta$. Z values indicated on graph. Circles: deformations calculated from isotope shifts, all relative to assumed zero deformation of nuclei with 82 neutrons. ($_{60}$Nd points, connected by dashed line, are only known relatively to each other and therefore contain an additional normalizing factor.) Crosses: deformations calculated from quadrupole moments, assuming cylindrically symmetric deformation and strong-coupling projection factor. Squares: deformations calculated from $E2$ transition rates, assuming rotational states. Triangles: *one-fourth* of deformations calculated from energy of first excited states of even-even nuclei, assuming strong coupling rotational states. All points use $R=1.20\times10^{-13}A^{\frac{1}{3}}$ cm. Isotope shifts and rotational energies determine $R^2\beta^2$. Quadrupole moments and transition rates determine $R^4\beta^2$.

deformation and the mass deformation as measured by the rotational energy.

Either of the following possibilities could increase the moment of inertia of the rotational state and thereby help to explain the existing discrepancy.

(1) Neutron structure different from proton structure. This could mean deformation of neutron structure greater than deformation of proton structure, for which there is no known reason, or neutron radius greater than proton radius, for which a qualitative argument has been advanced.[18] Such effects, together with possible effects of nonuniform density distribution, cannot reasonably account for all the factor four.

(2) Breakdown of the idea that the kinetic energy of collective motion is correctly described by an equivalent irrotational fluid flow. The observed moments of inertia lie, as they should, between the limiting moments for rigid-body rotation and for irrotational fluid flow. Independent evidence for a long mean path of nucleons within nuclear matter leads to the expectation that moments of inertia should lie close to the (small) moments of irrotational flow. The large observed moments, therefore, provide evidence against a complete independent particle picture.

It should be stressed that the very close agreement of theoretical and experimental energy ratios provides strong evidence for rotational states, (i.e., energy proportional to square of angular momentum), but does not check the unified (or collective) model in any fundamental way. Only the magnitudes of the energies test the independent particle assumption which underlies these models. The approximate proportionality of the moments of inertia to the square of the nuclear deformation favors the independent particle picture. The large values of the moments show that the picture is not wholly adequate.

Some helpful comments by Professor M. G. Mayer and Professor J. A. Wheeler caused this note to be written. The remarks here are based on earlier work with L. Wilets and D. L. Hill (reference 2).

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PHYSICAL REVIEW VOLUME 95, NUMBER 5 SEPTEMBER 1, 1954

Scattering of 1.32-Mev Neutrons by Protons*

C. L. Storrs† and D. H. Frisch
Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received April 29, 1954)

The total cross sections of polyethylene and graphite for scattering of neutrons of 1.315 ± 0.003 Mev have been measured and are $\sigma(CH_2)=(9.542\pm0.035)\times10^{-24}$ cm^2 and $\sigma(C)=(2.192\pm0.020)\times10^{-24}$ cm^2. From these the hydrogen cross section is $\sigma(H)=(3.675\pm0.020)\times10^{-24}$ cm^2. If the best current values of the binding energy of the deuteron (see reference 3) the coherent scattering length (see reference 4) and the epithermal cross section (see reference 5) are used, the singlet effective range in the shape-independent approximation is $(2.4\pm0.3)\times10^{-13}$ cm. This is to be compared with the proton-proton singlet effective range of $(2.7\pm0.1)\times10^{-13}$ cm.

I. INTRODUCTION

It seems probable that consideration of the singlet and triplet zero orbital angular momentum states suffices for an accurate description[1] of neutron-proton scattering from the bound state up to neutron bombarding energies of the order of 15 Mev. If so, four independent measured quantities may determine[2] the strength and effective range of both the singlet and triplet S-wave interactions, and six independent quantities may begin to differentiate in both cases between interactions of qualitatively different shapes. Three accurate independent quantities known at present are the binding energy of the deuteron[3] and the thermal neutron-proton singlet and triplet scattering amplitudes.[4,5] If applied to a description of the interactions by static potentials, these may be used to determine the strength and range of a triplet potential of a given shape and to determine the approximate product of the magnitude times the square of the range for a singlet potential of a given shape.

Other independent measurements which may be made are the total neutron-proton cross sections for fast neutrons of different energies. The region of neutron

* Submitted to the Department of Physics, Massachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Philosophy for C. L. Storrs. This work has been supported in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

† Now at General Electric Company, ANP Project, Evendale, Ohio.

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