Second-Order Corrections to Beta Spectra

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(Received December 1, 1953; revised manuscript received March 23, 1954)

The origin of small correction terms to beta spectra is discussed. The second-order corrections to allowed spectra are presented for both pure interactions and linear combinations.

RECENT measurements[1,2] of the beta spectra of C^{14} and Ni^{63} which, according to the nuclear shell model,[3,4] should show an allowed shape, have indicated deviations at low energies. An attempt to explain these deviations[2] is not consistent with the parity and spin changes predicted by the shell model.

In the present paper, second-order correction terms to allowed beta spectra will be presented. An attempt to explain the experimental results on the basis of these correction terms will be the subject of a future communication.

In the Fermi theory of beta radioactivity,[5] the relative probability $N(W)dW$ for the emission of a beta particle into the energy range dW at W is written

$$N(W)dW = pWK^2F_0(W,Z)C_{nx}dW, \tag{1}$$

where C_{nx} is the spectral correction factor for the nth forbidden decay, the subscript x specifying the form of interaction (i.e., S, V, T, A, or P).

The quantities C_{nx} as computed by Greuling[6] are incomplete, a fact which was pointed out for the case of allowed spectra by Konopinski and Uhlenbeck.[7] Actually, C_{nx} should be written as the square of an infinite sum

$$C_{nx} = |a_n + a_{n+2} + \cdots|^2, \tag{2}$$

where parity selection rules forbid the presence of terms involving subscripts $n+1$, $n+3$, etc.

For an allowed scalar decay, for example, $|a_0|^2 \sim |\int\beta|^2$, while $|a_2|^2 \sim |\int\beta r^2|^2$. Presumably, $|\int\beta r^2|^2$ is a fourth-order small quantity compared to $|\int\beta|^2$, and is indeed negligible. However, the cross term involving $[(\int\beta)(\int\beta r^2)^* + \text{c.c.}]$ is of second order, and may be observable, particularly if some poorly understood effect, such as l forbiddenness, decreases the size of $\int\beta$ without acting on $\int\beta r^2$.[8]

The quantities $[a_0{}^* a_2 + \text{c.c.}]$ are given below for the allowed spectra.

Scalar:

$$-[(\int\beta r^2)^*(\int\beta) + \text{c.c.}](\tfrac{1}{6}K^2L_0 + \tfrac{1}{3}KN_0).$$

Polar vector:

$$i[(\int 1)(\int \boldsymbol{\alpha}\cdot\mathbf{r})^* - \text{c.c.}](\tfrac{1}{3}KL_0 - N_0) + [(\int 1)(\int r^2)^* + \text{c.c.}](-\tfrac{1}{6}K^2L_0 + \tfrac{1}{3}KN_0).$$

Tensor:

$$\tfrac{1}{2}[(\int\beta\boldsymbol{\sigma})\cdot(\int\beta\boldsymbol{\alpha}\times\mathbf{r})^* + \text{c.c.}](-\tfrac{1}{3}KL_0 + N_0) + [(\int\beta\boldsymbol{\sigma})\cdot(\int\beta\boldsymbol{\sigma}r^2)^* + \text{c.c.}](-\tfrac{1}{3}K^2L_0) + [(\int\beta\boldsymbol{\sigma})\cdot(\int(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})^* + \text{c.c.}][-(1/15)KN_0].$$

Axial vector:

$$i[(\int\boldsymbol{\sigma})\cdot(\int\gamma_5\mathbf{r})^* - \text{c.c.}](\tfrac{1}{3}KL_0 - N_0) + [(\int\boldsymbol{\sigma})\cdot(\int\boldsymbol{\sigma}r^2)^* + \text{c.c.}](-\tfrac{1}{3}K^2L_0) + [(\int\boldsymbol{\sigma})\cdot(\int(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})^* + \text{c.c.}][(1/15)KN_0].$$

Although ordinarily selection rules and the transformation properties of the nuclear matrix elements preclude any linear combinations of interactions in the allowed spectra, there are second-order contributions for combinations of S with T and V with T. These are

$$ST:\ -i[(\int\beta)(\int\beta\boldsymbol{\alpha}\cdot\mathbf{r})^* - \text{c.c.}](N_0 + \tfrac{1}{3}KL_0),$$

$$VT:\ -i[(\int 1)(\int\beta\boldsymbol{\alpha}\cdot\mathbf{r})^* - \text{c.c.}](R_0 + \tfrac{1}{3}KP_0),$$

where R_0 and P_0 are the radial function combinations defined by Pursey.[9]

I should like to acknowledge the valuable contributions of J. Mize, who first called the experimental results to my attention, and of Professor H. Brysk. I wish particularly to express my appreciation to H. Hurwitz, Jr., for many illuminating discussions.

* Operated for the U. S. Atomic Energy Commission by the General Electric Company.

1 J. P. Mize and D. J. Zaffrano, Phys. Rev. **91**, 210 (1953).

2 Kobayashi, Miyamoto, and Mori, J. Phys. Soc. Japan **8**, 684 (1953).

3 Mayer, Moskowski, and Nordheim, Revs. Modern Phys. **23**, 315 (1951).

4 L. W. Nordheim, Revs. Modern Phys. **23**, 322 (1951).

5 E. Fermi, Z. Physik **88**, 161 (1934).

6 E. Greuling, Phys. Rev. **61**, 568 (1942).

7 E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941). We adopt the notation of this paper.

8 The experiments of references 1 and 2 involve decays predicted by the shell model to be l forbidden.

9 D. L. Pursey, Phil. Mag. **42**, 1193 (1951).