Measurement of the Current during the Formative Time Lag of Sparks in Uniform Fields in Air*†

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The current in a parallel plane gap during the long formative time lags of sparks in air has been measured from 10^{-6} amp a few μsec after application of the voltage up to 10^{-2} amp just before breakdown, for time lags between 10 and 100 μsec. Calculations for the increase of $\exp(\int_0^d \alpha dx)$ due to field distortion by positive ions are in good agreement with an observed rapid increase of current shortly before breakdown. The Townsend discharge is shown to be spread over the whole electrode surface in contrast to the filamentary nature of the final spark. The photosensitivity of the cathode surface has been found to decrease following collection of positive ions. The conditioning of the electrodes observed by Fisher and Bederson is believed to be due to such a decrease in γ_p. Some gain has been made toward a theoretical solution for the growth of the Townsend discharge. Working with an integral equation and taking due care with integration limits, has established the range of validity of the existing solution and yielded another approximate solution; from these, exact solutions for the limiting cases of only one γ are obtained. There is over-all agreement between theory and experiment within limits of error except for an observed delay in the initial current rise. To explain this requires the consideration of mechanisms, involving creation and transmission of active photons, which could cause a delay of the order of an electron crossing time.

INTRODUCTION

PRESENT understanding of spark breakdown depends mainly on two very different processes; the Townsend discharge[1] and streamer propagation.[2] For some years it was commonly believed that spark breakdown in uniform fields in air could be explained by applying the two theories separately; Townsend theory at low values of pd (pressure×gap length), and streamer theory at high pd. In an attempt to study the transition region Fisher and Bederson[3] measured time lags with low overvoltages from atmospheric pressure down to a few cm of Hg and found no transition from streamer to Townsend mechanism. Moreover, with their improved voltage stability they were able to work at overvoltages down to 0.02 percent and observed time lags varying continuously from 1 up to 100 μsec long at atmospheric pressure. Thus they observed filamentary sparks characteristic of streamer breakdown with time lags at least a thousand times too long for a streamer crossing time. In view of the work of Varney, White, Loeb, and Posin,[4] and similar contributions by Steenbeck,[5] which had shown that $\exp(\int_0^d \alpha dx)$ could be increased by field distortion due to the accumulation of space charge, Loeb,[6] and Fisher and Bederson,[3] proposed that the discharge starts as a Townsend type mechanism which builds up space charge until the field distortion is such that a streamer can start and then the breakdown is by streamer mechanism. (α is the average number of ionizing acts by one electron traveling 1 cm in the field direction.)

The long time lags observed offered the possibility of measuring the current buildup during the formative time as a means of further studying the mechanisms involved; that was the object of this work.

APPARATUS

The purpose of the apparatus was to suddenly overvolt a parallel plane gap and then measure the current flowing in the gap while the discharge built up to a spark. To accomplish this a positive approach voltage V_a was applied to the gap while the pulser side of the 0.005-μf condenser (see Fig. 1) was held at 1000 volts negative to ground. Thus the voltage across the condenser was V_a+1000. The opening of S_1 triggered the pulser which then raised the negative side of the condenser up to ground potential so that the full voltage across it was then also across the gap; at the same time the sweep was triggered on the synchroscope. S_2 was adjusted to open about 3 μsec after the pulse was applied and thereafter the voltage developed across the signal resistor R was amplified and displayed on the synchroscope, which was photographed.

Due to the capacity of the gap a charging current had to flow to the cathode when the pulse was applied. This was 10^6 or more greater than the discharge currents to be measured just afterward and had to be prevented from blocking the amplifiers. As a switch to short it out, a hard tube has too much resistance when closed and a thyratron cannot be opened fast enough. If the non-overloading amplifier circuit[7] had been available at the

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† This work has been presented in more detail, especially as regards apparatus, in the Ph.D. thesis of H. W. Bandel, Department of Physics, University of California, Berkeley, California, April 1954.

[1] L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939).

[2] L. B. Loeb and J. M. Meek, *The Mechanism of the Electric Spark* (Stanford University Press, Stanford, 1941).

[3] L. H. Fisher and B. Bederson, Phys. Rev. **81**, 109 (1951).

[4] Varney, White, Loeb, and Posin, Phys. Rev. **48**, 818 (1935).

[5] A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen* (Julius Springer, Berlin, 1934), Vol. II, p. 48 ff.

[6] L. B. Loeb, Phys. Rev. **81**, 287 (1951).

[7] R. L. Chase and W. A. Higinbotham, Rev. Sci. Instr. **23**, 34 (1951).

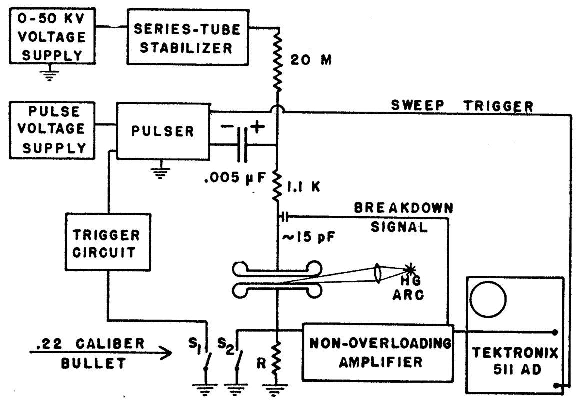

FIG. 1. Block diagram of apparatus.

start of the work a dummy system with a pulse of opposite sign might have been used, but balancing such a system to a part in a million did not seem feasible so the rifle operated switch was developed. It was completely shielded, contact separation was normal, and the motion stopped after ~ 1 μsec. With $R=1000$ ohms, switch noise was measured up to 1 millivolt.

The parallel plane electrodes were of brass polished with rouge and tin oxide. They were 10 cm in diameter with the edges having a radius of curvature of 1 cm. Fine fiducial lines were turned on the outer edge when they were made so that adjustments for parallelism and gap length could be made by measuring the separation of these lines with a cathetometer; accuracy was about ± 0.01 mm. The cathode was mounted on a screw thread so that it could be raised or lowered while in the chamber. The threads were lubricated with graphite. Clearance between the outer edge of the electrodes and the chamber walls was about 9 cm.

The chamber was of brass with a Dural lid. The windows were of quartz and all gaskets of neoprene. The chamber was originally cleaned with nitric acid before assembly. Later, after collapse of an oil-filled lead-in insulator, it was disassembled and washed with detergent; then scoured with whiting, washed with water, and rinsed several times with distilled water before the final assembly. After this treatment a trap cooled with dry ice in alcohol was kept on at all times that the chamber was connected to the vacuum system in order to exclude mercury and stopcock greases.

Whenever the chamber was pumped it soon came down to 2×10^{-4} mm Hg and then stayed about the same for a day or more after which it would drop to 10^{-5} mm Hg. The source of this virtual leak is not known but in this work on air it didn't matter. No difference was noticed for gas fillings made before the final drop to 10^{-5} occurred, or for those left in the chamber many days and used for a number of runs. Room air was drawn in through glass wool, calcium chloride, glass wool, and then a trap made of ~ 4 meters of 7-mm glass tubing cooled by dry ice in alcohol.

The light from a Hanovia quartz mercury arc was focused on the cathode through a side window and a quartz lens. Intensity was adjusted by moving screens into or out of the beam. For determining i_0 the cathode could be connected to a Keithley electrometer and a set of Victoreen resistors to measure currents down to 10^{-12} amp.

The high voltage was furnished by a 0–50 kv supply which was regulated until measurements of the output showed voltage fluctuations and ripple to both be less than $\frac{1}{2}$ volt at 32 kv except for thermal drifts which could not be controlled or measured.

The pulse voltage came from a 0 to -6 kv series tube regulated supply, although for the work reported here only pulses of 1000 volts were used. The pulse came from the cathode of a 677 thyratron; to reduce noise from the thyratron a capacitor was used instead of a resistor in the cathode lead. The pulse rose from -1000 volts and overshot to $+1$ volt in less than a μsec; it then dropped back to $+\frac{1}{2}$ volt at 2 μsec, rose to $+\frac{3}{4}$ volt at 5 μsec, and decayed to its final value of $-\frac{1}{3}$ volt by 30 μsec. Approximate integration of the measured discharge current showed that in the worst case it would have lowered the gap voltage by only a small fraction of a volt during the time of measurement. As far as the development of the discharge was concerned, the deviations of the pulse from a perfect step function could be neglected but they did place a limitation on the measurement of small currents, because any dv/dt of the applied pulse resulted in a charging current flowing to the gap and thus a signal to the amplifier.

The non-overloading amplifier used the basic circuit of Chase and Higinbotham.[7] Its maximum output did not cause blocking of the amplifiers of the Tektronix 511 AD when using 1 stage of gain so the system as a whole was non-overloading except for the first grid, and S_2 protected that until after the pulse. The amplifier input was protected from damage when breakdown occurred by first a spark gap and then an illuminated Ne bulb preceded by 2000 ohms. With 4 stages of gain the sensitivity could be varied by factors of 10 from 2.5×10^{-4} to 2.5×10^{-1} volt/cm. In addition, the signal resistor could be varied; 1000 ohms was the largest commonly used. With this, the maximum current sensitivity was 1 μa for full scale deflection of 4 cm with a noise level usually of about 2 or 3 mm and a low-frequency ripple of about twice that. Although the available sensitivity would have allowed measurement of currents from about 10^{-7} up to 1 amp the limits of measurement at both ends of the time lag were set in general by the rate of rise of the synchroscope trace being too fast to register photographically. In the early part of the time lag there was also a limit set by the dv/dt of the applied pulse.

The time of breakdown was recorded by taking a signal capacitatively from the high-voltage lead-in insulator and coupling it to the amplifier output through a germanium diode in such a way that normal operation of the amplifier was not affected by quite large fluctuations of the anode potential, but when breakdown occurred the sudden drop by thousands of volts drove the amplifier output to full negative signal. This gave an indication of the time of spark that was independent of the gain of the amplifier or the value of R.

EXPERIMENTAL PROCEDURE AND RESULTS

The voltage supplies and arc were turned on and allowed to warm up for at least an hour, and the light intensity was adjusted to give the desired value of i_0. Then without firing the rifle, the pulser circuit was triggered by tripping S_1 by hand and the time lags observed visually on the synchroscope so the approach voltage could be adjusted to give the desired time lag. Then with everything adjusted the camera shutter was opened and the rifle fired. A number of such photographs were taken for different time lags at each sensitivity setting. The negatives were then projected on a screen; deflections were measured, converted to amperes, and plotted on a semilogarithmic plot. Each trace was labeled with the time lag measured from the film. When enough data was collected to have such sections of curves close together all over the plot a continuous curve for any particular time lag could be drawn in. Further, because of the statistical variations in the discharges, such a mean curve is more representative than any single one of the measured data.

In this work as in that of Fisher and Bederson,[3] i_0 was determined by measuring the multiplied current at a voltage roughly 10 percent below sparking and then using Sanders'[8] values of α in the equation $i=i_0e^{\alpha d}$. This method is not very good because $e^{\alpha d}$ varies so much for small uncertainties in α, but it seems to be the only method at present available. When the data of Harrison and Geballe[9] became available, a comparison was made between using their data for α and η (coefficient for attachment) and that of Sanders. In general, neither could be said to fit better than the other so the use of Sanders' data was continued.

Fisher and Bederson[3,10] found that each time after starting, the sparking potential kept rising until the electrodes had been conditioned by the passage of a large number of sparks, after which it became definite at any time but still changed slowly with time. Essentially the same thing was observed in this work, and it was found that the passage of sparks also decreased i_0; this was found to be a change in the photosensitivity of the cathode, probably a change in the gas film on the surface following collection of positive ions. Starting with a chamber that had been evacuated for a day or more and then filled, the photosensitivity was always high. Then even the small current passed by the approach voltage caused a gradual, measurable decrease, and sparking often caused a decrease by orders of magnitude. After the photosensitivity had been much depressed by sparking it recovered slowly by just standing idle. It seems likely that γ_p may decrease along with the photosensitivity to light from the Hg arc, and this would account for the observed increase in sparking potential as the electrodes are conditioned. (γ_p is the average number of electrons released from the cathode by photons generated in the gas per ionizing event in the gas.)

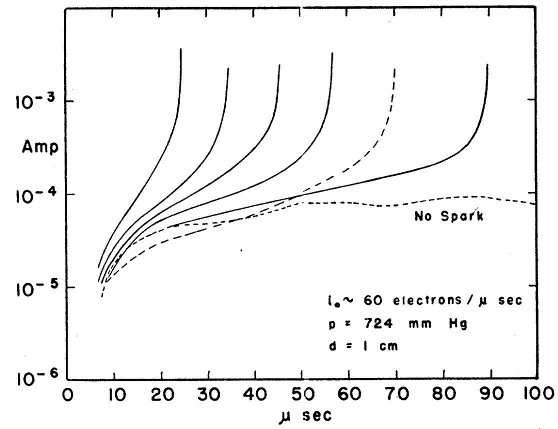

FIG. 2. Current *vs* time with large i_0. The 70-μsec curve shows maximum scatter. Curves terminate within one μsec of breakdown.

By the nature of Fisher and Bederson's apparatus,[3] it seems likely that they would have kept sparking it frequently while taking data so that some saturation of the effect might have been maintained. In this experiment, that could have been done while conditioning the electrodes, or observing time lags visually to get the voltage adjusted, but there was an unavoidable delay each time the camera, rifle switches, and voltages were all reset and the data recorded. After this idle time the first time lag was always shorter than previous ones at the same voltage, and the data taken showed a very large scatter. All of the data finally used were taken by sparking once every 5 minutes. This regular sparking was usually carried out for an hour or more until i_0 seemed comparatively constant before any data were taken, and then i_0 was checked, and if necessary the light intensity readjusted, occasionally during a run. With this, measured values of i_0 usually varied by less than ± 10 percent. There was still nearly always a drift of the sparking potential up or down, sometimes over a range of nearly 200 volts out of about 29 kv in the course of a day's run.

Figure 2 shows some plots of data taken with $i_0 \approx 60$ electrons per μsec, $d=1$ cm, and $p=724$ mm Hg. (All pressures are reduced to 22°C.) In general the current builds up rapidly in the early part of the formative time, then increases roughly exponentially in the middle of the time lag, and finally increases much faster than exponentially shortly before breakdown. The upper ends of the curves terminate within one μsec of breakdown.

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[8] F. H. Sanders, Phys. Rev. **41**, 667 (1932); **44**, 1020 (1933).
[9] Melvin A. Harrison and Ronald Geballe, Phys. Rev. **91**, 1 (1953).
[10] L. H. Fisher, Elec. Eng. **69**, 613 (1950).

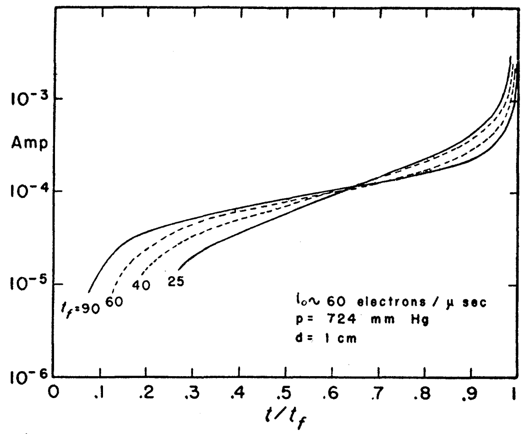

FIG. 3. Some of the same data as in Fig. 2 with current plotted *vs* fraction of the formative time lag.

A rough qualitative picture of the general shape of these curves is as follows. The current measured by the external circuit is due to the motion of all of the ions and electrons in the gap at any particular time. Because the electron velocity is so much higher, electrons make a much larger contribution to the current for the short time they are in the gap, but the positive ions remain in the gap for a correspondingly longer time. In a steady state the contribution of the positive ions and that of the electrons are in the ratio of the average distances they each travel in the field, which is approximately $(\alpha d-1)/1$. For the values of αd at onset this would be a contribution by the positive ions of about ten times as great as that of the electrons. When any change occurs in the rate of ion (and electron) production, the electron current follows the change quickly while the ion current requires an ion crossing time to adjust to the new value. Thus, whenever the measured current is increasing, a larger fraction of it is due to electrons than would be the case for steady state; and the faster it is increasing, the larger is the fraction of it due to electrons. This makes direct interpretation of the measured current difficult.

In the absence of any secondary mechanisms the measured current would increase rapidly at first as the constant rate of production increased the number of positive ions in the gap. It would level off gradually and become constant at one ion crossing time when the rate of loss became equal to the rate of creation. With secondary mechanisms acting, this initial buildup is of course greater, and the current cannot become constant by one ion crossing time. The more or less exponential increase in the middle of the time lag is due to secondary mechanisms, and the final up-curving which leads to breakdown is due to space-charge distortion of the field, as will be shown later.

Figure 3 shows some of the same data as in Fig. 2 plotted against fraction of the formative time, t/t_f. Two of the curves are dashed merely to aid the eye in following them through. This method of plotting spreads out the beginnings of the curves which were nearly the same for all time lags but it reveals the similarity in shape of all the curves.

There was always some statistical scatter in the development of the discharges but with this relatively heavy i_0 it was not very great. In general a section of curve obtained with one sensitivity would join quite smoothly onto another section for the same time lag although they had been taken for different sparks. The lower end of the dashed curve in Fig. 2 is actual data for a time lag of 70.5 μsec and is included because it was the widest variation observed in the whole run.

This variation is to be compared with Fig. 4 which presents some data for the same pressure and gap length but with i_0 an order of magnitude lower. These were all for time lags within a few percent of 40 μsec and when plotted against t/t_f should have all fallen practically on the same curve were it not for variations in the development of the discharges. The scatter here is much greater than with the heavier i_0 and the different traces do not plot smoothly together. However, a sort of mean curve drawn through the center of these (dashed line in Fig. 4) passes 10^{-6} amp at about the same time that the 40 μsec curve crosses 10^{-5} amp in Fig. 3. In general, changes in i_0 merely raised or lowered the early part of the curves, because in this work the independent variable was time lag instead of overvoltage. Voltage was always recorded but, with the sparking limited to one every 5 minutes, V_s (sparking threshold voltage) could not be checked conveniently or frequently so the overvoltage was not usually known. There can be no question that if V_s were constant, changes in i_0 would move the entire curves up or down at least up to the point where space charge distortion starts the final up-curving, and the time lag for a given overvoltage would be changed. Fisher and Bederson did not observe this because their variations in i_0 were small and the resultant changes in time lag *vs* overvoltage were less than the experimental accuracy.

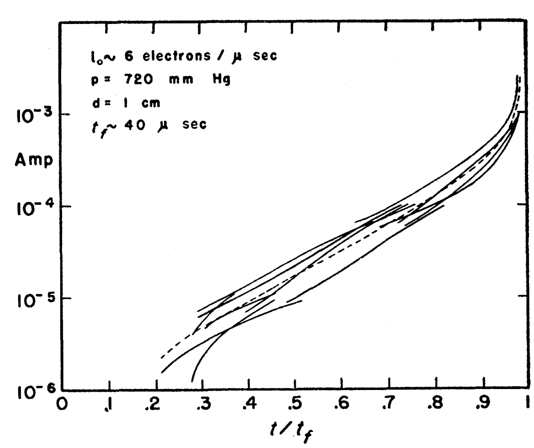

FIG. 4. Current *vs* fraction of the time lag with small i_0 for lags of $\sim$40 μsec, showing scatter.

Figure 5 includes a set of experimental curves (dashed lines) for $d=1$ cm, $p=722$ mm Hg, and $i_0 \approx 12$ electrons per μsec. The scatter in this data was appreciably less than with $i_0 \approx 6$ electrons/μsec but was still considerable.

In Figs. 2 and 5, examples are included of actual data for cases in which sparks just failed to develop. These are fairly typical of many such events observed either by chance or by intentionally lowering the voltage to remain a few volts below threshold. The early build-up, which is due largely to filling the gap with positive ions, is nearly the same as for the longer time lags but then the curves level off, although there was always more or less fluctuation about the final values.

A considerable amount of data was taken at different gap lengths and pressures, but the results were inconclusive because the variations observed were all such that they could be ascribed to errors in determining i_0.

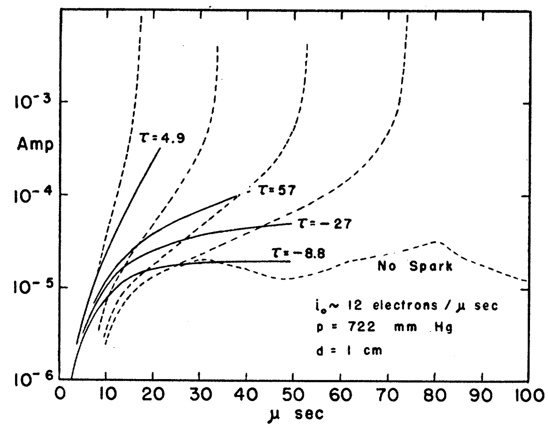

FIG. 5. Comparison of theory and experiment (dashed lines experimental).

THEORY AND APPLICATION

Since the work of Fisher and Bederson, there have been three published attempts at theoretical explanation of the dependence of time lag on overvoltage which they observed. The first was that of Kachikas and Fisher[11] in the extension of the work to N_2, for which the experimental results were "almost identical" with those in air. This was a simplified and approximate theory which considered only a photon γ, and assumed all the photons to come approximately from the anode. It was not intended to apply for times greater than an ion crossing time. Although they recognized the role of space charge distortion and the streamer mechanism in the final breakdown they had no way of taking it into account so used a critical current at the cathode as the criterion for breakdown.[12] This simple theory was remarkably successful for the results in N_2 but less so in A and O_2.[13]

Dutton, Haydon, and Jones[14] showed no recognition of the streamer as the final breakdown mechanism or of the consequent necessity of expressing a true criterion in terms of the necessary field distortion by positive ions. They used an approximate solution for a Townsend discharge due to Bartholomeyczyk[15] and Davidson,[14] and took a critical current at the cathode as the criterion for breakdown. (This solution will be discussed later.) By varying the overvoltage, that is α, they obtained curves of roughly the same general shape as those of Fisher and Bederson.

[11] G. A. Kachikas and L. H. Fisher, Phys. Rev. **88**, 878 (1952).

[12] As published, the criterion is expressed in terms of number of electrons per electron transit time. This theory was an approximation to single electron triggering of the original theory presented in Kachikas' thesis (see reference 13) which expressed the criterion in terms of current explicitly.

[13] G. A. Kachikas, thesis, Department of Physics, College of Engineering, New York University (unpublished).

[14] Dutton, Haydon, and Llewellyn-Jones; Mathematical Appendix by P. M. Davidson, Brit. J. Appl. Phys. **4**, 170 (1953).

[15] W. Bartholomeyczyk, Z. Physik **116**, 235 (1940).

Raether[16] has been the only one to attempt a criterion based on space charge distortion of the field. His picture of the physical situation is good (the spreading of the discharge receives verification later in this work), but he was forced to make rather crude approximations to carry the work through.

If we consider the case of a uniform field, taking $x=0$ at the cathode and $x=d$ at the anode the balance equations are:

$$\frac{1}{v_-}\frac{\partial i_-}{\partial t} = -\frac{\partial i_-}{\partial x} + \alpha i_-, \qquad \frac{1}{v_+}\frac{\partial i_+}{\partial t} = \frac{\partial i_+}{\partial x} + \alpha i_-, \tag{1}$$

where $i_-(x,t)$ and $i_+(x,t)$ are the electron and positive ion currents and v_- and v_+ the electron and ion velocities, assumed constant for the range of overvoltages involved. Considering only α, γ_p, and γ_i mechanisms the boundary conditions are:

$$i_-(0,t) = i_0 + \gamma_i \alpha i_+(0,t) + \gamma_p \alpha \int_0^d i_-(x,t)dx, \qquad i_+(d,t) = 0. \tag{2}$$

(γ_i is the average number of electrons released from the cathode per impinging positive ion). The γ_p term in (2) is sometimes written

$$\frac{\delta}{\omega}\int_0^d e^{-\mu x} i_-(x,t)dx$$

to include an absorption coefficient explicitly but this is likely to be more misleading than helpful. The photons do not go directly to the cathode in the minus x direction but go off in all directions so that some may reach the cathode after traveling a distance many times x. Thus μ would itself have to be a function of x. Further, very

[16] Heinz Raether, Z. angew. Phys. **5**, No. 6, 211 (1953).

little is known about the wavelengths involved; quite probably there would have to be a number of different $\mu(x)$. In view of this it seems just as satisfactory at the present time to use a γ_p outside the integral sign.

Bartholomeyczyk[15] solved these equations omitting i_0 from the boundary conditions. Davidson[14] modified Bartholomeyczyk's solution to include i_0. He also pointed out that it does not satisfy the initial conditions and that it would be necessary to obtain an accurate solution in order to know how good or bad that one was. He then proceeded to get an exact solution; unfortunately this last is too complex to be of any practical help.

It has proven worth while to attack this problem from a different direction. If it is possible to get a solution for $i_-(0,t)$, the electron current at the cathode, then

$$i_-(x,t)=i_-(0,\,t-x/v_-)e^{\alpha x}. \tag{3}$$

The positive ion current created in dx' at x' and t is

$$di_+=\alpha i_-(x',t)dx'=\alpha i_-(0,\,t-x'/v_-)e^{\alpha x'}dx'.$$

This reaches x at a time $(x'-x)/v_+$ later so the current at x and t coming from dx' is

$$di_+(x,t)=\alpha i_-\left(0,\,t-\frac{x'-x}{v_+}-\frac{x'}{v_-}\right)e^{\alpha x'}=\alpha i_-\left(0,\,t+\frac{x}{v_+}-\frac{x'}{v}\right)e^{\alpha x'},$$

where $1/v=1/v_+ + 1/v_-$ or $v=v_+v_-/(v_+ + v_-)$. Then

$$i_+(x,t)=\int_x^{v(t+x/v_+)\text{ or }d}\alpha i_-\left(0,\,t+\frac{x}{v_+}-\frac{x'}{v}\right)e^{\alpha x'}dx'. \tag{4}$$

The upper limit on this integral assumes that no electrons leave the cathode until $t=0$ (time of application of the voltage) and then takes into account the fact that x' makes no contribution to the positive ion current at x until there has been time for an electron to go up to x' and a positive ion to go back to x. So the upper limit is $v(t-x/v_+)$ until this quantity equals d and then remains d thereafter. Similarly there can be no contribution by γ_p from any x until electrons have had time to reach x from the cathode. With this, (3) and (4) may be used in (2) to obtain the integral equation

$$i_-(0,t)=i_0+\gamma_i\alpha\int_0^{vt\text{ or }d}i_-\left(0,\,t-\frac{x'}{v}\right)e^{\alpha x'}dx' +\gamma_p\alpha\int_0^{v_-t\text{ or }d}i_-\left(0,\,t-\frac{x}{v_-}\right)e^{\alpha x}dx. \tag{5}$$

Now, trying

$$i_-(0,t)=i_0(A-Be^{t/\tau}) \tag{6}$$

in (5) gives

$$\begin{aligned}i_-(0,t)=i_0+\gamma_i\alpha i_0\Bigg[&\frac{A}{\alpha}\{\exp[\alpha(vt\text{ or }d)]-1\} -\frac{Be^{t/\tau}}{\alpha-1/v\tau}\left\{\exp\left[\left(\alpha-\frac{1}{v\tau}\right)(vt\text{ or }d)\right]-1\right\}\Bigg]\\ +\gamma_p\alpha i_0\Bigg[&\frac{A}{\alpha}\{\exp[\alpha(v_-t\text{ or }d)]-1\} -\frac{Be^{t/\tau}}{\alpha-1/v_-\tau}\left\{\exp\left[\left(\alpha-\frac{1}{v_-\tau}\right)(v_-t\text{ or }d)\right]-1\right\}\Bigg].\end{aligned} \tag{7}$$

For times shorter than $vt=d$ this equation cannot be satisfied for any constant τ or A, and for longer times it is just the equation obtained by Davidson. However, we now know that the solution is exact for longer times and can see when it begins to fail. Unfortunately, since $v\simeq v_+$, it does not become good until one ion crossing time, so the whole solution is not of much help. On the other hand, in case $\gamma_i=0$ the remaining equation can be satisfied for times greater than one-electron crossing time. In this limiting case, A and τ are given by

$$A=\frac{1}{1-\gamma_p(e^{\alpha d}-1)}, \tag{8}$$

and

$$\gamma_pe^{\alpha d}e^{-d/v_-\tau}=1-1/\alpha v_-\tau+\gamma_p. \tag{9}$$

I am indebted to Dr. Wulf Kunkel for another solution of the integral Eq. (5). If $(di_-/dt)\cdot(d/v_-)\ll i_-$, then $i_-(0,t)$ can be used in the γ_p term instead of $i_-(0,\,t-x/v_-)$, and in this case he was able, by means of a Laplace transform, to obtain the solution, for $d/v_-\leq t\leq d/v$,

$$i_-(0,t)=\frac{i_0}{1+\gamma_i-\gamma_p(e^{\alpha d}-1)}\left[1+\frac{\gamma_i}{1-\gamma_p(e^{\alpha d}-1)}\times\exp\left(\frac{1+\gamma_i-\gamma_p(e^{\alpha d}-1)}{1-\gamma_p(e^{\alpha d}-1)}\alpha vt\right)\right]; \tag{10}$$

and for $t\geq d/v$,

$$i_-(0,t)=\frac{i_0}{1-(\gamma_i+\gamma_p)(e^{\alpha d}-1)}-i_0Be^{t/\tau}, \tag{11}$$

where τ is the root of

$$\gamma_ie^{\alpha d}e^{-d/v\tau}=\frac{-1}{\alpha v\tau}[1-\gamma_p(e^{\alpha d}-1)]+1+\gamma_i-\gamma_p(e^{\alpha d}-1).$$

The condition $(di_-/dt)\cdot(d/v_-)\ll i_-$ says that the current change during one electron transit time is much less than the current itself. If we consider the case of $i_0=1$ electron per electron transit time (i.e., $\sim$10 electrons per μsec) this is approximately the same as saying $\gamma_p(e^{\alpha d}-1)\ll 1$. This does not hold at the sparking potential, so again the whole solution is not good for the present case, but this solution is exact for the limiting case of $\gamma_p=0$. In this case the constant B, evaluated by equating (10) and (11) at $t=d/v$, can be written in a simple form by neglecting γ_i compared to 1 and 1 compared to $e^{\alpha d}$:

$$B \simeq \frac{\gamma_i e^{\alpha d}(1-1/\alpha v\tau)}{1-\gamma_i e^{\alpha d}}. \tag{12}$$

Moreover, this can be used to evaluate the constant B in the case of $\gamma_i=0$ above. From (5) it is seen that the integral equation for the two limiting cases is the same with only the change of subscripts on γ and v. Therefore letting $\gamma_p=0$ in (10) and then changing γ_i to γ_p and v to v_- gives the solution for $\gamma_i=0$ and $t\leq d/v_-$. It follows that with this change of subscripts (12) gives the constant B in the case of $\gamma_i=0$. Further, this solution for $t\leq d/v_-$ is good even if γ_i is not zero because the ions cannot contribute in so short a time.

At this point, solutions have been obtained for the case when both γ_i and γ_p are important for $t\leq d/v_-$ and for $t\geq d/v$, though for the latter B has not been evaluated, and for the limiting cases $\gamma_i=0$ or $\gamma_p=0$ for all times.

For $d/v_-<t<d/v$ an approximate solution for both γ's might be obtained by taking the one for $\gamma_i=0$ in the early times before many ions can reach the cathode and somehow joining it to the one for both γ's which becomes valid at $t=d/v$. If no mathematical expression could be obtained the two could still both be plotted into the intermediate region where they both lose validity and then possibly be joined graphically. However this would have to be studied carefully because there seems to be no *a priori* reason why the first derivative need be continuous at one ion crossing time. With both γ's quite unknown, both velocities a bit indefinite, and even α and the required overvoltage of doubtful applicability, as will be seen, the use of this method to try to fit the experimental data would require more trial and error calculation than seems worth while at present.

The data taken for the cases when sparks did not occur indicates that steady state (if it may be called that in spite of the fluctuations) was achieved in 50 μsec or less. To account for the steady-state value of the current with the known voltage and Sanders' data for α requires that secondary mechanisms increase the current by one or two orders of magnitude. If the major part of this were due to γ_i it would take thousands of microseconds to reach steady state, therefore calculations have been made here only for $\gamma_i=0$.

The above solutions are equally good for $\gamma e^{\alpha d}<$ or >1 because τ changes sign with the denominator.

Having once obtained an expression for $i_-(0,t)$, then (3) and (4) give $i_-(x,t)$ and $i_+(x,t)$. The current measured by the external circuit is due to the motion of all the charges in the gap, and for infinite parallel planes is

$$i(t)=\frac{1}{d}\int_0^d \{i_-(x,t)+i_+(x,t)\}dx. \tag{13}$$

The assumption of infinite electrodes is to assure not only uniformity of field but also that all lines of force from a charge in the gap end on either the electrodes or other charges in the gap, not on chamber walls, but it is a reasonable approximation for this experiment.

For the case of $\gamma_i=0$, when (8) and (12) (with change of subscripts) are used in (6) and this is used with (3) and (4) in (13), neglecting 1 compared to $e^{\alpha d}$ and taking $v=v_+$, the final result for the measured current is, for $t\leq d/v_+$,

$$i(t)=\frac{i_0}{\alpha d(1-\gamma_p e^{\alpha d})}\left[\left\{1-\gamma_p e^{\alpha d}\frac{\varphi}{\psi}(1-\alpha v_+\tau)\right\}e^{\alpha d} +\alpha v_+ e^{\alpha d}t-\left\{1-\gamma_p e^{\alpha d}\frac{\varphi}{\psi}\right\}e^{\alpha v_+ t} -\gamma_p e^{\alpha d}\frac{\varphi}{\psi}e^{\varphi d}\left\{\frac{\alpha(v_+\tau)^2\varphi-1}{v_+\tau\varphi}\right\}e^{t/\tau}\right],$$

and for $t\geq d/v_+$,

$$i(t)=\frac{i_0}{\alpha d(1-\gamma_p e^{\alpha d})}\left[\alpha d e^{\alpha d}-\gamma_p e^{\alpha d}\frac{\varphi}{\psi} \times\left\{e^{\varphi d}\left(\frac{\alpha(v_+\tau)^2\varphi-1}{v_+\tau\varphi}\right)-\alpha v_+\tau e^{\psi d}\right\}e^{t/\tau}\right], \tag{14}$$

where $\varphi=(\alpha-1/v_-\tau)$, $\psi=(\alpha-1/v_+\tau)$, and τ is given by (9). In order to use these expressions to plot curves for comparison with the experimental data values are needed for α, v_-, v_+, and γ_p. The expression,

$$\alpha=p\times 1.048\times 10^{-4}(E/p-27.38)^2, \tag{15}$$

was adjusted to fit Sanders' data at $E/p=34$ and 40, and is quite close at $E/p=50$. E_s is not definitely known because of the way the sparking potential varied, but was nearly 28.8 kv/cm for the data to be compared with it. For breakdown E/p is about 40 volts/cm×mm Hg; if the work of Bradbury and Nielsen[17] for electron velocity in air is extrapolated to this E/p, it gives $v_-\simeq 1.6\times 10^7$ cm/sec. From Varney's[18] work on drift velocities in N_2 and O_2, $v_+=6\times 10^4$ cm/sec was taken as a reasonable value for air at $E/p\simeq 40$.

Using these values, and $p=722$ mm Hg and $i_0=12$ electrons/μsec, some curves were calculated for comparison with the data of Fig. 5 as follows. From E_s and p, (15) gives α. If $\gamma_p e^{\alpha d}<1$, then for long times the solution (14) goes over into the familiar steady-state expression,

[17] See reference 2, p. 191.
[18] R. N. Varney, Phys. Rev. **89**, 708 (1953).

$i=i_0e^{\alpha d}/(1-\gamma_p e^{\alpha d})$. Taking the steady-state value of the current for the "no spark" curve in Fig. 5 as 2×10^{-5} amp, this equation gives γ_p. Then τ can be obtained from (9). γ_p is then assumed constant for the small range of overvoltages studied, and α and τ are calculated for successively higher voltages. The curves calculated using these values were too widely separated vertically at the first shoulder, for a given change of final slope. A better fit was obtained by taking $E_s=28.5$ kv/cm and $v_+=8\times10^4$ cm/sec. (These gave $\gamma_p=1.5\times10^{-5}$.) This change is greater than the experimental uncertainty in E_s but α is really the quantity being varied; and it is more uncertain. The curves drawn in heavy lines in Fig. 5 were calculated using these values.

The calculated curves have a similar shape and change with overvoltage in a similar manner but there are two main differences which cannot be reconciled by adjustment of the parameters. The first is a difference in time for the initial current rise; the fit of the early part of the curves would be quite good if the experimental curves could be translated 5 μsec to the left. The second is that the final slope of the calculated curves increases much faster with overvoltage than that of the experimental curves if Fisher and Bederson's data for overvoltage is applied to the time lags observed here using Sanders' data for α. The curve labeled $\tau=4.9$ μsec was calculated for an overvoltage of only 0.03 percent [percent overvoltage $=100(V-V_s)/V_s$]. From the data of Fisher and Bederson this overvoltage would give a time lag of 50 μsec. However, it is somewhat doubtful that their data applies because they probably had a lower γ_p due to "conditioning" of the electrodes by repeated sparking, as discussed above. In fact, it was noted in the course of this work that the overvoltages necessary seemed less than theirs, but with sparks 5 minutes apart and V_s as capricious as it was, it was nearly impossible to get data on this. Further, although it was argued above that positive ions cannot contribute the major portion of the secondary electrons because of the apparent leveling off of the "no spark" curves, these curves fluctuate far too much to be able to rule out the positive ion mechanism entirely. If γ_i does contribute appreciably in the longer formative times, then the overvoltage must be correspondingly larger for the short time lags since then the ions cannot contribute significantly. All in all, this point is not too serious because both theory and experiment are still too inexact and incomplete.

The difference in time at the beginning is not so easily disposed of, however. If the parameters of the calculated curves are adjusted to make the shoulder of the curve come at a late enough time the initial slope is less than the experimental; it is as though the experimental current rise were somehow delayed. The operation of the apparatus was rechecked as to start of the sweep relative to the pulse, linearity of the sweep, and rise time of the amplifiers; 1 μsec seems an outside limit for experimental error.

Anything which would delay the development of an avalanche, or of following secondary avalanches, by a time comparable to an electron transit time could slow up the whole buildup process about as observed. The delay may be due to one, or a combination, of several effects. The first is photoionization in the gas. Cravath[19] has measured absorption coefficients of 2 and 10 cm^{-1} for radiations from a discharge in air. He found the second radiation was "more effective in ionizing air than in releasing photoelectrons from brass." Dechene[20] has measured absorption coefficients from about 2 to 6 cm^{-1} for ionizing radiation in air. Now Sanders' measurements were for steady-state currents and such photoionization in the gas merely looked like a larger α, but when this α is then used to calculate transients the rise will be too fast. That is, by using Sanders' α, avalanches crossing the gap in one-electron transit time are calculated to yield a certain amount of ionization, whereas actually the original avalanches are smaller but are followed by other avalanches starting at various places in the gap and yielding their ionization at later times. Due to absorption, more of the secondary avalanches would start near the anode, but those which started near the cathode would have much greater amplification in the gas and these would be about an electron transit time late. The only estimate of yield for such photoionization in air seems to be that of Cravath for the 10-cm^{-1} radiation that there was about one such photon for every 10^4 ions created by collision. Therefore it is impossible to estimate how much average delay might be thus introduced, but it doesn't seem likely that all of the observed delay could be due to this effect.

There could also be a delay of the γ_p action by entrapment of resonance radiation, by creation of atomic excited states by metastables, by excitation of atomic states by secondary impacts, or by chemical interaction such as between N and O to yield excited states of NO, the radiation from which is not absorbed and can liberate photoelectrons but arrives at the cathode after some delay. From Holstein's[21] theory for imprisonment of resonance radiation, one would expect times for this of the order of 10^{-5} to 10^{-4} sec, which is much too long for the observed discrepancy. However, molecular radiations are much less liable to entrapment and if such occurs at all it would be for much shorter times. In O_2, photons above 6-ev energy are heavily absorbed to give dissociation to O. E. Huber (work to be published), using cylindrical geometry with a Ni cathode, has found that pure N_2 or O_2 give no γ_p pulses at pressures from 50 to 600 mm Hg, presumably because all the radiation capable of photoemission is absorbed in the gas. The addition of a little O_2 to the N_2 does give γ_p pulses, either by increasing γ_p or by decreasing

[19] A. M. Cravath, Phys. Rev. **47**, 254 (1935).
[20] G. Dechene, J. phys. et radium **7**, 533 (1936).
[21] T. Holstein, Phys. Rev. **72**, 1212 (1947).

γ_i so that some previously masked γ_p action can be observed.

A metastable process has been observed in argon by L. Colli and U. Facchini (work to be published) with lifetimes about right to account for the difference observed here. They found $A^m+2A \rightarrow A_2^*+A$ and $A_2^* \rightarrow A_2+h\nu$. The photons should have an energy of 6.5 to 11 ev and are not absorbed in the gas.

None of these processes could be expected to delay more than a small portion of the total radiation from the discharge, but the γ_p process may be entirely due to just a few such wavelengths; too little is known yet for definite conclusions.

As earlier stated, it was proposed that space charge distortion of the field is required to make streamer propagation possible; no mechanism considered in the foregoing analysis could account for the hyper-exponential increase in current observed just before breakdown. It is thus essential to see whether the observed increase in the rate of rise of the current can be shown quantitatively to be due to space charge.

In order to get an expression for the field due to space charge in the gap, consider infinite parallel plane geometry with charge distributed uniformly in the y and z directions in a slab of thickness dx' at x' and both planes at zero potential. If the volume charge density is given by $q_v(x)$, then the charge per unit area of this slab is $q_v(x')dx'$. If a Gaussian surface is taken in the form of a cylinder parallel to the field, the use of the facts that with no variation in the y and z directions the field lines must be parallel, and that the integral of $E_{sc}dx$ across the gap must be zero, yields, in mks units, $E_{sc}=-q_v(x')(d-x')dx'/\kappa_0 d$, and $E_{sc}=q_v(x')x'dx'/\kappa_0 d$ for the regions $x\leq x'$ and $x\geq x'$, respectively. Assuming the discharge to be spread uniformly over the electrodes and neglecting the few electrons present, $q_v(x)=i_+(x)/v_+A$, where A is the area of the electrodes; so, finally,

$$E_{sc}(x)=\int_0^x \frac{i_+(x')x'}{v_+A\kappa_0 d}dx'-\int_x^d \frac{i_+(x')(d-x')}{v_+A\kappa_0 d}.$$

This will be in volts/cm if we use amp, cm, sec, and $\kappa_0=8.85\times10^{-14}$ farad/cm.

Now to show whether or not space charge distortion of the field is enough to appreciably affect $\exp(\int_0^d \alpha dx)$, some expression for the current must be used in the above expression for E_{sc} and the integrations performed; then the resulting $E_{sc}(x)$ plus the applied field are used in (15) for α where the whole quantity is squared and again integrated across the gap. To do this with the rather complicated expression for $i_+(x,t)$ used above involves a prohibitive amount of labor. However, in the expression for the final steady-state current when $\gamma e^{\alpha d}<1$, i.e., $i=i_0e^{\alpha d}/(1-\gamma e^{\alpha d})$, the effect of the γ action is the same as an increased i_0, and for the longest time lags studied when the current is increasing slowly the distribution of charge in the gap should not be so very different from what it would be for a steady current of that value. Thus the expression $i_+(x)=i_0(e^{\alpha d}-e^{\alpha x})$ should give a reasonable approximation for that case. When the integrations are carried through using this expression for i_+, the final result for the increment in the exponent is

$$\Delta\int_0^d \alpha dx \simeq \frac{1.05\times10^{-4}}{p}\left(\frac{i}{v_+\kappa_0 A}\right)^2 \times\left(\frac{-1}{\alpha^4d^4}+\frac{2.5}{\alpha^3d^3}-\frac{1}{\alpha^2d^2}+\frac{1}{12}\right)d^3,$$

where $i=i_0e^{\alpha d}$ is the measured current with the assumed expression for $i_+(x)$.

For the long time lags, values of α near threshold are of interest; the change in α between the lower two calculated curves in Fig. 5 was $\sim$0.015. Certainly a smaller change than this would give an appreciable change in the current. However, since the effect of a change in α on the measured current appears gradually over one ion crossing time, a change of say one-tenth this much could hardly give a visible change of slope. Something around 0.005 seems a reasonable figure. Equating the above expression to this figure and solving for i, the current at which field distortion should cause a noticeable up curving, the extent of variation in i for v_+ from (6 to 8)$\times10^4$ cm/sec and α from 11.1 to 11.8 (i.e., V_s from 28.5 to 28.8 kv) is from 2.8 to 3.7×10^{-4} amp. Experimentally, the semilog plots of current for the long time lags begin to curve upward around (2 to 4)$\times10^{-4}$ amp. This agreement not only supports the proposed space charge mechanism but also shows that the discharge is spread more or less uniformly over the electrodes as assumed in the calculations. If the discharge were localized to an electrode area of a few cm² the up curving would have had to appear for currents more than an order of magnitude smaller. As seen above, the dependence on v_+ and α are not critical, and since i is proportional to the square root of the value chosen as a sufficient change in α, this is not overcritical either; there cannot be any order of magnitude error. This is also in agreement with the experimental observation that when the light was focused on the center of the cathode, sparks appeared to occur randomly over the whole electrode, independent of the illuminated area.

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