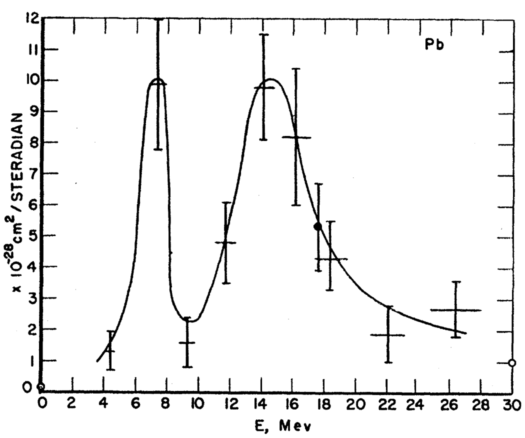

FIG. 1. Differential cross section at 120° for the elastic scattering of photons by lead. The energy spread indicated for the points is the width of the differential discriminator channel and the standard deviations on the intensities are based only on the number of counts registered. The open circles at the low- and high-energy end of the graph show, respectively, the magnitudes of the Thompson scattering cross section by the whole nucleus and by the Z free protons. The point at 17.6 Mev is taken from the data of M. B. Stearns [Phys. Rev. 87, 706 (1952)].

section given here by as much as a factor of four. Self-absorption corrections are necessary for the primary beam only, since the Doppler shift should prevent nuclear resonance absorption of the scattered photons. Experiments are now being planned to study the self-absorption effects in detail.

In spite of the crudeness of the experiment, there are significant differences in the elastic scattering cross section as a function of photon energy for the various nuclei studied. The differences depend on the level structures and oscillator strength distributions for the individual nuclei.

Bethe and Ashkin[2] have predicted the qualitative features of the scattering cross section reported here. For heavy nuclei the scattering cross section just below the (γ,n) threshold is expected to approach the value obtained by extrapolating the (γ,n) cross section into this energy region. For all of the nuclei studied except Au the general features of Fig. 42 of reference 2 are well reproduced when the present scattering data are combined with the available data on $\sigma(\gamma,n)$. The fact that in the case of Au the neutron yield curve does not join smoothly with the elastic scattering curve is probably an indication that the inelastic scattering cross section in Au is considerably larger than the elastic scattering cross section for energies just below the (γ,n) threshold. A large inelastic scattering cross section peaking near the (γ,n) threshold has been observed for gold.[3]

TABLE I. Elastic scattering cross section in cm²/sterad.

Energy (Mev)	Cu $\times 10^{-29}$	Mn $\times 10^{-29}$	Sn $\times 10^{-28}$	Au $\times 10^{-28}$	Pb $\times 10^{-28}$	Bi $\times 10^{-28}$
4.2– 4.7	2.1 ±1.1	5.7±5.7	0.57±0.32	2.3±1.3	1.3±0.6	3.8±2.2
6.5– 7.3		9.1±2.5		1.6±0.7		13.2±2.9
7.0– 7.8	9.4 ±1.2		7.4 ±0.6		9.9±2.1	
8.9– 9.9	2.8 ±1.2	30 ±5	1.4 ±0.4	1.1±0.4	1.6±0.8	0.57±0.33
11.1–12.2	0.79±0.55	<0.42	0.17±0.17	2.6±0.7	4.8±1.3	0.25±0.07
13.5–14.7	0.95±0.54	1.3±0.7	1.1 ±0.4	6.0±1.0	9.8±1.7	7.6 ±1.6
15.2–17.0	4.4 ±1.2	2.5±0.9	1.9 ±0.4	5.8±1.0	8.2±2.2	6.7 ±1.5
17.3–19.5	8.9 ±1.3	3.5±1.0	1.9 ±0.4	3.2±0.6	4.4±1.1	4.3 ±1.2
21.0–23.0	5.6 ±1.1	5.0±1.4	1.1 ±0.3	1.1±0.4	1.9±0.9	3.9 ±1.7
24.9–28.2	4.3 ±1.0		1.2 ±0.3		2.7±0.9	

* This research was supported by the U. S. Air Force, through the Office of Scientific Research of the Air Research and Development Command.

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Theory of Multiple Coulomb Scattering from Extended Nuclei*

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UNCERTAINTIES in the analysis of recent μ-meson scattering experiments[1,2] have emphasized the necessity for a reasonably accurate estimate of the modification in the Coulomb multiple scattering distribution required to properly take into account the finite extension of the nucleus.

In particular, cosmic-ray experiments[2] performed recently have been interpreted as indicating the existence of an anomalous (i.e., nonelectromagnetic) μ-nuclear interaction which cannot be explained in terms of known μ-meson interaction processes. In most of these experiments the multiple scattering distribution of relativistic μ mesons from 2- or 5-cm lead plates is measured; then the experimental results are compared with the predictions of the Olbert[3] and Molière[4] multiple scattering theories. Although by no means the only difficulty arising in the interpretation of such experiments, one of the most striking is the absence of a reliable estimate of the expected electromagnetic multiple scattering distribution at large angles.

In the Molière multiple scattering theory the nucleus is treated as a point charge. The single scattering cross section is taken to be the Rutherford cross section modified at small angles due to electron shielding. In the Olbert theory an attempt is made to estimate the effect of the nuclear extension by multiplying the single scattering law for projected angles by a step function which cuts off all single scattering beyond a certain projected angle. This gives a very great underestimate of the multiple scattering at angles beyond the cutoff angle where the Olbert distribution has a Gaussian dropoff and soon falls greatly below any reasonable single scattering curve. It is easily seen that the correct multiple scattering curve should always fall above the single scattering curve at large angles (for a reasonable choice of the single scattering law).

We have, therefore, attempted to develop procedures for solving the multiple scattering problem starting with single scattering cross sections of the form

$$f(\varphi)d\varphi=\tfrac{1}{2}QF_N(\varphi/\varphi_0)d\varphi/(\varphi^2+\varphi_m{}^2)^{\frac{3}{2}}.$$

φ is the projected angle and φ_m is the screening angle; $F_N(\varphi/\varphi_0)$ is the nuclear form factor,[5] where $F_N(0)=1$ and $F_N(\varphi/\varphi_0)$ decreases approximately as φ^{-4} for large values of φ; $\varphi_0=\hbar/pR$ and R is the nuclear radius.[6]

Two completely independent methods for solving this kind of problem have been developed and applied to the case of the multiple scattering of relativistic μ mesons from 2 cm of lead. The two methods give results that are in agreement with one another.

The first method might be characterized as a "brute force" numerical folding together of several partial

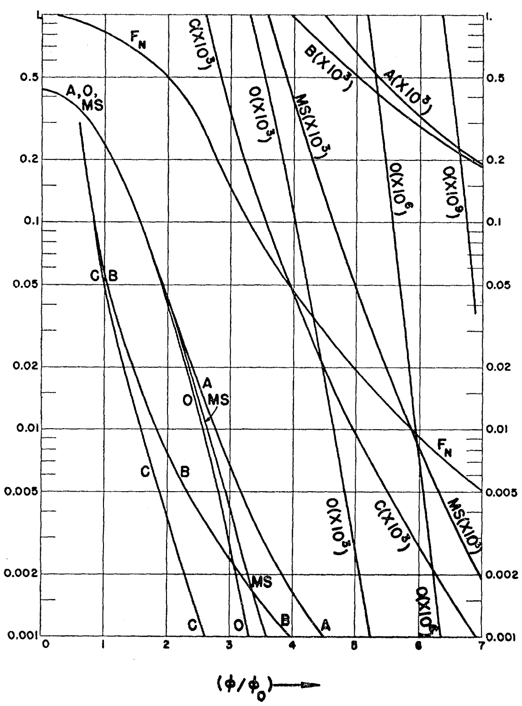

FIG. 1. Curves appropriate to $cp=1$ Bev and 2-cm Pb. Multiply (φ/φ_0) by 1.74 for Bev degrees. Curve $A=$Molière; $B=$point nucleus single scattering; $F_N=$assumed nuclear form factor, $C=$single scattering law including F_N; $MS=$resulting multiple scattering law; $O=$Olbert distribution with single scattering cutoff at $(\varphi/\varphi_0)=1.1$.

distributions which together add up to the selected distribution of single scatterings. Such a method at first seemed hopelessly tedious, but several approximations were used which greatly speeded up the calculations without introducing excessive errors. The final calculation required about two days of slide rule and desk calculator time to obtain results which are estimated to be good to 1–2 percent at small angles and 5–10 percent at large angles (aside from errors in the assumed form factor). This method has the advantage of giving a good insight into the way the final result develops in terms of the physical processes.

The second method is an extension of the Molière theory in which $F_N(\varphi/\varphi_0)$ is included as part of the single scattering cross section. (The derivation will be given in an article to follow.) The multiple scattering law deduced by this second method can be expressed analytically and the difficult part of the calculation reduces to the evaluation of one integral which can be done readily by numerical methods.

We obtain:

$$M(x)=\frac{\exp(-x^2)}{\sqrt{\pi}}+\frac{1}{4G}\left\{f'(x,\infty)-\frac{1}{\sqrt{\pi}}\int_0^\infty\frac{d\xi[1-F_N(\xi/x_0)]}{(\xi^2+x_m{}^2)^{\frac{3}{2}}}T(x,\xi)\right\},\qquad(1)$$

$$M(x)=\frac{\exp(-x^2)}{\sqrt{\pi}}\left[1+\frac{(2x^2-1)}{4G}q\right]+\frac{1}{4G}\frac{1}{\sqrt{\pi}}\int_\kappa^\infty\frac{d\xi F_N(\xi/x_0)}{(\xi^2+x_m{}^2)^{\frac{3}{2}}}T(x,\xi),\qquad(2)$$

where

$$T(x,\xi)=\{\exp[-(x+\xi)^2]+\exp[-(x-\xi)^2]-2\exp(-x^2)\},$$

$$q=\ln(\kappa/1.26)^2,\quad 0.1\leq\kappa\leq 0.5,$$

and where x and x_0 are proportional to φ and φ_0.

Formula (1) is the same as Molière's formula except for the last term which gives the correction due to the nuclear extension. It is seen that if $F_N(\xi/x_0)=1$ (point nucleus) the correction term is zero. For large values of x, formula (1) is inconvenient because the correction term is large compared to the net value of $M(x)$. In this case, however, (2) is convenient; especially so where x is large enough so that the first term in (2) can be neglected due to the smallness of $\exp(-x^2)$. The two formulas can be used to evaluate $M(x)$ for all values of x. It is also to be noticed that a single calculation will serve for all momenta in the relativistic region as the integral depends upon $\beta\approx 1$. The integrals were evaluated numerically by use of Weddle's[7] rule and grid spacings and values of κ of $\frac{1}{2}$ and $\frac{1}{4}$. Comparison of the results for the two grid spacings indicates that errors resulting from the numerical integrations are quite small.

We carried through calculations for the case (Fig. 1) of 2 cm Pb and $cp=1$ Bev and used $F_N(\varphi/\varphi_0)=1.00$, 0.82, 0.50, 0.15, and $12(\varphi/\varphi_0)^{-4}$ for $(\varphi/\varphi_0)=0$, 1, 2, 3, and $\geqslant 4$. This particular choice of F_N is intended to slightly underestimate the nuclear size effects, but otherwise to represent our best guess as to the "correct" form factor on the basis of recent experiments. We chose $R=1.1A^{\frac{1}{3}}\times 10^{-13}$ cm and applied F_N to the law for projected scattering. This should give nearly the same result as choosing $R=1.0A^{\frac{1}{3}}\times10^{-13}$ cm with F_N applied to the law for total angle scattering. Inelastic scattering was not included here, but will be discussed in the article to follow.

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Polarization of Nucleons Elastically Scattered from Nuclei*

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THE polarization of high-energy nucleons elastically scattered from spin-zero nuclei must be a consequence of an effective spin-orbit potential in the nucleon-nucleus interaction. This suggests a simple generalization of the conventional optical model of the nucleus by the addition of a spin-orbit potential.[1] Calculations of the polarization to be expected from this model as applied to beryllium and carbon have been made in several instances.[1] The results are in qualitative agreement with the experimental measurements in that they show that a small spin-orbit potential, of the order of 1 Mev, can lead to the large polarizations observed. However, the calculations indicate that in the region of the diffraction minima, the polarization shows a double reversal of sign within an angular region of a few degrees. This double reversal, or dip, is not experimentally observed in Be or C.[2] It has been suggested that this dip as it appears in the calculations is a reflection of the use of a square-well central potential.[3] Our calculations for the polarization of 290-Mev neutrons elastically scattered from carbon indicate that if the real central potential is taken to be zero or sufficiently small compared to the imaginary potential, the dip is not eliminated by rounding off the edges of the square well.[4] However, if for a given potential-well shape the real central potential is increased sufficiently, relative to the imaginary central potential, the dip in the polarization becomes of less significance, so that experimentally it would not be observed. Both the shape of the well and the magnitudes of the potentials should of course be fixed by a comparison with the experimental scattering measurements. It is felt, though, that the measurements at 300 Mev are not sufficiently extensive or accurate to fix all the parameters involved.

In addition to the above considerations there is another still unconsidered point that has a bearing on the interpretations of the calculations. A formal analysis of the justification of the optical model of high-energy nucleon-nucleus scattering has been made, which leads to the conclusion that in general the predictions of this model are valid only for small scattering angles. The exact line between small- and large-angle scattering is, of course, not precise. For the light elements, Be and C, however, the predicted first diffraction minima and the associated polarization phenomena occur at an angle $\simeq 20°$. One notes that not only is the dip in the polarization not observed, but also that the first diffraction minimum is not observed. Now if the absence of these phenomena in Be and C can be ascribed to the lack of validity of the model for large scattering angles, one would expect that for the heavier nuclei, where the diffraction and polarization occur at smaller angles, these phenomena would manifest themselves according to the predictions of the calculations. Such indeed is the case. Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis[5] have found that a dip in the polarization occurs for the elements Al, Ca, and Fe, in the region around the diffraction minima. The calculated polarization of 290-Mev neutrons elastically scattered from Al, assuming a parabolic-shaped central nuclear potential,[6] is shown in Fig. 1, along with the experimentally observed polarization for 290-Mev protons elastically scattered from aluminum. The second dip that is predicted by the model would not be expected to be experimentally observed because of the probable lack of validity of the model for such large angles of scattering.[7] The effect of including the Coulomb potential in the calculations to describe the scattering of protons will be to decrease the maximum polarization and to widen the region of the dip in the polarization.

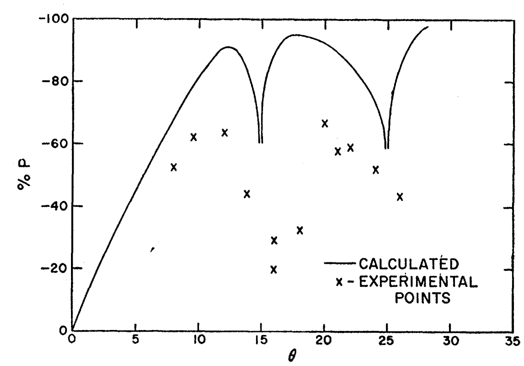

FIG. 1. The calculated polarization of 290-Mev neutrons elastically scattered from aluminum. The crosses, ×, are the experimentally measured polarizations for protons elastically scattered from aluminum as given by reference 5.