Scattering of 187-Mev Negative Pions by Hydrogen*

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The differential cross sections for scattering of 187-Mev negative pions in hydrogen have been observed at six angles. The scattering is well represented by the angular distributions in the barycentric system: $\pi^-+p\rightarrow\pi^-+p$: $d\sigma/d\omega=0.81\pm0.13+(0.35\pm0.20)\cos\chi+(3.08\pm0.38)\cos^2\chi\times10^{-27}$ cm^2/sterad, $\pi^-+p\rightarrow\pi^0+n$: $d\sigma/d\omega=1.46\pm0.24-(0.16\pm0.30)\cos\chi+(5.63\pm0.88)\cos^2\chi\times10^{-27}$ cm^2/sterad. A transmission measurement of the total cross section gave $63.5\pm1.6\times10^{-27}$ cm^2. An analysis of these data in terms of phase shifts is discussed.

THE scattering of negative pions of energy 187 Mev in liquid hydrogen has been investigated using the methods previously applied at 217 Mev.[1] The equipment and experimental technique were the same as those described in A. The pions passed through two scintillation counters, Nos. 1 and 2, and from there into the liquid hydrogen Dewar. For observation of the ordinary scattering, $\pi^-+p\rightarrow\pi^-+p$, two large counters, Nos. 3 and 4, were placed at various angles with respect to the incident beam to detect the scattered pions. Aluminum absorbers were required between counters Nos. 3 and 4 at the forward angles to remove the recoil protons. These amounted to 9.45 g/cm^2 at 25.7° and 2.59 g/cm^2 at 51.4°. The double coincidences D of counters Nos. 1 and 2 recorded the pions entering the liquid hydrogen, while the quadruple coincidences Q of all four counters gave the number of scattered pions detected. In the charge exchange scattering, $\pi^-+p\rightarrow\pi^0+n$, the neutral pion decays promptly into two photons. To observe one of these photons, three counters, Nos. 5, 3, and 4, formed a telescope to detect electrons produced by the photon in a lead sheet placed between counters Nos. 5 and 3. Counter No. 5 was counted in anticoincidence with the quadruple coincidences Q' of the other four counters so as not to detect charged particles produced in the scattering process. The details of the arrangement and description of equipment may be found in Secs. I and II of A.

The energy of the pions was calculated from their observed range in copper. The mean pion energy in the center of the hydrogen was 187 Mev, and the beam had a spread in energy estimated from the range curve as ±6 Mev. The beam contained 97 ± 1 percent pions.

The procedure followed in reducing the data was the same as in A. The values of the ratio Q'/D observed with and without hydrogen in the scattering cell are given in Table I. The errors quoted there are the standard deviations. The differential cross section for production of gamma rays is then given by

$$\frac{d\sigma}{d\omega}=\frac{Q'/D}{\epsilon_\gamma(0.97)(0.993)\Delta\omega N}=\frac{2.89\times10^3}{\epsilon_\gamma\Delta\omega}Q'/D,$$

in units of 10^{-27} cm^2/sterad, where ϵ_γ is the efficiency for detecting gamma rays emitted into the solid angle $\Delta\omega$, 0.97 is the fraction of the number D which consists of pions, 0.993 is a correction factor due to the attenuation of the beam in the cell, N is 3.59×10^{23}, the number of hydrogen atoms per square centimeter, and $\Delta\omega$ is the solid angle subtended by counter No. 4 in steradians. The gamma-ray efficiency ϵ_γ was calculated, making corrections[2] for the scattering and absorption of the electrons produced in the 7.30-g/cm^2 lead converter. The lateral distribution of the pion beam was measured, and this distribution was used to calculate the mean distance traveled by the pions in the hydrogen and therefore the value of N. Corrections were made to the solid angle for the finite extent of the counters and the target. The values used, along with the calculated differential cross sections in the laboratory and barycentric systems, are presented in Table II. The last column, the barycentric system differential cross section, has the contribution of the reaction $\pi^-+p\rightarrow n+\gamma$ (estimated from the process[3] $\gamma+p\rightarrow\pi^++n$ and the ratio[4] of positive to negative pion photoproduction in deuterium as 0.05×10^{-27} cm^2/sterad) subtracted. The errors quoted include estimates of the uncertainty in the pion content and the number of hydrogen atoms N.

The values of the ratio Q/D observed with and without hydrogen in the target are given in Table III. From these, the differential cross section for ordinary

TABLE I. Observed values of the fraction Q'/D of pions interacting to give a gamma ray, in units of 10^{-6}.

Laboratory angle (deg)	With hydrogen	Without hydrogen	Net
25.7	410.4±14.3	124.8±8.5	285.6±16.6
51.4	207.9± 9.2	50.9±4.8	157.0±10.4
77.1	135.4± 8.2	36.0±6.0	99.4±10.1
102.8	141.0± 8.4	31.6±5.1	109.4± 9.8
128.6	149.0± 7.0	43.0±6.6	106.0± 9.6
154.3	208.7± 9.5	93.6±8.6	115.1±12.8

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[1] M. Glicksman, Phys. Rev. 94, 1335 (1954), quoted here as A.
[2] Anderson, Fermi, Martin, and Nagle, Phys. Rev. 91, 155 (1953).
[3] J. Steinberger and A. S. Bishop, Phys. Rev. 86, 171 (1952).
[4] White, Jacobson, and Schulz, Phys. Rev. 88, 836 (1952).

TABLE II. Differential cross sections for $\pi^- + p \rightarrow \gamma$.

Laboratory angle θ (deg)	Angle in the barycentric system χ (deg)	Solid angle $\Delta\omega$ (sterad)	Gamma-ray detection efficiency ϵ_γ	Differential cross section in laboratory (10^{-27} cm^2/sterad)	Differential cross section in barycentric system for γ from π^0 (10^{-27} cm^2/sterad)
25.7	32.3	0.107	0.585	13.19±0.82	8.63±0.54
51.4	62.8	0.107	0.561	7.56±0.53	5.79±0.41
77.1	90.7	0.106	0.540	5.02±0.52	4.72±0.49
102.8	115.6	0.106	0.521	5.73±0.52	6.65±0.61
128.6	138.4	0.107	0.506	5.66±0.52	7.78±0.73
154.3	159.6	0.107	0.507	6.13±0.69	9.43±1.07

scattering was calculated:

$$\frac{d\sigma}{d\omega}=\frac{2.89\times10^3}{\epsilon_\pi\Delta\omega}\left[(Q/D)_\pi-\frac{\epsilon_\gamma'\Delta\omega}{2.89\times10^3}(d\sigma/d\omega)_\gamma\right].$$

The symbols ϵ_π and ϵ_γ' represent the efficiencies of the counters for detecting scattered pions and gamma rays produced in the hydrogen, respectively, and $(d\sigma/d\omega)_\gamma$ is the measured gamma-ray cross section given in the second last column of Table II. These factors and the resulting differential cross sections are collected in Table IV.

The differential cross sections in the laboratory system can be integrated to give the total cross sections for ordinary and charge-exchange scattering. Such an integration gives $22.5\pm1.3\times10^{-27}$ cm^2 for $\pi^-+p\rightarrow\pi^-+p$, $82.3\pm2.9\times10^{-27}$ cm^2 for $\pi^-+p\rightarrow\gamma$, and $64.0\pm2.0\times10^{-27}$ cm^2 for the total cross section, including the contribution estimated above for $\pi^-+p\rightarrow n+\gamma$. During the course of the measurements of the differential cross sections, the total cross section was observed directly. Counters Nos. 3 and 4 were placed at 0° and the values of Q/D were recorded with and without hydrogen in the target. The total cross section was then calculated from these observations, and corrected for the scattered pions and protons detected by counters Nos. 3 and 4 using the angular distribution inferred from the measured differential cross sections. This transmission measurement yielded a total cross section of $63.5\pm1.6\times10^{-27}$ cm^2, in excellent agreement with the value obtained by integration. Both of these measurements are consistent with an interpolation

TABLE III. Observed values of the fraction Q/D of elastically scattered pions, in units of 10^{-6}.

Laboratory angle (deg)	With hydrogen	Without hydrogen	Net
25.9	882.0±14.2	674.5±13.9	207.5±19.9
51.4	210.3± 6.5	137.7± 5.9	72.6± 8.8
77.1	133.0± 4.7	94.4± 4.3	38.6± 6.4
102.8	121.3± 4.9	72.5± 4.3	48.8± 6.5
128.6	141.2± 5.3	82.5± 4.5	58.7± 7.0
154.3	249.6± 7.1	174.7± 6.2	74.9± 9.4

of the recent results[5] of Ashkin, Blaser, Feiner, Gorman, and Stern, which would give about $65\pm2.5\times10^{-27}$ cm^2 at this energy.

The differential cross sections for the ordinary and charge exchange scattering can be fitted by angular distributions of the type $a+b\cos\chi+c\cos^2\chi$ which assume that the scattering can be described in terms of s- and p-wave scattering alone. The π° angular distribution was calculated[2] from the gamma-ray distribution as in A. Least squares values for the coefficients are given in Table V. Both fits are good. There is thus no evidence, at this level of experimental error, that higher angular momentum states, for example d waves, also need to be included in analyzing either of the reactions at this energy.

On the assumption that only s and p waves are important in the interaction at this energy, the scattering can be described in terms of six phase shifts[2] of angular momentum $\frac{1}{2}$ and $\frac{3}{2}$, isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$. However, as has been pointed out,[1] there are a large number of sets of these phase shifts which fit the negative data alone at one energy. De Hoffman, Metropolis, Alei, and Bethe[6] have used the negative data at energies in the range 120 to 217 Mev to calculate these sets, and reduced considerably the multiplicity of solutions by introducing the requirement that the phase shifts should predict a positive pion total cross section which agrees with experiment. The largest number of acceptable solutions occurs with the 194-Mev data.[7] With the additional condition that the solutions vary smoothly with energy, they find that the data used in this way allow only three different sets of phase shifts.

For a number of reasons,[6] de Hoffman *et al.* conclude that the set of solutions in which[8] α_{33} passes through 90° at about 195 Mev is probably the correct one. This solution is the one in which the three phase shifts α_{31}, α_{13}, and α_{11} are small throughout this energy region, and corresponds to the phase shift set reported

[5] Ashkin, Blaser, Feiner, Gorman, and Stern, Phys. Rev. **93**, 1129 (1954).
[6] De Hoffman, Metropolis, Alei, and Bethe, Phys. Rev. (to be published).
[7] Fermi, Glicksman, Martin, and Nagle, Phys. Rev. **92**, 161 (1953).
[8] The notation of reference 2 is used.

TABLE IV. Differential cross sections for $\pi^- + p \rightarrow \pi^- + p$.

Laboratory angle θ (deg)	Angle in the barycentric system χ (deg)	Solid angle $\Delta\omega$ (sterad)	Pion detection efficiency ϵ_π	Gamma-ray detection efficiency ϵ_γ'	Differential cross section in laboratory (10^{-27} cm^2/sterad)	Differential cross section in barycentric system (10^{-27} cm^2/sterad)
25.7	33.3	0.113	0.858	0.025	5.80 ± 0.60	3.60 ± 0.37
51.4	64.7	0.112	0.938	0.025	1.80 ± 0.24	1.34 ± 0.18
77.1	93.0	0.111	0.968	0.024	0.94 ± 0.18	0.89 ± 0.17
102.8	118.0	0.111	0.968	0.024	1.20 ± 0.18	1.45 ± 0.22
128.6	140.3	0.105	0.968	0.024	1.48 ± 0.19	2.21 ± 0.28
154.3	160.7	0.113	0.968	0.024	1.90 ± 0.26	3.25 ± 0.45

in A which was calculated on the assumption that these three phase shifts were actually all zero. The data reported here were analyzed under these assumptions.

The phase shifts α_{33}, α_3, and α_1 were calculated employing the same procedure as in A. Possible sets of phase shifts were found from a geometrical analysis suggested by Ashkin and Vosko.[9] This analysis uses five of the available seven independent data (the six coefficients of the negative pion angular distributions, and the total cross section for the scattering of positive pions in hydrogen taken[10] as $190\pm10\times10^{-27}$ cm^2), but under the simplifying assumptions made ($\alpha_{31}=\alpha_{13}=\alpha_{11}=0$), only three relations for the three phase shifts are used. As a result there are phase-shift "solutions" which are very poor fits to all the data. These were dropped and the remaining sets were improved in their fit by numerically varying the phase shifts to decrease the least-squares sum

$$M=\sum_{i=1}^{7}(\Delta_i/e_i)^2.$$

In M, Δ_i is the difference between the calculated and experimental value for each of the coefficients and for the total positive pion cross section, and e_i is the experimental error assigned to these quantities.

Minimizing the least squares sum gave the phase shift set: $\alpha_{33}=83$ deg, $\alpha_3=0$ deg, and $\alpha_1=8$ deg, with a value for M at the minimum of about 4.5, which indicates a good fit to the data. However the phase-shift values are quite sensitive to the data.[11] Indeed, if each of the phase shifts is varied (holding the other two constant) to note the effect upon M, the fit becomes poor only for values beyond the limits: α_3 to $+10$ and -15 deg, α_1 to $+12$ and -2 deg, and α_{33} to 98 and 70 deg. It appears that the experimental data would need to be greatly improved to allow one to calculate more definite values of the phase shifts in this very sensitive energy region.

TABLE V. Coefficients of $d\sigma/d\omega=a+b\cos\chi+c\cos^2\chi$ in units of 10^{-27} cm^2/sterad, for the various processes.

Process	a	b	c
$\pi^-\rightarrow\pi^-$	0.81 ± 0.13	0.35 ± 0.20	3.08 ± 0.38
$\pi^-\rightarrow\gamma$	4.91 ± 0.35	-0.23 ± 0.42	5.28 ± 0.83
$\pi^-\rightarrow\pi^0$	1.46 ± 0.24	-0.16 ± 0.30	5.63 ± 0.88

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[9] J. Ashkin and S. H. Vosko, Phys. Rev. **91**, 1248 (1953).

[10] Ashkin, Blaser, Feiner, Gorman, and Stern (private communication).

[11] This has been noted in previous analyses of the data in this energy region. See reference 6, and R. L. Martin, Phys. Rev. (to be published).