Meson Production in *n-p* Collisions at Cosmotron Energies*

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(Received May 3, 1954)

154 analyzable three-prong events were observed in collisions between Cosmotron-produced neutrons, with energies up to 2.2 Bev, and protons in a hydrogen-filled diffusion cloud chamber. These events were classified as a result of the reactions $n+p\rightarrow p+p+\pi^-$, $p+p+\pi^-+\pi^0$, and $p+n+\pi^++\pi^-$ with frequencies in the ratio $(0.8\pm0.3):(1\pm0.35):(3.2\pm0.7)$, respectively. The observed ratio of the probability for double meson production to that for single meson production is more than 20 times higher than the ratio predicted from Fermi's statistical model. No triple meson production is observed, in agreement with the statistical model. The momentum distributions of the observed mesons are also in rough agreement with the theory. In the center-of-mass system, in the reaction $n+p+\pi^++\pi^-$, protons and π^+ show a tendency to be emitted backward, while neutrons and π^- tend towards forward emission. However, the data suggest that protons and π^+, and also neutrons and π^-, tend to be emitted more frequently in opposite directions than protons and π^- or neutrons and π^+. All of these results may be qualitatively in agreement with a meson production model where each nucleon is excited separately to an intermediate, possibly resonant, state which subsequently decays by emission of a meson.

THE Brookhaven Cosmotron provides a source of particles with energies up to 3 Bev and permits experiments on nuclear interactions at these energies under controlled conditions. A considerable number of experiments have been performed with synchrocyclotrons on nucleon-nucleon interactions at energies up to about 450 Mev and pion-nucleon interactions at energies up to about 250 Mev.[1] At these energies most nucleon-nucleon interactions result in elastic scattering, and inelastic scattering leads to the production of pions in at most a few percent of the cases. Similarly, the scattering of pions by nucleons has been almost entirely elastic (if this term is understood to include the possibility of charge-exchange scattering). At Cosmotron energies mesons are produced in a considerable fraction of the cases, and even "elementary" particle collisions may lead to a large number of different reactions.

This group has undertaken a preliminary cloud chamber survey of nucleon-nucleon and pion-nucleon interactions in the Bev energy range in an attempt to determine the general nature of these reactions. The results reported here have to do with neutron-proton interactions, in particular the production of pions in *n-p* collisions when the neutron energy is about 2 Bev.

There is, of course, a very large amount of data on nuclear interactions at these energies derived from cosmic ray experiments.[2] The interpretation of these results is difficult because the energy and even identity of the particle causing an interaction is in most cases somewhat uncertain. In addition most experiments involve interactions in emulsions or absorbers in which elements of high atomic weight predominate, so that complicated cascade processes may develop in the nucleus which is struck. It is difficult to determine any details of nucleon-nucleon or meson-nucleon interactions from such data. In some cases interactions occurring in hydrogen have been isolated,[3] but the results have not permitted a detailed comparison with theoretical predictions.

At the Cosmotron, it has been possible to achieve improvements in both of these respects. We have employed a diffusion cloud chamber filled with hydrogen gas at a pressure of about 20 atmospheres and have concentrated on interactions occurring in the cloud chamber gas. Under these conditions almost all interactions are with hydrogen.[4] The cloud chamber has been operated in the neutron beam emerging at 0° from an internal target. There is little quantitative information concerning the purity of this beam. It seems safe, however, to assume that the events studied are due to neutrons. There is, unfortunately, no reason to believe that the neutrons are monoenergetic, and little is known about their energy distribution. (This question will be discussed in Part IV.)

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] For summaries of the results of these experiments see R. E. Marshak, *Meson Physics* (McGraw-Hill Book Company, Inc., New York, 1952), and Henley, Ruderman, and Steinberger, Ann. Rev. Nuc. Sci. **3**, 1 (1953).

[2] For summaries of these results see R. E. Marshak, *Meson Physics* (McGraw-Hill Book Company, Inc., New York, 1952), B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., New York, 1952), and Camerini, Lock, and Perkins, *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1952), p. 1.

[3] M. L. Vidale and M. Schein, Phys. Rev. **84**, 593 (1951); G. W. Rollosson, Phys. Rev. **87**, 71 (1952); W. Bosley and H. Muirhead, Phil. Mag. **43**, 783 (1952); Weaver, Long, and Schien, Phys. Rev. **87**, 531 (1952); A. B. Weaver, Phys. Rev. **90**, 86 (1953); Kusumoto, Miyake, Suga, and Watase, Phys. Rev. **90**, 998 (1953); McCusker, Porter, and Wilson, Phys. Rev. **91**, 384 (1953); S. Miyake (private communication).

[4] There is a small contamination of carbon and oxygen in the alcohol vapor (about 1 alcohol molecule in 800 hydrogen molecules) and, in this experiment, a contamination of 0.3 percent oxygen in the hydrogen, but the occasional interactions involving carbon or oxygen are easily recognized. Eleven events were observed to occur in these heavier nuclei. These were recognized either by the fact that their prong number was even, which is not possible for an *n-p* collision, or by the occurrence of more than two protons, or of protons which passed further backwards than is kinematically possible in *n-p* collisions. The prong numbers observed were 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, and 8. These events had the typical appearance of collisions with heavy nuclei, showing slow "evaporation prongs" in addition to the fast prongs.

I. OBJECTIVES OF THE EXPERIMENT

While the experiment described here is exploratory in nature, it was intended to provide information on a number of specific questions as well as to give a qualitative picture of the nature of n-p interactions. The following questions are of special interest.

1. Multiplicity of Meson Production

Cosmic-ray results have not provided a clear picture as to whether a nucleon-nucleon collision frequently (or perhaps ever) leads to the emission of more than one meson.[2] The uncertainty arises from the complicated nature of the cascade process in a large nucleus, which permits the observed meson showers to be explained in terms of single meson production at each of many nucleon-nucleon collisions ("plural theory").[5] Results on n-p collisions in hydrogen gas should decide unambiguously between single and multiple production. Fermi has proposed a statistical theory of meson production which is based on the idea that pion interactions are strong, but involves no specific meson theory.[6] It is of particular interest to compare our observations with this theory.

2. Energy, Angle, and Charge of Emitted Mesons

Not only meson multiplicity but also further details of the process can be compared with theoretical predictions, for example, ratios of numbers of positive to negative to neutral mesons, and angular and energy distributions for the emitted mesons and nucleons. The charge ratios are related to ideas of the charge independence of nuclear phenomena, while the energy distributions are related to the degree of inelasticity of the collisions.

3. Angular Correlations between Emitted Particles

The presence or absence of angular correlations between emitted particles may give evidence of interactions between them, such as meson-nucleon forces, or meson-meson forces when two (or more) mesons are produced. If the forces are strong enough to produce bound intermediate states, these might also be recognized.

4. Production of "New Unstable Particles"

In addition to n-p collisions involving meson production, other events of interest might well be observed in the chamber. In particular it was hoped that V-events or other decays of members of the "new unstable particle" family would be observed. A few such V events were indeed found, which are reported elsewhere,[7] but none were produced in n-p collisions occurring in the cloud chamber gas.

5. Neutron Beam Energy

A final objective was that of obtaining some information about the neutron beam. Since measurements of the momenta of the particles emitted from n-p interactions would often determine the energy of the neutron that produced them, they should give some idea of the energies of the neutrons emitted from the Cosmotron target.

II. EXPERIMENTAL PROCEDURE

The observations were made with a diffusion cloud chamber filled with hydrogen at a pressure of about 20 atmospheres employing methyl alcohol as the condensable vapor. It was operated in a magnetic field of about 10 500 gauss. Details of the cloud chamber, its operation with the Cosmotron, projection method, and accuracy of measurements have been given elsewhere.[8]

The cloud chamber was placed in the external neutron beam emerging at 0° from a carbon target struck by the circulating proton beam of the Cosmotron at a time when the protons had energies of 2.2 Bev. A lead collimator in the Cosmotron shield defined a beam approximately 1 in.$\times$2 in. in size. Charged particles coming from the target were deflected out of this beam by the magnetic field of the Cosmotron. The beam emerging from the collimator contained neutrons and photons from the target and charged particles from the wall of the Cosmotron vacuum tank. A 1.5-inch lead converter served to reduce the photon intensity. The beam then passed through a permanent magnet, which deflected charged particles out of the beam, and finally into the cloud chamber.

About 20 000 Cosmotron pulses were photographed. The pictures were scanned for 3-prong and 5-prong events. Any n-p interaction results in an odd number of emitted charged particles as long as all of them have unit charge and electric charge is conserved. Elastic recoil protons and other 1-prong events were not counted. They would appear as single tracks starting in the cloud chamber gas, usually with density of ionization fairly close to minimum. Such tracks are difficult to find in scanning because the limited sensitive depth and gaps in the sensitive region cause many other tracks to look similar. In principle it is possible to distinguish between them, but in practice it was not feasible to do so. Furthermore, even if 1-prong events had been counted, their interpretation would be quite ambiguous since they could be due to elastic n-p collisions or inelastic collisions resulting in production

[5] W. Heitler and L. Janossy, Proc. Phys. Soc. (London) **62**, 669 (1949). For a recent summary see H. Messel, *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1954), Vol. 2, p. 135.

[6] E. Fermi, Progr. Theoret. Phys. (Japan) **5**, 570 (1951).

[7] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **90**, 1126 (1953).

[8] Fowler, Shutt, Thorndike, and Whittemore, Rev. Sci. Instr. (to be published).

TABLE I. Types of n-p reactions.

Number of pions produced	Products of reaction	Abbreviation	Number of charged prongs
0	$p+n$	pn	1
1	$p+n+\pi^0$	$pn0$	1
1	$p+p+\pi^-$	$pp-$	3
1	$n+n+\pi^+$	$nn+$	1
2	$p+n+\pi^0+\pi^0$	$pn00$	1
2	$p+n+\pi^++\pi^-$	$pn+-$	3
2	$p+p+\pi^-+\pi^0$	$pp-0$	3
2	$n+n+\pi^++\pi^0$	$nn+0$	1
3	$p+n+\pi^0+\pi^0+\pi^0$	$pn000$	1
3	$p+n+\pi^++\pi^-+\pi^0$	$pn+-0$	3
3	$p+p+\pi^-+\pi^0+\pi^0$	$pp-00$	3
3	$p+p+\pi^-+\pi^-+\pi^+$	$pp--+$	5
3	$n+n+\pi^++\pi^0+\pi^0$	$nn+00$	1
3	$n+n+\pi^++\pi^++\pi^-$	$nn++-$	3

of neutral mesons. Accordingly it was decided to concentrate on events with three or more prongs.

A total of 185 3-prong events were recorded, but no 5-prong events were seen. Two photographs of 3-prong events have already been published.[9] On each 3-prong event the following measurements were made:

1. Angles in space were measured for emitted tracks relative to the neutron direction by means of a projector which reproduced the geometry of camera and cloud chamber. The neutron direction was known within about $\pm 1°$.
2. Densities of ionization were estimated for emitted tracks by using the usual subjective methods.
3. Curvatures of tracks were measured by means of a microscope with micrometer stage.

Since the depth of the sensitive layer was only about 2 inches, the emitted tracks were often too short for accurate determination of momentum, and in some cases even the angle of the track was not well determined. In these cases, therefore, events could not be analyzed with certainty.

III. CLASSIFICATION OF n-p COLLISION EVENTS

The different reactions leading to 0, 1, 2, or 3 pions are given in Table I. It is assumed that only pions are produced, and all the following analysis is based on this assumption. One cannot prove that no heavy mesons were involved, but no decay events were seen to be associated with the n-p interactions, no track had density and momentum which identified it as a heavy meson, and few heavy unstable particles were observed to be produced in the cloud chamber walls. It is, therefore, unlikely that many heavy mesons were produced in the n-p collisions, and probably there were none. As will be seen, the interactions can be analyzed on the assumption that only pions were emitted.

Table I shows that 3-prong events can be either $(pp-)$, $(pn+-)$, $(pp-0)$, $(pn+-0)$, $(pp-00)$, or $(nn++-)$. Since no 5-prong events $(pp--+)$ were seen, it was assumed that also few, if any, of the 3-prong events involved triple meson production. The data on each 3-prong event were analyzed to determine whether it should be classified as $(pp-)$, $(pn+-)$, or $(pp-0)$.

The first step in the classification of an event was to determine which tracks were protons and which pions. Measured momenta and estimated ionization densities of negatively charged particles were consistent throughout with those of π^--mesons.[10] Positive tracks were consistent with those of π^+ mesons or protons. Where a track was too short, its sign could not be determined directly but could often be inferred from the known signs of the two other particles and conservation of charge.

In 20 percent of the cases it was possible to identify all three particles from measured momenta and estimated ionization densities. However, the laws of conservation of energy and momentum afford a number of criteria which often helped to determine an event completely. Sternheimer[11] has calculated that for 2.2-Bev nucleons incident on nucleons the maximum angle at which a nucleon can be emitted in the laboratory system with respect to the incident direction is 68° for single meson production and 58° for double meson production. Mesons can be emitted in all directions. For incident neutrons of lower energies these angles become smaller. Tracks at angles larger than the maximum nucleon angle could be identified as pions. Furthermore, a proton emitted near the maximum angle should ionize heavily, so that a minimum ionization track at an angle near the maximum had to be a pion.

Where two of the three tracks could be due to protons the classification $(pp-)$ was first investigated. If P_z is the algebraic sum of the forward components of momentum of the 3 particles, $\mathbf{P}_r$ the vector sum of their components perpendicular to the direction of flight of the incident neutron, and E_s the sum of their total energies (including rest energies), then for the reaction $(pp-)$ the two conditions for momentum and energy balance, $\mathbf{P}_r=0$ and $E_s-M=(P_z^2+M^2)^{\frac{1}{2}}$, must be satisfied, where $M=0.93$ Bev is the nucleon mass. $E_s-M(=E_0)$ is, of course, equal to the total energy of the incident neutron.

If a π^+ has been identified, the event is provisionally classified as $(pn+-)$. If both positive particles are identified as protons, but the conditions for $(pp-)$ are not satisfied, the event is provisionally classified as $(pp-0)$. In each case the classification is checked

[9] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **91**, 758 (1953).

[10] Only one case, not counted here, was found which has to be interpreted as due to the reaction $n+p\rightarrow p+n+\pi^0\rightarrow p+n+e^+ +e^-+\gamma$. The mode of mesonic conversion [Steinberger, Sachs, and Lindenfeld, Phys. Rev. **90**, 343 (1953); Cornelius, Sargent, Rinehart, Lederman, and Rogers, Phys. Rev. **92**, 1583 (1953)] into 2 electrons is known to occur in about 1 percent of all π^0 emissions. One estimates that only one or two such cases can be expected in this work.

[11] R. M. Sternheimer, Phys. Rev. **93**, 642 (1954).

by the calculations described below. If the event is not $(pp-)$, and positive particles are not identified in either of the above ways, then either $(pn+-)$ or $(pp-0)$ is a possibility, but a definite classification may be arrived at from the following calculations of momentum and energy balance.

We now can attempt to calculate the momentum of the neutral particle, assuming that it is a neutron or a π^0 of mass $m_4=0.93$ Bev or 0.14 Bev, respectively. Its transverse momentum component is given by $\mathbf{p}_r+\mathbf{P}_r=0$. Conservation of energy and forward momentum provide two equations which can be solved to give the momenta of incoming neutron and emitted neutron or π^0. The forward component, p_z, of the emitted particle is given by

$$p_z=-\{P_z\pm E_t[1+(m_4^2+P_r^2)Y^2/X]^{\frac{1}{2}}\}/Y, \quad (1)$$

where $E_t=E_s-M$, $X=P_z^2-E_t^2$, and $Y=2/[1+(M^2-m_4^2-P_r^2)/X]$. The sign before the second term in (1) must be chosen so that

$$E_t+(P_r^2+p_z^2+m_4^2)^{\frac{1}{2}}=[(p_z+P_z)^2+M^2]^{\frac{1}{2}}. \quad (2)$$

The latter two expressions represent the total energy of the incident neutron E_0. If a solution has been found for p_z, then a test for its validity is, of course, that $E_0-0.93\leq 2.2$ Bev, the kinetic energy of the protons circulating in the Cosmotron. However, a solution for p_z does not necessarily exist for any given combination of P_r, P_z, E_t, and m_4. It follows from (1) that for a solution to exist the value of X must lie *outside* the interval,

$$-[M-(m_4^2+P_r^2)^{\frac{1}{2}}]^2\geq X\geq -[M+(m_4^2+P_r^2)^{\frac{1}{2}}]^2. \quad (3)$$

The existence of this forbidden region often affords a quick and decisive test whether $m_4=0.93$ Bev [$(pn+-)$ classification] or $m_4=0.14$ Bev [$(pp-0)$ classification].

For events where all three masses and momenta are known with some certainty, the procedure described above is straightforward. Where not all momenta could be measured several assumptions can be made, consistent with the estimated ionization densities of the tracks, each assumption to be tested separately. Sometimes several different assumptions were kinematically possible, leading to different classifications of the event and different calculated values for the momenta of the neutral particles.

Uncertainties in momentum and angle measurements also have to be taken into account. Varying a measured quantity within its range of experimental uncertainty may lead to different classifications of the same event. One might restrict the analysis to well-measured events, but much information would then be ignored, and, in addition, selecting only well-measured events may lead to a bias of the results since the geometry of the events partially depends on their type. For instance, proton tracks can only pass in a forward direction, as mentioned above. Thus many of them have a good chance to remain for some distance inside the sensitive layer of the cloud chamber, which is advantageous for angle and momentum measurements. On the other hand, their momenta are most probably high, making momentum measurements more difficult. Mesons, particularly π^+, often appear to be emitted at rather large angles. Thus, because of the limited thickness of the sensitive layer in the cloud chamber, their tracks may become too short for curvature measurements and for accurate angle measurements. For these reasons only events for which no momentum measurements at all could be performed were excluded. It is felt that no bias is introduced by their omission. (A few events occurring outside the region where the direct neutron beam passed through the chamber were excluded since these could only have been secondary events produced by previously scattered neutrons of unknown direction.)

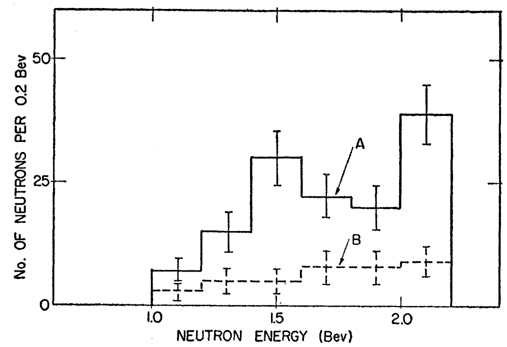

FIG. 1. Energy spectrum of the incident neutrons. Graph A was obtained making use of all events, while graph B represents a selection of 38 events which could be determined with some accuracy.

The remaining total of 154 events could be analyzed with varying degrees of certainty. In some cases, for example, the event may be classified with good certainty, while momenta of certain tracks are not measured, but given assumed values which are consistent with the classification. In the following discussion of results some are well-established, while others are uncertain and speculative because of the limitations of the basic data, as will be pointed out in the discussion.

IV. ENERGY SPECTRUM OF THE NEUTRON BEAM

The energy spectrum inferred for the neutrons producing the 3-prong events is shown in Fig. 1. Here, graph A gives the result from all events where the total energy could be estimated. Since often only upper or lower limits could be given for the measured quantities, many of the plotted energies represent only limits. Graph B gives the spectrum from 38 events where all experimental quantities could be determined, uncertainties on individual neutron energy determinations varying between ± 0.1 and ± 0.3 Bev. One sees that the apparent maximum at 1.5 Bev in graph A is probably not real, but that the general shape of this spectrum agrees with graph B. The spectrum starts at 1 Bev and rises gradually up to 2.2

TABLE II. Numbers of 3-prong events classified as single or double pion production.

Quality of event	Single pion production (pp−)	Double pion production (pn+−) or (pp−0)	Total
"certain"	18	115	133
"probable"	11	10	21
Total	29	125	154

Bev. The median energy for graph A is 1.72 Bev, and for graph B it is 1.76 Bev. 80 percent of the spectrum lies above 1.4 Bev.

Figure 1 probably does not represent the true spectrum of the incident neutrons since it also depends on the cross sections for producing 3-prong events. Thus the apparent absence of neutrons with energies below 1 Bev may either mean that indeed none are incident or that the cross sections for $(pp-)$, $(pp-0)$, and $(pn+-)$ are quite small below $\sim$1 Bev. Yang[12] has calculated that 2.2-Bev protons colliding with light nuclei should produce a neutron spectrum in the forward direction with a median value near 0.8 Bev, using the statistical theory[6] whose validity is not yet certain. Hill *et al.*[13] have observed a mean neutron energy of 1.4 Bev in a similarly produced neutron beam, indicating the presence of a number of neutrons below 1 Bev. Finally, the incident neutrons must have been produced in reactions similar to those observed in our cloud chamber. The nucleon momentum spectrum from our 154 events given in Part VI contains many nucleons emitted in a forward direction with energies much lower than those of the incident neutrons. Therefore the incident neutron beam should also contain a fairly large number of neutrons with energies far below the 2.2-Bev proton energy producing them.

The laboratory threshold energies for single and double meson production are 0.29 and 0.60 Bev, respectively. Near 0.4 Bev the (single) meson production cross section in nucleon-nucleon collisions is 0.1 to 1.0 millibarns. At energies above 1 Bev the total nucleon-nucleon cross section is $\sim$43 mb.[13] Probably most of this cross section is responsible for meson production. One might expect that after its threshold single production rises to a maximum and then declines as double production becomes more probable, the relative multiplicities perhaps being governed by a statistical model such as considered by Fermi.[6] Thus, if many neutrons were incident below 1 Bev some $(pp-)$ events should have been observed giving rise to inferred neutron energies <1 Bev in our neutron spectrum (Fig. 1). Since none were observed, the cross section for $(pp-)$ below 1 Bev would have to be quite small.

[12] C. N. Yang (private communication).

[13] Hill, Coor, Hornyak, Smith, and Snow, Phys. Rev. **94**, 791 (1954).

V. PION MULTIPLICITIES AND CHARGE DISTRIBUTION

Each 3-prong event was classified as single pion production, $(pp-)$, or double pion production, $(pp-0)$ or $(np+-)$. The numbers of single and double production events are given in Table II. The events whose multiplicities are considered to be established with fair to good certainty, called "certain" events, are tabulated in the first row. Twenty-one additional "probable" cases have been noted, which may fit either classification, but with one or the other the more probable. Each event is counted once. The "probable" group contains relatively many single production events, mostly because separation of $(pp-)$ from $(pp-0)$ is not always definite unless the momenta of all three tracks are well known. One sees that, for our 3-prong events, double production predominates over single production in the ratio 4:1. Since no 5-prong events were seen, triple production has been assumed to be negligible.[14] These results are compared with theoretical predictions in Part VIII.

Each 3-prong event was further classified as $(pp-)$, $(pp-0)$, or $(pn+-)$. The numbers of events in each class are given in Table III. The classifications "certain" and "probable" are defined as before. The number of "probable" events is now larger than in Table II because of the added uncertainties in separating $(pp-0)$ from $(pn+-)$. One sees, as a further result, that production of a positive-negative meson pair is by far the most frequent. The ratio for $(pn+-)$: $(pp-0)$:$(pp-)$ is about 3:1:1. In Part VII a reclassification of events is made which changes these ratios slightly.

To investigate the energy dependence of the multiplicity the events were divided into low- and high-energy groups. The first had incident neutron energies below the median of 1.72 Bev; the second had incident

TABLE III. Numbers of events with different charge distributions observed in n-p collisions.

Quality	(pp−)	(pp−0)	(pn+−)	Total
"certain"	18	5	86	109
"probable"	11	22	12	45
Total	29	27	98	154

TABLE IV. Numbers of events with different charge distributions for low and high energy incident neutrons.

Median energy of incident neutrons	(pp−)	(pp−0)	(pn+−)	Selected events (pp−)	Selected events (pp−0)	Selected events (pn+−)
1.46 Bev	16	11	40	6	3	10
2.04 Bev	13	8	46	6	1	12

[14] As far as the experimental observations are concerned, some 3-prong events could be classified as $(pn+-0)$, $(pp-00)$, or $(nn++-)$. Such a classification, however, would force one to conclude that the reaction leading to $(pp+--)$ is forbidden for some unforeseen reason. It seems more reasonable to conclude that all triple meson production is infrequent.

neutron energies above that value. The median energy for the incident neutrons in the low-energy group is 1.46 Bev; in the high-energy group it is 2.04 Bev. The numbers of $(pp-)$, $(pp-0)$, and $(pn+-)$ events in each group are given in Table IV. The selected events for which neutron energies are known best (as used for graph B in Fig. 1) are tabulated separately.

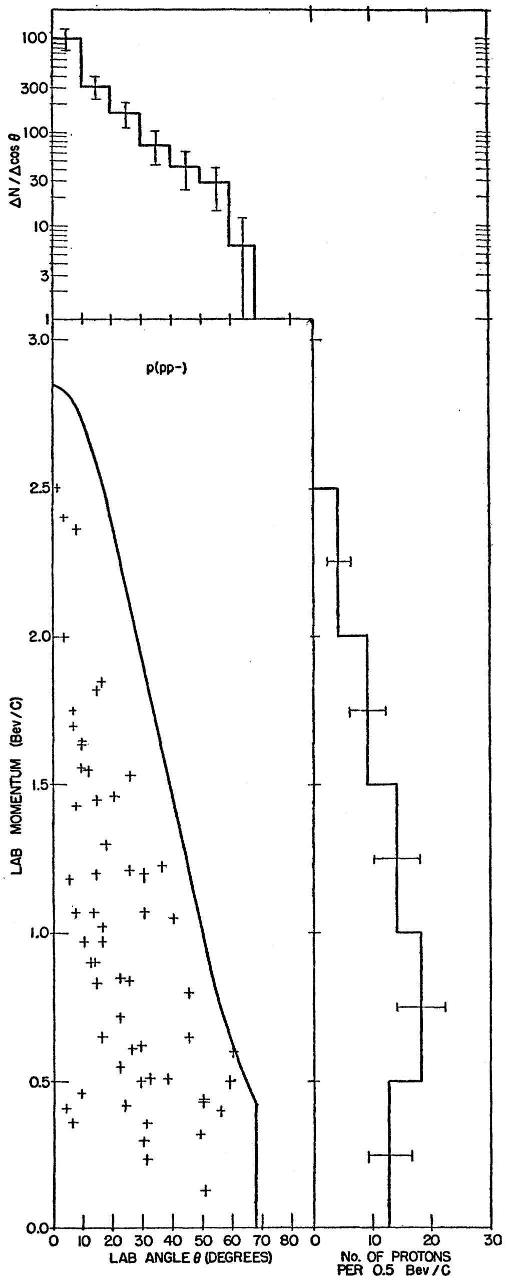

FIG. 2. Laboratory scatter diagram of the protons from the reaction $(pp-)$. At the top the differential angular distribution is plotted for the protons in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. Note the cut-off angle at 68°.

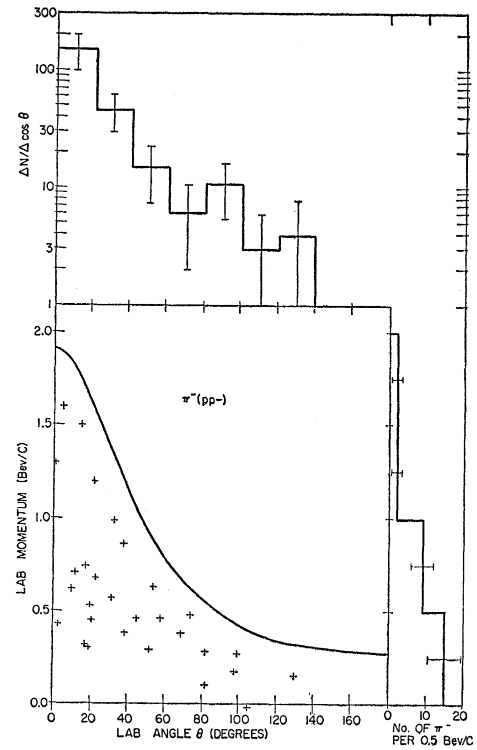

FIG. 3. Laboratory scatter diagram of the π^- from the reaction $(pp-)$. At the top the differential angular distribution is plotted for the π^- in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined.

[One sees that for these selected events the multiplicities are apparently changed in favor of single production. The reason is that incident neutron energies can be calculated for $(pp-)$ more easily than for the other reactions because, for only 3 emitted charged particles, the momentum of one particle can be inferred if the momenta of the two others are known in addition to all of the angles.] We find that below and above 1.72 Bev the relative multiplicities agree within statistical errors for both selected and unselected events, although the slight relative increase of double production events over single production events from 3.2:1 to 4.2:1 may be real. It is surprising that the relative multiplicities have not changed more with energy. This will be discussed further in Parts VIII and IX.

Four of the $(pn+-)$ cases actually were identified as due to the reaction $n+p\rightarrow d+\pi^++\pi^-$. If the "Fermi energy" between bound nucleons is 20 Mev, corresponding to a momentum of 200 Mev/c, then one indeed expects some small percentage of the $(pn+-)$ cases to result in this reaction, particularly since the median momentum of the resulting nucleons in the center-of-mass system (c.m.) given in Part VIII is

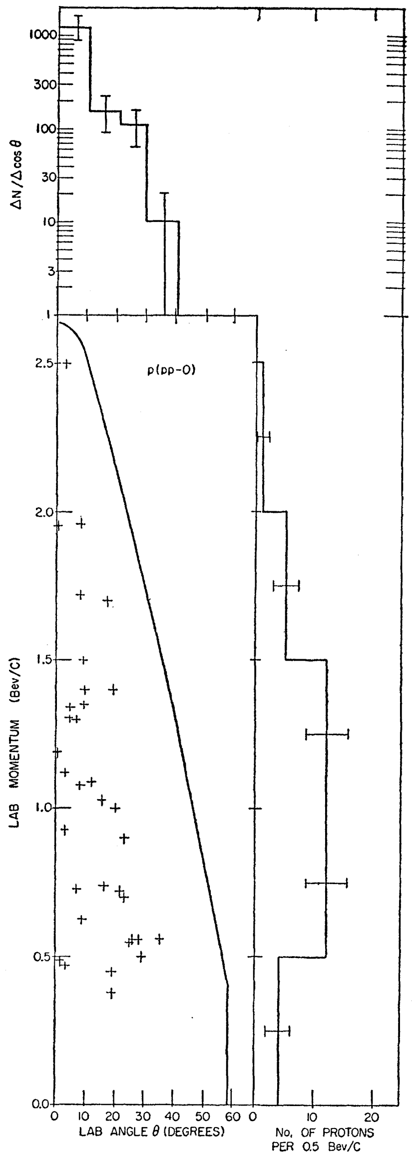

FIG. 4. Laboratory scatter diagram of the protons from the reaction ($pp-0$). At the top the differential angular distribution is plotted for the protons in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. Note the cut-off angle at 58°.

only about 400 Mev/c. Thus a fair amount of phase space should be available where the relative momentum of the two nucleons is ≤ 200 Mev/c, which should be favorable for deuteron formation. This has been treated theoretically by Brueckner and Kovacs.[15] The observed frequency of deuteron formation seems to be less than predicted.

[15] K. A. Brueckner and J. S. Kovacs, Phys. Rev. **94**, 726 (1954).

VI. ANGULAR AND MOMENTUM DISTRIBUTIONS IN LABORATORY SYSTEM

Figures 2 to 10 give scatter diagrams which show laboratory momentum *vs* laboratory angle and angular and momentum distributions for all particles involved in the different reactions. At the top of each scatter diagram the differential angular distribution $\Delta N/\Delta \cos\theta$ is plotted on a logarithmic scale, while at the right side the momentum distribution is given on a linear scale. The statistics are insufficient to warrant angular distributions for certain ranges in momentum, or momentum distributions for certain ranges in angle. *Indications* of such details in structure can be inferred from the scatter diagrams. A curve showing the maximum momentum obtainable at each angle for 2.2-Bev incident neutrons is drawn in each scatter diagram, which also shows the cut-off angle for the nucleons.

Figures 2 to 10 are labeled by writing the reaction from which a particle originates in parenthesis after the symbol for the particle, such as $\pi^-(pn+-)$ for a π^- coming from a $(pn+-)$ event. Furthermore, we will refer to angular distributions by the subscript a and to momentum distributions by the subscript m. Particles for which no momentum could be measured are plotted below the angle coordinate.

Table V gives median values inferred from Figs. 2 to 10. The class $(pn+-)$ contains most of the events

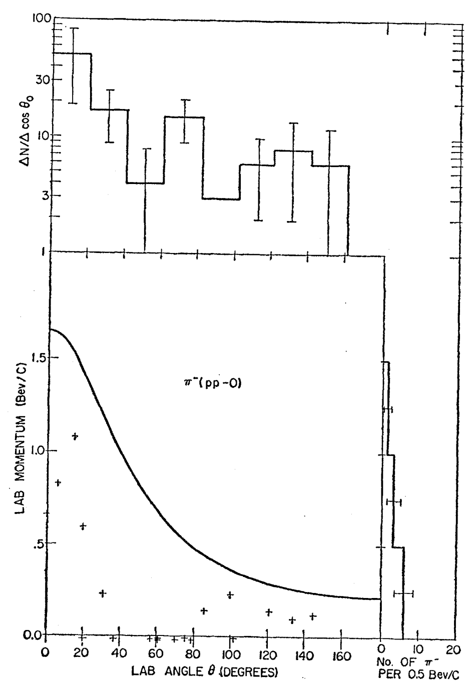

FIG. 5. Laboratory scatter diagram of the π^- from the reaction ($pp-0$). At the top the differential angular distribution is plotted for the π^- in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined.

TABLE V. Median values for momenta and angles observed in the laboratory system.

Particle and reaction	Median momentum (Bev/c)	Median angle
$p(pp-)$	0.95	20°
$\pi^-(pp-)$	0.46	34°
$p(pp-0)$	1.00	10°
$\pi^-(pp-0)$	0.23	61°
$\pi^0(pp-0)$	0.18	61°
$p(pn+-)$	0.82	22°
$n(pn+-)$	1.12	16°
$\pi^+(pn+-)$	0.26	43°
$\pi^-(pn+-)$	0.33	29°

and therefore its median values should be most meaningful. One sees that $p(pn+-)$ has a lower median momentum but higher median angle than $n(pn+-)$. The difference between the angular distributions $p_a(pn+-)$ and $n_a(pn+-)$ will be discussed in Part VII. A similar difference exists between $\pi_a^+(pn+-)$ and $\pi_a^-(pn+-)$. The median momentum for $p_m(pn+-)$ and $n_m(pn+-)$ taken together is 0.96, which agrees well with the corresponding values for $p_m(pp-)$ and $p_m(pp-0)$. As one might expect, the median momentum for $\pi_m^-(pp-)$ is higher than for the doubly produced mesons, since more kinetic energy is available when only one pion is produced.

The median angle for the products of $(pp-0)$ disagree with those for the other reactions. The number of events in this class is relatively small so that the disagreement may be partially due to statistical uncertainties. As mentioned in Part V the $(pp-0)$

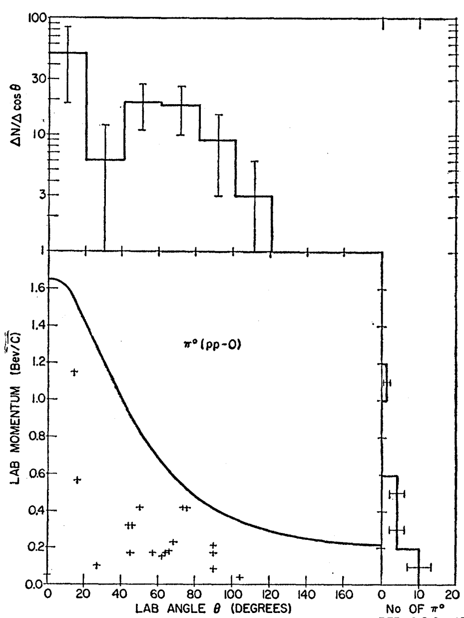

FIG. 6. Laboratory scatter diagram of the π^0 from the reaction $(pp-0)$. At the top, the differential angular distribution is plotted for the π^0 in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev.

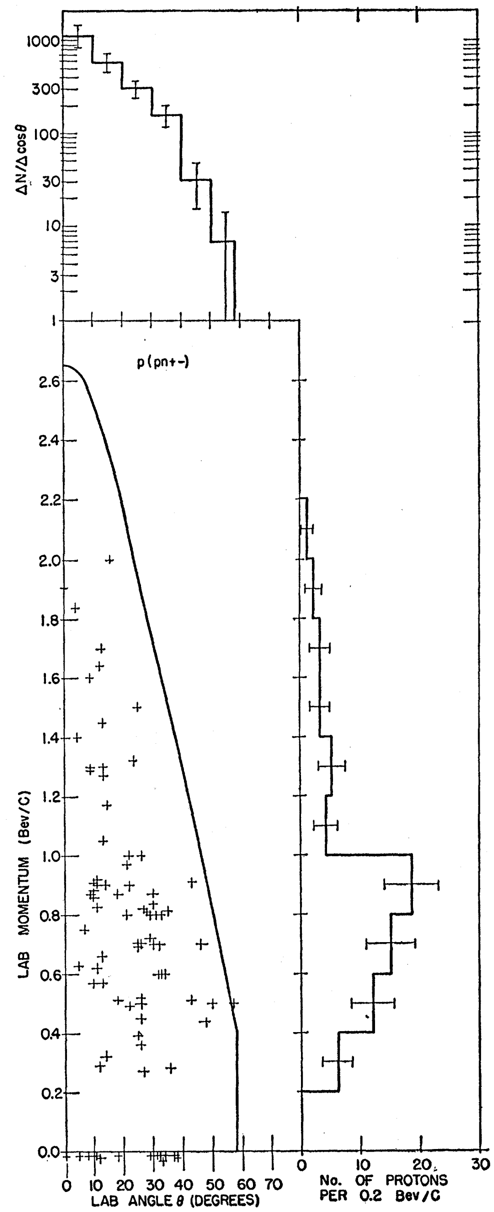

FIG. 7. Laboratory scatter diagram of the protons from the reaction $(pn+-)$. At the top the differential angular distribution is plotted for the protons in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. Note the cut-off angle at 58°. For points plotted under the abscissa the momenta could not be determined.

group contains mostly rather uncertain events. The present disagreement probably indicates that some of the "probable" events in Table III have been misidentified. One might assume that $\pi^-(pn+-)$ and $\pi^-(pp-0)$ should have identical distributions, and also $\pi^+(pn+-)$ and $\pi^0(pp-0)$. Making use of this assumption we shall later reclassify some of the "probable" events. The effect on the general conclusions reached

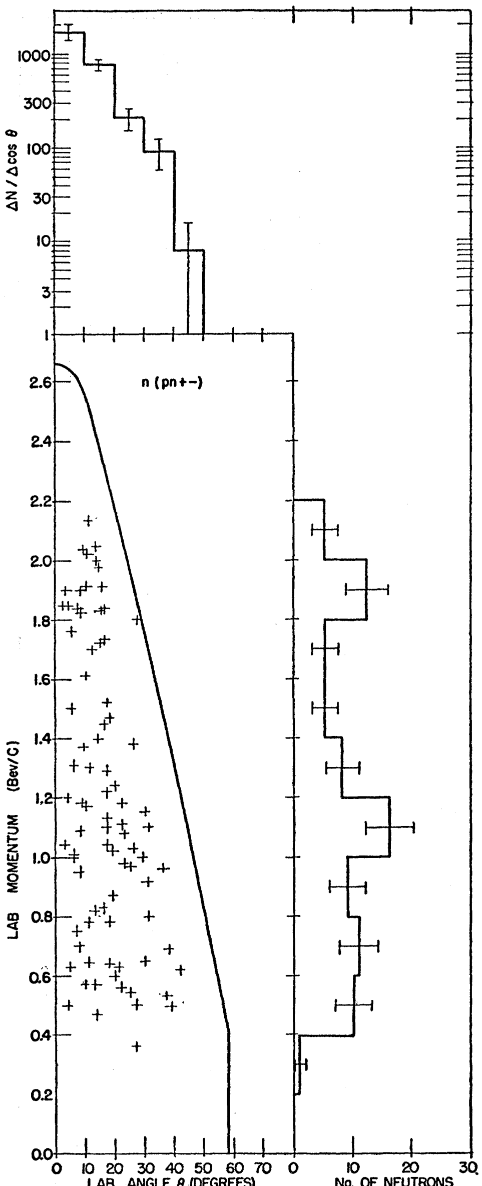

FIG. 8. Laboratory scatter diagram of the neutrons from the reaction ($pn+-$). At the top the differential angular distribution is plotted for the neutrons in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. Note the cut-off angle at 58°.

will be negligible, since the net changes are within the statistical errors.

Since these interactions should be similar to those by which the neutron beam is produced in the Cosmotron target, it is of interest to compare these momentum distributions with those inferred for the neutron beam. For 64 nucleons from all 3-prong events with angles of emission $<10°$, the median momentum is 1.35 Bev/c. For 21 nucleons with angles of emission $<5°$ the median momentum is the same, corresponding to a kinetic energy of 0.70 Bev. Since these values do not differ for the two angular intervals considered one might assume that nucleons emitted at very small angles ($<1°$) have similar momenta. A measure of the amount by which the energy of the incident neutrons is degraded into energy of the nucleons emitted forwards from the 3-prong events would be obtained by dividing this inferred value of 0.70 Bev by the median energy of 1.72 Bev of the incident neutrons. One obtains a ratio r of 0.41. The 2.2-Bev circulating proton beam in the Cosmotron is degraded into forward-going neutrons of median energy of 1.72 Bev,[16] or $r=0.78$, which is almost twice as large as the corresponding ratio for events in the cloud chamber. Neutrons in the beam may, however, have been produced by elastic events with $r=1.0$, while these were not counted in the cloud chamber. Yang has estimated that these might comprise 20 percent of the neutrons in the beam.[12] If these elastically produced 2.2-Bev neutrons are removed from the beam spectrum, its energy is lowered and one obtains $r=0.70$ (in place of 0.78) for comparison with the ratio of 0.41 found for 3-prong events.

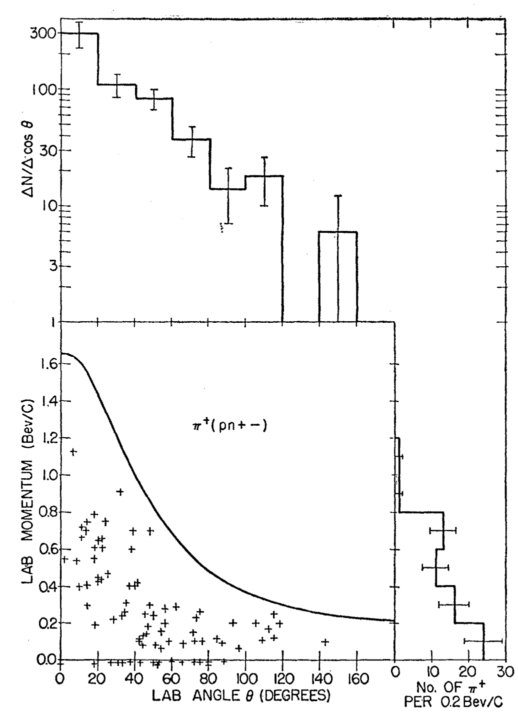

FIG. 9. Laboratory scatter diagram of the π^+ from the reaction ($pn+-$). At the top the differential angular distribution is plotted for the π^+ in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined.

[16] By analogy with the momenta in Table V one might expect that the protons from the interactions in the Cosmotron target would have slightly *higher* momenta than the neutrons.

This argument indicates that the neutron beam spectrum obtained in this way is biassed in favor of high energies and that there should indeed be neutrons of lower energies (less than 1 Bev) in the incident beam, as was mentioned in Part IV.

One therefore infers that the lack of neutrons with energies less than 1 Bev in Fig. 1 is due to the small cross section for producing 3-prong events at such energies. Thus even the single-meson production cross section for $(pp-)$ events must be small for neutrons of less than 1 Bev. Since above 1 Bev the reactions $(pn+-)$ and $(pp-0)$ appear to be more probable than $(pp-)$ there is evidence in these qualitative arguments that the reaction $(pp-)$ is not very likely at any energy. The evidence is only qualitative, however, and rather indirect, so that this conclusion cannot be considered very definite until confirmed by more direct experiments.

VII. ANGULAR AND MOMENTUM DISTRIBUTIONS IN THE CENTER-OF-MASS SYSTEM

From the velocity and angle of emission of a particle in the laboratory system, one can calculate the corresponding quantities in the center-of-mass system (c.m.s.) if the velocity of the incident neutron is known.

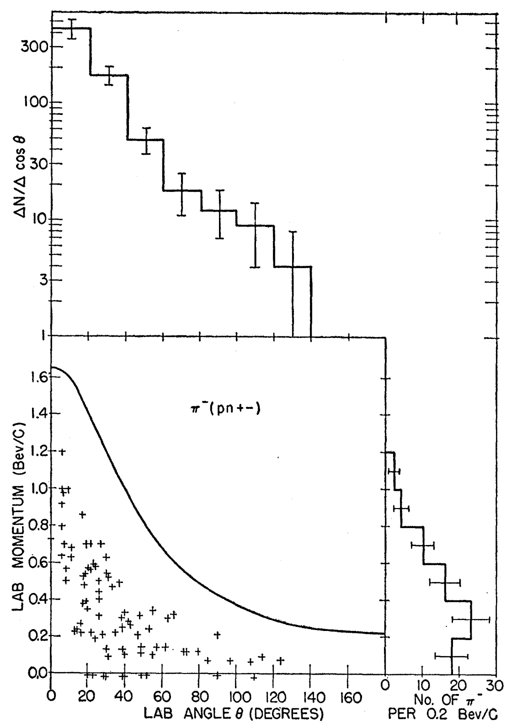

FIG. 10. Laboratory scatter diagram of the π^- from the reaction $(pn+-)$. At the top the differential angular distribution is plotted for the π^- in the laboratory system. At the right side their momentum distribution is shown. The curve drawn in the scatter diagram represents the maximum momenta obtainable at different angles for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined.

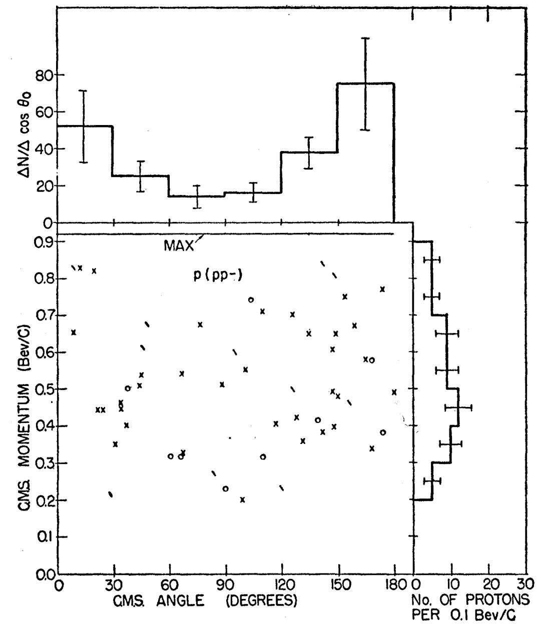

FIG. 11. Center-of-mass scatter diagram of the protons from the reaction $(pp-)$. At the top the differential angular distribution is plotted for the protons in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\).

This transformation is usually quite insensitive to the energy of the incident neutron, since the velocity of the c.m.s. varies quite slowly with this energy. The transformation does become sensitive only when the laboratory velocity of a particle is nearly equal to the velocity of the c.m.s., and simultaneously the laboratory angle of emission is small ($<10°$). The transformation could be carried out with fair certainty for most particles here considered.

Figures 11 to 19 give the c.m.s. scatter diagrams with angular and momentum distributions attached in the same way as in Figs. 2 to 10. The diagrams show the quality of the events in the following manner. Where the energy of the incident neutron could be calculated with some certainty, open circles are shown. (These events had also been used to obtain graph B in Fig. 1.) Where the energy is not known with as good a degree of certainty but the interpretation of the event is certain, vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. Finally, the least certain events are shown as oblique lines (\). Particles whose momenta are not known are again indicated below the angle coordinate. Estimates for the angles for these latter particles could be obtained because from the estimated ionization densities of these particles lower velocity

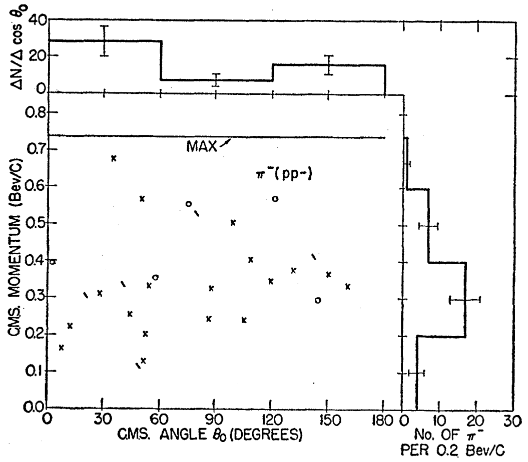

FIG. 12. Center-of-mass scatter diagram of the π^- from the reaction ($pp-$). At the top the differential angular distribution is plotted for the π^- in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\).

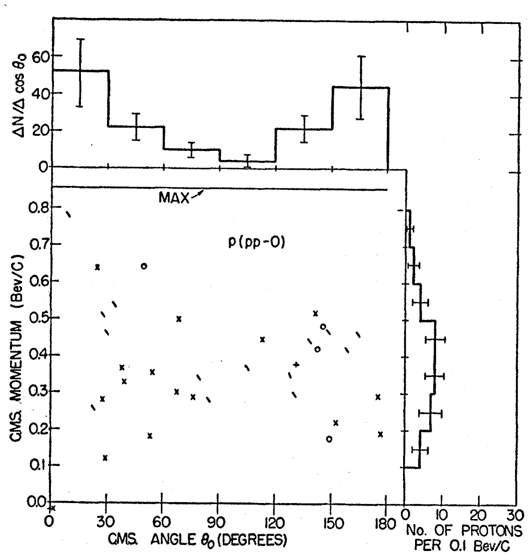

FIG. 13. Center-of-mass scatter diagram of the protons from the reaction ($pp-0$). At the top the differential angular distribution is plotted for the protons in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\).

limits could be inferred. Table VI gives the angular distributions numerically.

We shall first discuss the ($pn+-$) angular distributions which have the best statistics. The distribution

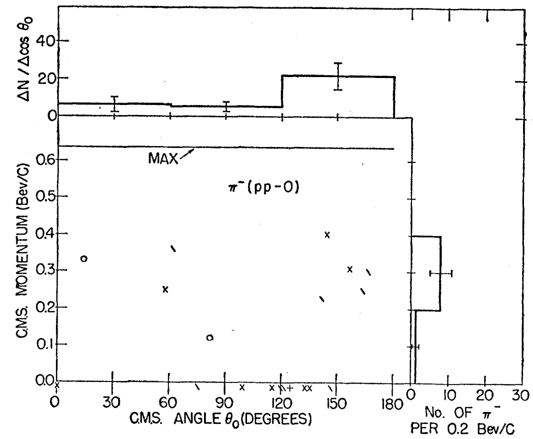

FIG. 14. Center-of-mass scatter diagram of the π^- from the reaction ($pp-0$). At the top the differential angular distribution is plotted for the π^- in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\).

$p_a(pn+-)$ (Fig. 16) indicates that the protons are preferably emitted backward in the c.m.s. On the other hand, forward emission of the neutrons predominates in $n_a(pn+-)$ (Fig. 17). Within the statistical uncertainty the two distributions can be called antisymmetrical with respect to each other.[17] Since it turns out that the nucleons usually carry most of the momentum after emission, conservation of momentum tends to make their angular distributions antisymmetrical. The two distributions are thus statistically dependent. Since $n_a(pn+-)$ is derived from a number of more or less well determined quantities it is not as reliable as $p_a(pn+-)$.

Next, we see that $\pi_a^+(pn+-)$ and $\pi_a^-(pn+-)$ (Figs. 18 and 19) are also antisymmetrical, with the π^+ preferably emitted backward, and the π^- forward, although the effect is not very pronounced. Since the mesons carry only $\frac{1}{2}$ as much momentum on the average as the nucleons, their angular distributions are only slightly statistically dependent. For better statistics we have plotted $\pi_a^-(pn+-)$ against $(180°-\theta_0)$ instead of θ_0 and have added it to $\pi_a^+(pn+-)$, as shown in Fig. 20. On the same figure $p_a(pn+-)$ has been replotted for comparison. The hypothesis of charge independence requires antisymmetry of $\pi_a^+(pn+-)$ and $\pi_a^-(pn+-)$. (Actually only charge symmetry is required for this conclusion.) Antisymmetry of $p_a(pn+-)$ and $n_a(pn+-)$ would also be required on the basis of charge independence, but this feature

[17] By "antisymmetrical" we mean that the value of the first distribution at angle θ_0 is equal to that of the second at $(180°-\theta_0)$.

TABLE VI. Number of particles (ΔN) in each of 3 angular intervals in the center-of-mass system.

	0–60°	60–120°	120–180°
$p(pp-)$	17	16	24
$\pi^-(pp-)$	14	7	8
$p(pp-0)$	16	7	15
$\pi^-(pp-0)$	3	5	12
$\pi^0(pp-0)$	1	4	13
$p(pn+-)$	15	25	39
$n(pn+-)$	33	27	21
$\pi^+(pn+-)$	22	34	35
$\pi^-(pn+-)$	33	39	18
$\pi^-(pp-)$, reclassified	10	7	7
$\pi^-(pp-0)$, reclassified	11	7	10
$\pi^+(pn+-)$, reclassified	23	31	34
$\pi^-(pn+-)$, reclassified	29	37	21

would be expected for purely kinematic reasons, as mentioned above.

While the forward and backward asymmetries for $\pi_a^-(pn+-)$ and $\pi_a^+(pn+-)$, respectively, are not very pronounced, the differential cross section for emitting pions sideways (near 90°) definitely appears to be relatively low.

The distribution $\pi_a^-(pp-)$ appears to be peaked in the forward direction, although the statistical uncertainty is large and the classification of some of the events is uncertain. In that case $p_a(pp-)$ would have to be somewhat peaked backwards to balance the forward momenta of the π^-. This is also indicated. Again both distributions have few particles emitted sideways.

The distribution $p_a(pp-0)$ appears to be symmetrical (with itself) as expected, since here two identical nucleons are emitted which have to conserve most of the momentum, as previously mentioned. $\pi_a^-(pp-0)$ and $\pi_a^0(pp-0)$ are quite different from the other meson distributions inasmuch as both are very strongly peaked backward. Charge independence requires antisymmetry with $\pi_a^+(nn+0)$ and $\pi_a^0(nn+0)$, respectively, which are not known. But one might like to consider a model for double production of mesons where $\pi^-(pp-0)$ is similar to $\pi^-(pn+-)$, and $\pi^0(pp-0)$ is similar to $\pi^+(pn+-)$. Most of the $(pp-0)$ events are difficult to classify (Table III) and the above assumption provides an alternative basis for classifying the $(pp-0)$ events which may be preferable. We shall therefore try to reclassify a number of the "probable" events which are least certain so that consistent distributions are obtained. Table III shows that 11 of the $(pp-)$ events, 22 of the $(pp-0)$, and 12 of $(pn+-)$ are listed as "probable." We are essentially trying to lose backward π^- and to gain forward π^- for the distribution $\pi_a^-(pp-0)$. There are nine $(pp-0)$ events with backward π^-, which could also be $(pn+-)$, and two such events, which could also be $(pp-)$. Next, there are four $(pn+-)$ and four $(pp-)$ events with forward π^-, which could be $(pp-0)$. Transferring all of these events would change the forward-to-sideward-to-backward ratio of $\pi_a^-(pp-0)$

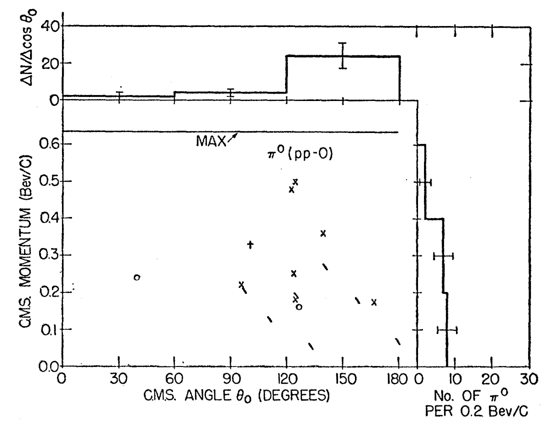

FIG. 15. Center-of-mass scatter diagram of the π^0 from the reaction $(pp-0)$. At the top the differential angular distribution is plotted for the π^0 in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\\).

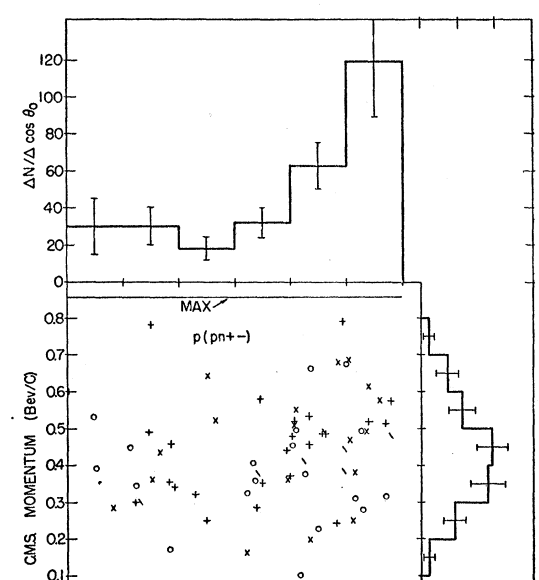

FIG. 16. Center-of-mass scatter diagram of the protons from the reaction $(pn+-)$. At the top the differential angular distribution is plotted for the protons in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\\).

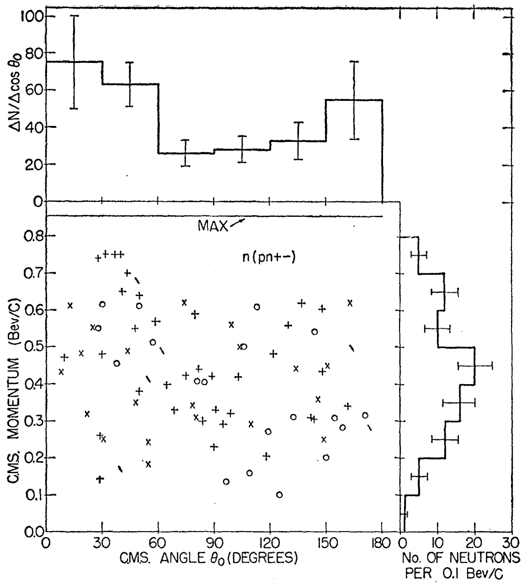

FIG. 17. Center-of-mass scatter diagram of the neutrons from the reaction $(pn+-)$. At the top the differential angular distribution is plotted for the neutrons in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\).

from 3/5/12 to 11/5/1. Thus more than necessary has been accomplished. Furthermore, the nine $(pp-0)$ events transferred to $(pn+-)$ would add nine π^+ (previously recorded as p) to $(pn+-)$ in a forward direction. All of these π^+ have momenta ≥ 1.0 Bev/c in the laboratory system. Figure 10 for $\pi_a^-(pn+-)$ in the laboratory system shows only 3 π^- with momenta >1 Bev, while Fig. 9 for $\pi^+(pn+-)$ shows only 1 with momentum ≥ 1 Bev. Adding nine π^+ with momentum ≥ 1 Bev to $\pi_a^+(pn+-)$ therefore seems unreasonable and, particularly, would distort $\pi_a^+(pn+-)$ in the c.m.s. sufficiently to destroy the antisymmetry with $\pi_a^-(pn+-)$, which is required by the charge independence hypothesis. About the only sufficient compromise satisfying the latter condition is to transfer 3 $(pp-0)$ events to $(pn+-)$, 2 $(pp-0)$ events to $(pp-)$, 7 $(pp-)$ to $(pp-0)$ and 6 $(pn+-)$ to $(pp-0)$. Five of the events then classified as $(pp-0)$ are quite uncertain, however, and it seems wisest to discard them now as "unanalyzable." The resulting "reclassified" meson distributions are given in the last 4 rows of Table VI.

One sees that a reasonable distribution for $\pi_a^-(pp-0)$ has been obtained with negligible effect on the other meson distributions. The effect on the nucleon distributions has also been negligible.

$\pi_a^0(pp-0)$ has not been redetermined since the result is uncertain because of the indefiniteness of the $(pp-0)$ events and the difficulty of determining the angles of neutral particles.

Having omitted 5 of the least definite from our original 154 events we are now left with 24 $(pp-)$, 30 $(pp-0)$, and 95 $(pn+-)$, in the reclassified events. These agree with the original numbers given in Table III within statistics, but may be a better estimate of the true frequencies of interactions leading to $(pp-)$, $(pp-0)$, and $(pn+-)$. This reclassification uses assumptions suggested by charge independence in addition to conservation of energy and momentum for determining the more doubtful events, but some uncertainties still remain.

VIII. COMPARISON WITH THEORY

Theoretical treatments of meson production at high energies have been given by several authors. Heitler and Janossy[5] predict no multiplicities >1 in nucleon-nucleon collisions, Heisenberg and Lewis *et al.*[18] predict multiplicities >1 at energies rather higher than those considered in this experiment. Fermi[6] assumes that the interactions taking part in meson production are so strong that complete thermal equilibrium is obtained before mesons are emitted. Thus it is assumed that energy is distributed statistically between the emitted particles. The interaction volume is given a radius equal to the meson Compton wavelength, but is

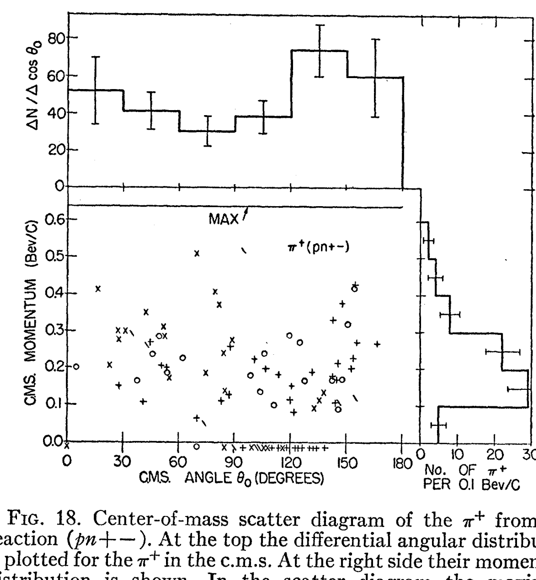

FIG. 18. Center-of-mass scatter diagram of the π^+ from the reaction $(pn+-)$. At the top the differential angular distribution is plotted for the π^+ in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\).

[18] W. Heisenberg, Z. Physik **113**, 61 (1939); Nature **164**, 65 (1949); Z. Physik **126**, 569 (1949); Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).

contracted by the Lorenz transformation. The numbers of available states in phase space that conserve momentum and energy can then be calculated, resulting in relative multiplicities and momentum distributions for the emitted particles. Assuming charge independence Fermi[19] has also calculated the weights for the different charge distributions within each multiplicity state. [This gives, for example, the probabilities of $(pp-)$, $(pn0)$, and $(nn+)$, for single production.] Throughout the theory the probabilities are taken to be proportional to the squares of the individual matrix elements, so that no interference effects are considered. As pointed out to us by Dalitz,[20] if, for instance, the reactions $(pn+-)$ and $(pp-0)$ were to proceed only through certain isotopic spin states and not through all possible ones, different relative occurrences of $(pn+-)$ and $(pn-0)$ would be possible.[21]

Furthermore, angular momentum states are neglected, although Fermi[6] shows that they would have a small effect on the relative multiplicities. The experimental angular distributions discussed in the previous section might be analyzed in terms of emission in s and p states, but since higher angular momenta can also be involved, it does not seem appropriate.

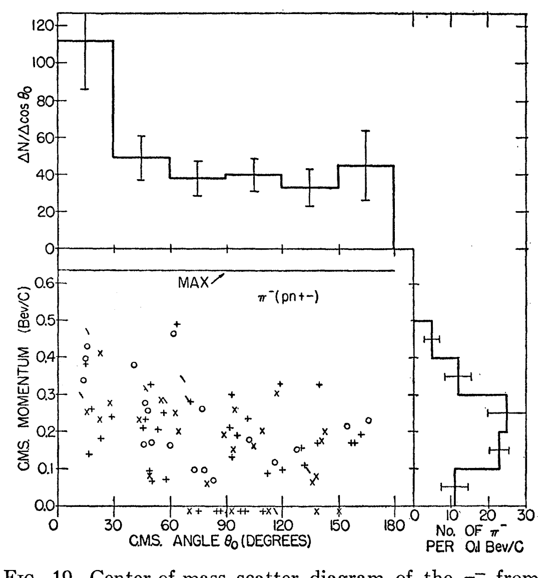

FIG. 19. Center-of-mass scatter diagram of the π^- from the reaction $(pn+-)$. At the top the differential angular distribution is plotted for the π^- in the c.m.s. At the right side their momentum distribution is shown. In the scatter diagram the maximum momentum obtainable is shown for the maximum incident neutron energy of 2.2 Bev. For points plotted under the abscissa the momenta could not be determined. Events where the energy of the incident neutron is known with some certainty are shown as open circles. Where the interpretation of the event is certain but incident neutron energy is less definite vertical crosses (+) are shown. Where the interpretation is less certain oblique crosses (×) are plotted. The least certain events are shown as oblique lines (\\).

[19] E. Fermi, Phys. Rev. **92**, 452 (1953); **93**, 1434 (1954).

[20] R. H. Dalitz (private communication).

[21] For general charge-independence restrictions see, for example K. M. Watson, Phys. Rev. **85**, 852 (1952); and Van Hove, Marshak, and Pais, Phys. Rev. **88**, 1211 (1952).

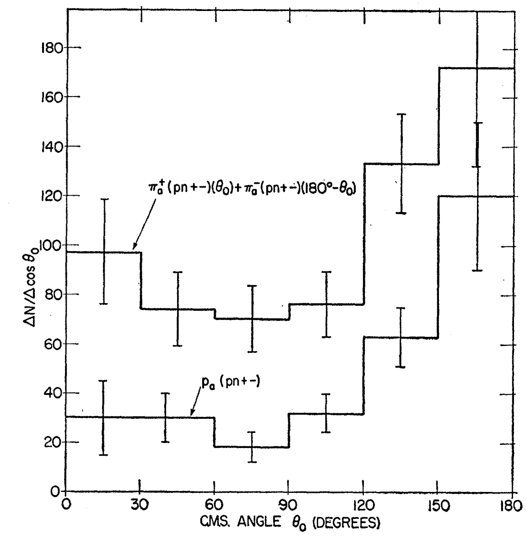

FIG. 20. Differential angular distribution in the center-of-mass system of the π mesons and protons from the reaction $(pn+-)$. For better statistics the π^- distribution has been plotted against 180° minus the c.m.s. angle θ_0 and added to the π^+ distribution.

Detailed predictions could be made only if specific information on angular momentum, spin, and isotopic spin states were available from some meson theory applicable here. In the absence of such a detailed theory any deviations from the general predictions of the statistical theory might point the way to a more detailed picture. Fermi's analytical expressions are valid either for classical or for relativistic energies of the particles involved. Neither case is completely applicable here. Yang and Christian[22] have calculated what would be the predictions of the theory at Cosmotron energies. Their result for the relative probabilities of $(pn+-)$: $(pp-0)$: $(pp-)$ is $\sim$3.3:1:12.2 for 2.2-Bev incident neutrons. One infers that at our median neutron energy of 1.7 Bev this ratio would be $\sim$3.3:1:20.5. Our best experimental ratio as obtained from the considerations in the previous section (reclassified events) is $95:30:24=(3.2\pm0.7):(1\pm0.35):(0.8\pm0.3)$, where the given uncertainties consist of double the statistical standard errors (square root of the number of events) to take account of the uncertainties in identification. One sees that, in the reactions studied here, the ratio of double to single meson production is about 20 times

[22] C. N. Yang and R. Christian, Brookhaven Internal Report. Their procedure consists of numerical evaluation of the expression

$$P_n(p)dp=p^2dp\int\delta(\mathbf{p}+\mathbf{p}_1+\cdots+\mathbf{p}_{n+1})\times\delta(E+E_1+\cdots+E_{n+1}-W)d\mathbf{p}_1d\mathbf{p}_2\cdots d\mathbf{p}_{n+1},$$

where $P_n(p)$ is proportional to the number of mesons emitted with c.m.s. momentum p if n mesons are produced. $\mathbf{p},\mathbf{p}_1\cdots\mathbf{p}_{n+1}$, $E,E_1\cdots E_{n+1}$ are the momentum vectors and energies, respectively, of the $(n+2)$ outgoing particles, and W is the total energy in the c.m.s. The two δ functions under the integral represent surfaces in phase space introduced by the conditions of momentum and energy conservation.

The relative probability for emission of n mesons is given by $f_n=(\Omega^n/n!h^{3n})\int P_n(p)dp$, where Ω is the volume of a sphere of radius $\hbar/\mu c$, contracted relativistically by a factor $2/W$.

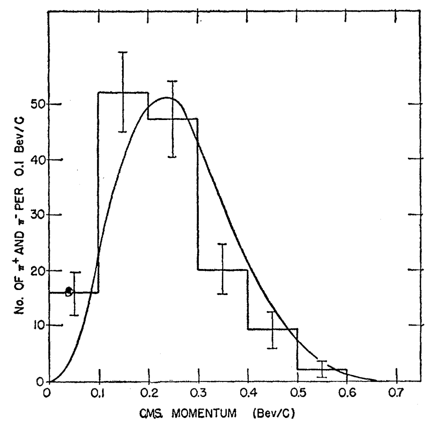

FIG. 21. Momentum distribution in the center-of-mass system of the π^+ and π^- from the reaction $(pn+-)$. The curve represents a theoretical prediction obtained from Fermi's statistical theory, having averaged over our neutron spectrum shown in Fig. 1.

as great as is predicted by the theory. On the other hand, the relative charge distribution for double production alone, as given by the ratio $(pn+-)$: $(pp-0)$, agrees very well with the predicted value. This good agreement is certainly much better than the agreement for the relative multiplicities, but it may be partially fortuitous.

Fermi's treatment[19] of charge independence gives for the relative cross sections for $(pp-)$: $(pn0)$: $(nn+)$ the ratio 1:1.5:1, and for $(pn+-)$: $(pp-0)$: $(nn+0)$: $(pn00)$ the ratio 3.3:1:1:1. From our experimental ratio of 4:1 for $(pn+-)$: $(pp-)$ we then would have for the ratio of the cross section for all double meson production, σ_2, to that for all single meson production σ_1 the value $\sigma_2/\sigma_1=2.2$, while the statistical model predicts $\sigma_2/\sigma_1=1/11$.

The energy dependence of the meson multiplicity seems to be less than expected. According to the statistical theory the ratio σ_2/σ_1 should increase by a factor of about 3 in passing from an incident energy of 1.4 Bev to 2.0 Bev. The experimental results shown in Table IV, however, indicate little change in the σ_2/σ_1 ratio. This conclusion is not certain, however, because the number of cases in each energy group is small and the determinations of incident neutron energy are indirect.

A further prediction of the theory is that only 0.6 percent of all collisions should result in triple production of mesons, or that only one five-prong event $(pp--+)$ should be observed in 300 three-prong events. This prediction follows from the small amount of phase space available for emission of 3 mesons. The fact that we have observed no five-prong events attributable to an n-p collision as against 182 three-prong events certainly agrees with this prediction. This fact makes it difficult to account for the frequent double meson production by effects which would also increase triple meson production, as, for example, by changing the size of the region in which the equilibrium of the statistical theory is reached, or by using the attraction between final state nucleons[15] to enhance multiple meson production.

Since explicit predictions of the statistical theory concerning angular distributions are not available, no comparison can be made. It seems unlikely, however, that these would involve asymmetry about 90° in the c.m.s. as seems to be shown in the experimental distributions.

Yang and Christian[22] have also calculated momentum distributions for the emitted mesons. These have been averaged over our neutron energy spectrum (graph B, Fig. 1) and reproduced in Fig. 21, together with the experimental distributions $\pi_m^+(pn+-)$ and $\pi_m^-(pn+-)$ added together. (As mentioned before, these two distributions are fairly independent kinematically because most of the momentum is carried by the nucleons.) One sees that fair agreement is obtained, although some experimental shift toward lower momenta may be indicated. Yuan and Lindenbaum have observed similar c.m.s. spectra for pions from the Cosmotron target.[23] They have interpreted the spectra and positive excess as evidence that multiple pion production predominates at 2.2 Bev, in agreement with our conclusions.

The median momentum of $\pi_m^+(pn+-)$ is 0.21 Bev/c, which agrees within statistics with the value of 0.22 Bev/c for $\pi_m^-(pn+-)$. For the protons as well as the neutrons from $(pn+-)$ one obtains a median momentum of 0.43 Bev/c. Thus the nucleons carry considerably more momentum than the mesons. The corresponding kinetic energies are 112, 120, 95, and 95 Mev for π^+, π^-, p, and n, respectively. Thus equipartition of the available energy is indicated for double meson production, in agreement with one of the original assumptions of the theory. The mean meson energy is

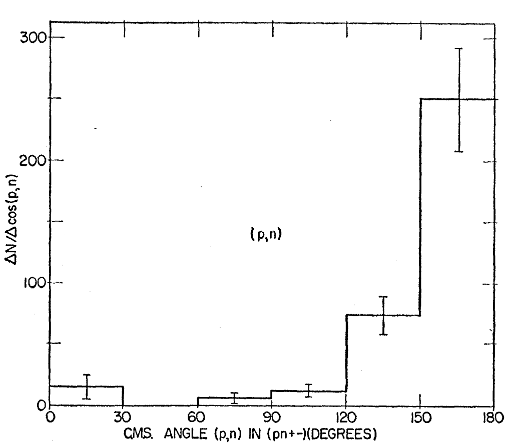

FIG. 22. Differential distribution of the angles between proton and neutron directions in the center-of-mass system, from the reaction $(pn+-)$.

[23] L. C. L. Yuan and S. J. Lindebaum, Phys. Rev. **93**, 1431 (1954).

expected to be higher than the mean nucleon energy, since the meson energy is somewhat relativistic. (Adding all of these energies to twice the rest energy of the meson one obtains 700 Mev, in sufficient agreement with the energy of 720 Mev available in a collision of a neutron of median energy of 1.72 Bev with a proton. Since the energies of neutral particles have been calculated from energy and momentum balance, this serves to check the consistency of the calculations. The two energy values were obtained as the result of very different averaging procedures so that some discrepancy is quite expected.)

The distribution $\pi_m^-(pp-)$ is statistically too poor to compare with any theoretical predictions. The median momentum of the π^- from $(pp-)$ is 0.33 Bev/c.

IX. DISTRIBUTIONS OF THE ANGLES BETWEEN INDIVIDUAL PARTICLES

We have seen that the only serious disagreement between the statistical theory and experiment is the large experimental preference for double meson production. In addition the apparent asymmetry of the c.m.s. angular distributions seems inconsistent with the ideas of the purely statistical theory. Perhaps these observations indicate that the interactions are really determined by certain specific forces between the particles or by certain specific intermediate states, rather than by statistical considerations. In order to investigate these possibilities and to gain more insight into the processes at hand we have calculated angles between individual particles (p,n), (π^+,π^-), (p,π^+), (p,π^-), (n,π^-), and (n,π^+) emitted in the reaction $(pn+-)$. The results are given in Figs. 22 to 25, and in Table VII.

The distribution (p,n) shown in Fig. 22 can be explained merely by kinematics. Since the nucleons carry most of the momentum they are also mostly responsible for conserving momentum. Thus usually a neutron must be emitted rather opposite to a proton. The distribution (π^+,π^-) given in Fig. 23 shows some preference for emission of the two mesons at relative angles greater than 90°. If the mesons were emitted independently (and isotropically) from an infinite mass, the distribution (π^+,π^-) should be independent of the relative angle. Since, however, the mesons carry an average of about one-half the momentum carried by the nucleons they are also somewhat responsible for momentum conservation, and therefore emission at relative angles >90° should be somewhat prevalent for kinematic reasons. This qualitative agreement with a purely statistical picture, however, suggests no particularly strong interaction between the mesons produced in the n-p collisions. If this interaction were very attractive the mesons should have a greater tendency to be emitted at relative (π^+,π^-) angles <90°, and if the interaction were very repulsive they should more often be emitted in opposite directions. Whether this means that mesons are not strongly coupled to mesons or that the presence of the two nucleons interferes sufficiently to prevent any obvious effects due to meson-meson interaction remains to be seen.

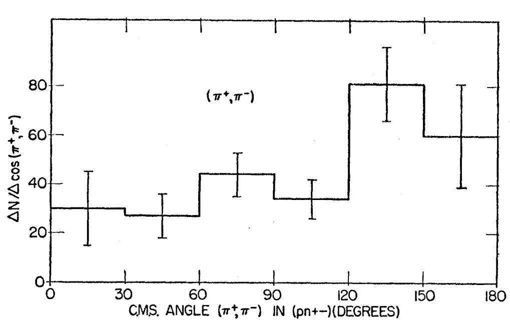

FIG. 23. Differential distribution of the angles between π^+ and π^- directions in the center-of-mass system, from the reaction $(pn+-)$.

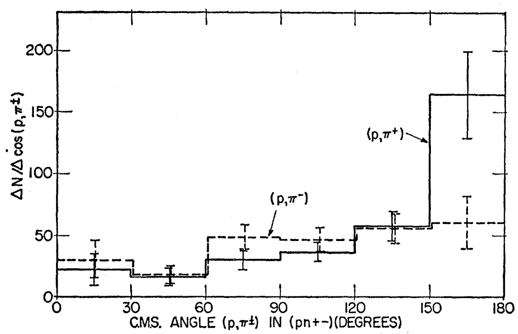

FIG. 24. Differential distribution of the angles between proton and π^+ directions, and proton and π^- directions, respectively, in the center-of-mass system, from the reaction $(pn+-)$.

That such an interference effect may indeed exist is indicated by Fig. 24, showing the distributions of the angles for (p,π^+) and (p,π^-). One sees that these two distributions differ from each other inasmuch as there appears to exist some preference for the π^+ to be emitted opposite the p, while no such preference beyond what might be expected purely statistically exists for the π^- relative to the p. The distribution (p,π^+) is all the more surprising because, as shown previously, both the p and the π^+ tend to be emitted backward in the c.m.s. Yet, by considering individual events in the present manner, we obtain the additional tendency for p and π^+ to be emitted in opposite directions. Charge independence requires that the distribution of the angles for (n,π^-), compared with that for (n,π^+), show the same preference for large angles as that for (p,π^+) when compared with (p,π^-). This is not evident from Fig. 25. However, one must remember that the momenta and angles for the neutrons were all derived from those for the charged particles. Therefore, the neutron angles are usually not very well known, except for the selected events where the momenta and angles of the charged particles are fairly well determined. Table VII gives the distributions con-

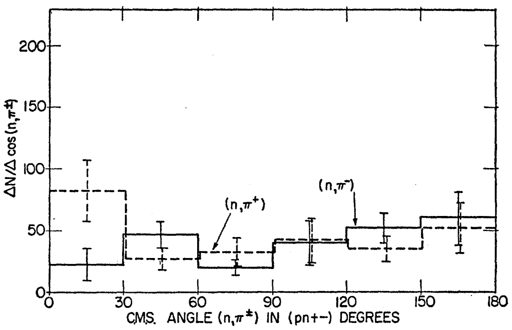

FIG. 25. Differential distribution of the angles between neutron and π^+ directions, and neutron and π^- directions, respectively, in the center-of-mass system, from the reaction $(pn+-)$.

sidered here for these selected events only. One sees that here the distributions for (n,π^-), (n,π^+) agree within statistics with the distributions (p,π^+), (p,π^-), respectively.

The present experiment is not good enough to ascertain that the effect discussed in the previous paragraph is not just due to some experimental systematic error or statistical fluctuation. Therefore deductions based on it are highly speculative until the experimental facts have been determined with greater certainty. If taken at face value, however, it would appear to indicate a strong pion-nucleon interaction or intermediate state in which pion and nucleon were combined. Observations on pion-nucleon scattering have given evidence for strong interaction in a state with angular momentum of $\frac{3}{2}$ and isotopic spin of $\frac{3}{2}$.[1] Such a state can be considered as an excited nucleon or compound state of the pion-nucleon system with an energy about 160 Mev (corresponding to the maximum cross section observed for π^+ with energies near 200-Mev laboratory energy scattered by protons) above the sum of nucleon and pion rest energies. One might imagine that pion production proceeds through such an intermediate state and that for double meson production two such excited nucleons are produced which separate by a distance of perhaps up to one nucleon diameter before each decays to nucleon and pion, thus often decaying almost freely. Peaslee has considered the consequences of such intermediate states for pion multiplicities, assuming charge independence.[24]

We can account for the angular correlations observed

TABLE VII. Relative distributions of c.m. angles (n,π^-) and (n,π^+), and (p,π^+) and (p,π^-), respectively, obtained from selected $(pn+-)$ cases only. Actual numbers of cases observed are given.

	0°–60°	60°–120°	120°–180°
$(n,\pi^-)/(n,\pi^+)$	3/7	6/6	8/4
$(p,\pi^+)/(p,\pi^-)$	2/5	5/8	11/5

[24] D. C. Peaslee, Phys. Rev. **95**, 717 (1954).

for the reaction $(pn+-)$ if the original proton then assumes the intermediate state of a doubly charged positive particle ("isobar" with charge component $+\frac{3}{2}$) which subsequently decays into a proton and a π^+, while the original neutron in its intermediate state becomes a singly charged negative particle (charge component $-\frac{3}{2}$) which decays into a neutron and a π^-. Both p and π^+ should then show a tendency to be emitted into the same hemisphere, while n and π^- should both be emitted into the opposite hemisphere. This agrees with the experiment (Figs. 16–19). Furthermore, if the excitation energies are high enough the observed average angles between n and π^-, and p and π^+, respectively, can be large, while the average angles between n and π^+, and p and π^-, respectively, may be smaller, the latter angles being determined only by statistical correlation between the decay planes of (n,π^-) and (p,π^+). This also agrees with some of the observations (Fig. 24, Table VII). As a matter of fact, considering that the observed large probability for double meson production may result from the possible resonance of the excited states one may say that this model fits all of the qualitative experimental features quite strikingly.

Peaslee deduces from this experiment that $\sigma_1 \approx \sigma_2$ at our energies. He also shows that in this model the cross section for the reaction $(pp-)$ is only $\frac{1}{6}$ of the single meson production cross section σ_1. This might help to explain the lack of $(pp-)$ events caused by low-energy neutrons noted in Part IV.[25]

An attempt was made to detect excitation energies for the nucleons in the intermediate states by calculating "Q values" for the combinations (n,π^-), (n,π^+), (p,π^+), and (p,π^-), in a manner identical to that used for calculating "Q values" for heavy unstable particles. The results are given in Figs. 26 and 27 and, for the selected events, in Table VIII. One might expect that the calculated "Q values" for (p,π^+) and (n,π^-) would be grouped around 160 Mev if the present model is valid, while the values for (p,π^-) and (n,π^+) should show no such grouping. No obvious differences are observed in the Q distributions, within the experimental uncertainties, but perhaps one should not expect any very definite indications. The observed maximum in the π^+-p scattering cross section is quite broad. Thus no sharp "Q values" should be expected here. Furthermore, of course, the present model is much over-

[25] For incident neutron energies <1 Bev double meson production is probably not likely, while single production may not be very likely either because elastic scattering becomes more probable at lower energies. One might also argue that σ_1 cannot be substantial until enough energy is available to supply excitation energies near 160 Mev in addition to the pion rest mass and some kinetic energy of the excited nucleon in the c.m.s. This may amount to 0.34 Bev, corresponding to 0.75 Bev for the incident nucleon. Therefore, the average value of σ_1 below 1 Bev may be fairly small, and, if only $\frac{1}{6}$ of the meson production events resulted in the reaction $(pp-)$, very few $(pp-)$ events should be expected below 1 Bev unless most incident neutrons have low energies. The fact that no $(pp-)$ cases were observed below 1 Bev, therefore, would not constitute a particular discrepancy any longer.

simplified, since states with isotopic spins other than $\frac{3}{2}$ are probably also possible and since the excited nucleons probably cannot travel far enough during their lifetime to enable them to decay as completely free particles. In addition, uncertainties in measured momenta and angles might lead to errors in the calculated "Q values." For these reasons any existing specific excitation energies would be much obscured.

X. COMPARISON WITH A PRELIMINARY p-p SCATTERING RESULT

For further comparison of the discussed models for meson production it may be of interest to state a preliminary result on p-p interactions obtained under similar conditions. Here a beam of 1.5-Bev protons was allowed to enter the hydrogen-filled cloud chamber.

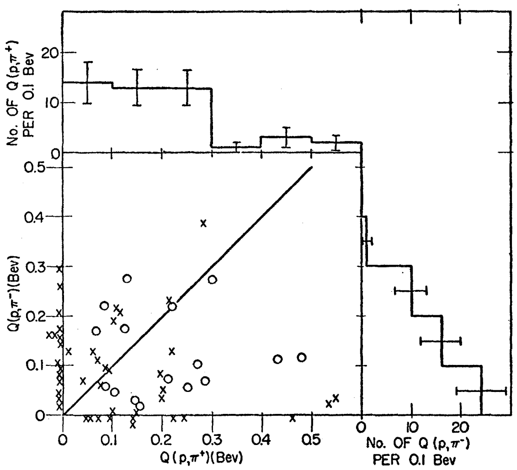

FIG. 26. Scatter diagram of "Q values" obtained for protons and π^+, and protons and π^-, respectively, from the reaction ($pn+-$). At the top of the scatter diagram the distribution of "Q values" for protons and π^+ is given, while at the right side the distribution of "Q values" for protons and π^- is plotted. Open circles represent the energetically most certain events while oblique crosses (✕) are indicated for the less certain events. Where "Q values" could be determined only for either proton and π^+ or proton and π^- alone, these are indicated under the abscissa or to the left of the ordinate, respectively.

The result to be considered is that out of 160 interactions in the gas only two resulted in 4 outgoing prongs to be interpreted as due to the reaction $p+p\rightarrow p+p+\pi^++\pi^-$, ($pp+-$). Unless the reaction ($pp+-$) is specifically excluded, this seems to indicate that for p-p interactions double meson production is much less likely than for n-p collisions.

Fermi's statistical model[6,12,22] predicts that for 1.5-Bev protons incident on protons the reaction ($pp+-$) should occur in 1 to 2 percent of all interactions. This would agree with the stated result. However, for n-p collisions we had found that double meson production is more than 20 times larger than predicted by the Fermi model.

From Fermi's weights for the different neutron-proton reactions within each state of meson multiplicity

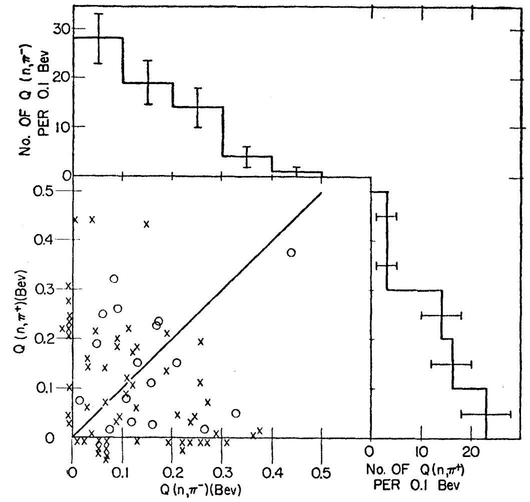

FIG. 27. Scatter diagram of "Q values" obtained for neutrons and π^- and neutrons and π^+, respectively, from the reaction ($pn+-$). At the top of the scatter diagram the distribution of "Q values" for neutrons and π^- is given, while at the right side the distribution of "Q values" for neutrons and π^+ is plotted. Open circles represent the energetically most certain events while oblique crosses (✕) are indicated for the less certain events. Where "Q values" could be determined only for either neutron and π^- or neutron and π^+ alone, these are indicated under the abscissa or to the left of the ordinate, respectively.

one expects for the ratio of the cross section for double meson production to that for single meson production the relation $\sigma_2/\sigma_1=[\sigma(pn+-)+\sigma(pp-0)]/2.3\sigma(pp-)$. From our experiment we obtain $\sigma_2/\sigma_1=2.2$ for incident neutrons with median energy of $\sim$1.7 Bev, while the Fermi model predicts $\sigma_2/\sigma_1=0.09$. This discrepancy has been discussed in Part VIII. If the cross sections for p-p collisions are assumed to be the same as those for n-p collisions, as is indicated experimentally,[13] we would expect that for 1.5-Bev protons σ_2/σ_1 might be $\approx$1.8, having made allowance for the theoretical energy dependence of the relative multiplicities.

Out of the 160 p-p interactions as many as 70 may be elastic, leaving 90 interactions resulting in meson production. From the assumed ratio $\sigma_2/\sigma_1=1.8$ it follows that 58 events should have resulted in double meson production. Since furthermore, from Fermi's treatment of charge independence, $\sigma(pp+-)/\sigma_2=0.3$ we should have observed 17 ($pp+-$) events, instead of 2, in the p-p experiment, if Fermi's treatment of charge independence is to apply consistently to n-p and p-p collisions.

TABLE VIII. Distributions of "Q values" for the combinations (n,π^-), (n,π^+), (p,π^+), and (p,π^-) for ($pn+-$), for selected events alone.

Q value (Bev)	0–0.1	0.1–0.2	0.2–0.3	0.3–0.4	0.4–0.5
(n,π^-)	6	7	2	1	1
(n,π^+)	7	4	4	2	0
(p,π^+)	3	5	5	1	2
(p,π^-)	7	5	4	0	0

Making use of the model discussed by Peaslee[24] we find the following. Here, for *n-p* collisions, σ_2/σ_1 is approximately given by $[\sigma(pn+-)+\sigma(pp-0)]/4.8\sigma(pp-)$. Furthermore, $\sigma(pn+-)/\sigma(pp-0)=41$ if the reaction proceeds through a state of isotopic spin $T=1$, and $\sigma(pn+-)/\sigma(pp-0)=5$ if $T=0$. We shall merely take an average of $\sigma(np+-)/\sigma(pp-0)\approx 11$ assuming that both states occur with equal probability. The first question we shall try to answer is whether it is possible to redistribute our events so that the latter condition is met. (Previously the result fitted the prediction of the Fermi model that $\sigma(np+-)/\sigma(pp-0)\approx 3$.) Because of the uncertainty of the $(pp-0)$ events it is possible to obtain for $(pn+-):(pp-0):(pp-)$ the distribution 107:14:33, and, by going to the limit of all uncertainties, even 104:9:41. (We are neglecting here the consequences of this redistribution on the angular distributions.) Thus a ratio $\sigma(pn+-)/\sigma(pp-0)\approx 10$ is experimentally not impossible in agreement with the theoretical ratio for the present model. From the given numbers for the *n-p* experiment we now obtain $\sigma_2/\sigma_1\approx 0.7$ at 1.7 Bev, or $\sigma_2/\sigma_1\approx 0.55$ at 1.5 Bev, allowing for a theoretical energy dependence of σ_2/σ_1. Using this for a prediction for the *p-p* experiment we obtain the result that 32 out of the 90 inelastic *p-p* events should have resulted in double meson production.

Finally, for the model under consideration one should have $\sigma(pp+-)/\sigma_2=0.2$. Therefore, for this model about six $(pp+-)$ events should have been observed in the *p-p* experiment, if consistency with the *n-p* experiment is postulated. The disagreement with the 2 events actually found is not bad, and this might perhaps be taken as further evidence in support of the discussed model.

XI. SUMMARY OF CONCLUSIONS

The following conclusions have been drawn from an analysis of 154 three-prong events due to *n-p* interactions with incident neutron energy in the range 1.0 to 2.2 Bev (median energy 1.7 Bev):

The ratio of double meson production to single meson production is about 20 times as great as predicted by Fermi's statistical theory. For example, the experimental ratio of $(pn+-)$ to $(pp-)$ is $(3.2\pm0.7)/(0.8\pm0.3)$, while the ratio from the statistical theory is 3.3/20.5. On the other hand no $(pp--+)$ events were found, so that triple meson production is probably negligible, as predicted by the statistical theory. These facts suggest that some "resonance" interaction may lead to double production of mesons. Indirect evidence suggests that the cross section for $(pp-)$ is quite low at energies below 1 Bev as well.

The experimental ratio of $(pn+-)$ to $(pp-0)$ is $(3.2\pm0.7)/(1\pm0.35)$, however, which agrees well with the statistical theory prediction of 3.3/1 based on an application of the hypothesis of charge independence. Similarly the momentum distributions found for π^- and π^+ from $(pn+-)$ events agree with the theory fairly well.

The angular distributions of the emitted particles show rather definite asymmetries (in the c.m.s.) which seem inconsistent with a purely statistical theory. In $(pn+-)$ reactions, which are the most numerous, protons show a tendency for backward emission while neutrons tend to be emitted forwards (in the c.m.s.). Similarly, the π^+ have a backwards tendency and the π^- forwards.

In the $(pn+-)$ reactions the angles between π^+ and π^- show no correlation beyond what can be explained by kinematics. Thus, no strong interaction between the mesons emitted here is indicated. The distribution of the angles between p and π^+ shows a preference for emission in opposite directions while the corresponding distribution for p and π^- does not show this preference. This is also indicated, although less definitely, by the corresponding distribution for n and π^- compared with that for n and π^+. This suggests the hypothesis that separate excited states of the two nucleons may be produced, with subsequent transition to the nucleon ground states by meson emission. This picture might be consistent with the observed strong preference for double meson production, as well as the angular distributions of the emitted particles. No specific excitation energy is found.

We are indebted to the Cosmotron staff for providing us with very reliable operation of the machine, and to the other members of the cloud chamber group for their effective help in operating the cloud chamber equipment. M. R. Burns and F. S. Keene have aided us considerably by scanning most of the photographs.