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The Ground States of Odd-Odd Nuclei

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Applications of techniques already developed for the study of atomic spectra to calculations on odd-odd nuclei give some particularly simple results. The spins of the ground states of nearly all odd-odd nuclei measured can be accounted for, and the validity of Nordheim's empirical rules is discussed.

INTRODUCTION

It was found by Nordheim[1] that the total spin J of the ground state of odd-odd nuclei could be given by the rules:

$$J=|j_p-j_n|, \quad \text{if } l_p+j_p+l_n+j_n \text{ is even,}$$
$$|j_p-j_n|<J\lesssim j_p+j_n, \quad \text{if } l_p+j_p+l_n+j_n \text{ is odd,}$$

where j_p, j_n are the individual spins of the odd proton and odd neutron involved, and l_p, l_n their respective orbital quantum numbers as determined from the shell model most successful in the study of odd-even nuclei. The calculation of de-Shalit[2] on nuclei with one odd proton and one odd neutron outside closed subshells has given a theoretical basis for Nordheim's rule in this restricted case. Other calculations in special cases have been carried out by Kurath[3] and by Flowers.[4]

To discuss the general odd-odd nucleus with n_1 equivalent particles of one kind and n_2 equivalent particles of the other kind all outside closed subshells, we shall adopt the "odd group model." In this approach it is assumed that the particles in the two groups, i.e., protons and neutrons, first interact among themselves to give in their lowest states some well-defined resultant group angular momenta J_1 and J_2. It is expected that this approach, neglecting all but the ground configurations of the separate groups, should be valid for the determination of only the very lowest levels of the combined configuration.[2]

THE INTERACTION MATRIX

We shall start with the zeroth-order wave function of the Mayer-Jensen j-j coupled independent particle scheme with the two groups separately coupled to well-defined resultant angular momenta. The configuration of the n_1 particles of the first group is characterized by the quantum numbers l_1, j_1, J_1; that of the n_2 particles of the second group by l_2, j_2, J_2.

The wave function for the combined configuration with total spin J and magnetic quantum number M is then written:

$$\psi(j_1^{n_1}J_1,j_2^{n_2}J_2,JM)=\sum_{M_1M_2}\psi_1(j_1^{n_1}J_1M_1)\psi_2(j_2^{n_2}J_2M_2) \times(J_1M_1J_2M_2|J_1J_2JM).$$

Here we write the sequence $j_1^{n_1}J_1$ as meaning n_1 equivalent particles each of spin j_1 coupled to a resultant spin J_1; $j_2^{n_2}J_2$ is similarly read. This form has no special symmetries regarding interchange of protons and neutrons, which are thus considered as two distinct particles. Whatever the actual relation between proton and neutron may be, our approximation is certainly valid for heavier nuclei where the total isotopic spin is not a good quantum number.

At this point all the different levels of the total spin J are degenerate. Now introducing an attractive short-range interaction between particles of group one and those of group two, we shall calculate by perturbation theory the first-order energy shifts and see which state of total J is pushed deepest.

Taking the interaction between protons and neutrons as a sum of two-body static potentials, we have

$$V_{12}=\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}V_{ij}, \tag{1}$$

where the indices i and j count particles in the first and second groups, respectively. Now, according to the techniques developed by Racah,[5] the two-particle interaction may be decomposed as

$$V_{ij}=-\sum_k f_k(r_i,r_j)t_i^{(k)}\cdot t_j^{(k)}. \tag{2}$$

Here $t_i^{(k)}$ is a tensor operator with respect to j_1 (thus also with respect to J_1 and J), of rank k, which operates only on the angle and spin coordinates of particle i. $t_j^{(k)}$ is similarly defined with respect to particle j, and $t_i^{(k)}\cdot t_j^{(k)}$ is the scalar product of the two tensors.

Since both groups contain equivalent particles, the radial part of the matrix element of every term in the double sum (1) and (2) will have the same value for a given k, namely

$$F_k=\int_0^\infty\int_0^\infty |R_1|^2|R_2|^2 f_k(r_1,r_2)r_1^2r_2^2dr_1dr_2, \tag{3}$$

where R_1 is the total radial part of the wave function characterized by l_1 and the principal quantum number of the state in question.

[1] L. W. Nordheim, Phys. Rev. **78**, 294 (1950).
[2] A. de-Shalit, Phys. Rev. **91**, 1479 (1953).
[3] D. Kurath, Phys. Rev. **87**, 218 (1952) and **91**, 1430 (1953).
[4] B. H. Flowers, Proc. Roy. Soc. (London) **A212**, 248 (1952).
[5] G. Racah, Phys. Rev. **62**, 438 (1942).

If we now set

$$T_1^{(k)}=\sum_{i=1}^{n_1} t_i^{(k)} \quad\text{and}\quad T_2^{(k)}=\sum_{j=1}^{n_2} t_j^{(k)}, \tag{4}$$

Eqs. (1), (2), (3) combine to give

$$V_{12}=-\sum_k f_k(r_1,r_2)T_1^{(k)}\cdot T_2^{(k)}. \tag{5}$$

For the first-order energy shifts we take the diagonal matrix element of (5). Using (3) we obtain:

$$E(j_1^{n_1}J_1,j_2^{n_2}J_2,J)=-\sum_k F_k(j_1^{n_1}J_1,j_2^{n_2}J_2,JM \times|T_1^{(k)}\cdot T_2^{(k)}|j_1^{n_1}J_1,j_2^{n_2}J_2,JM). \tag{6}$$

Now since $T_1^{(k)}$ operates only on particles of group one and $T_2^{(k)}$ only on particles of group two, we can use a well-known theorem of Racah[5] to decompose this matrix element (6) of $T_1^{(k)}\cdot T_2^{(k)}$ into the reduced matrix elements:

$$(-1)^{J_1+J_2-J}(j_1^{n_1}J_1\|T_1^{(k)}\|j_1^{n_1}J_1)(j_2^{n_2}J_2\|T_2^{(k)}\| \times j_2^{n_2}J_2)W(J_1J_2J_1J_2;Jk), \tag{7}$$

where W is a Racah coefficient.[5]

If, for the j-j coupled[6] configuration j^nJ, we define the Q operator and seniority number v in a manner completely analogous to that done for LS coupling,[7] it is not difficult to derive the pertinent equivalent of Racah's Eq. (69)III:

$$(j^nvJ\|T^{(k)}\|j^nvJ')=(j^vvJ\|T^{(k)}\|j^vvJ') \times\frac{(2j+1-n-v)-(-1)^k(n-v)}{2j+1-2v}, \tag{8}$$

except for $k=0$.

This equation gives for any irreducible tensor operator $T^{(k)}$ the matrix element in a configuration of n particles with seniority v in terms of the matrix element in the configuration of only v particles. For example, if we take $T^{(k)}$ to be the quadrupole moment operator $Q=(16\pi/5)^{\frac{1}{2}}Y_0^2$ (the normalized spherical harmonic is here written as Y_m^l) and v happens to be one $(J=j)$, Eq. (8) tells us that the quadrupole moment for the configuration $j^n(J=j)$ is $(2j+1-2n)/(2j-1)$ times the quadrupole moment for a single particle in that state with the same orientation.

Now it has been shown by Edmonds and Flowers[8] and also by Racah[9] that for a wide variety of attractive interactions among like particles the ground state of the configuration j^n for n odd has seniority $v=1$. This gives rise to Mayer's rule that an odd number of equivalent protons or neutrons couple to a resultant J equal to the j of the individual particles. With $v=1$, Eq. (8) reads

$$(j^nJ=j\|T^{(k)}\|j^nJ=j)=(j\|T^{(k)}\|j),\quad\text{for } k \text{ odd}; \tag{9a}$$

$$(j^nJ=j\|T^{(k)}\|j^nJ=j)=(j\|T^{(k)}\|j)\frac{2j+1-2n}{2j-1},\quad\text{for } k \text{ even}, \tag{9b}$$

except for $k=0$. With an interaction of the Wigner type, i.e., V_{ij} a function of $\mathbf{r}_i-\mathbf{r}_j$ only, it is well known that[5]

$$t_i^{(k)}=(4\pi)^{\frac{1}{2}}Y_i^{(k)}. \tag{10}$$

If we have a spin force of the form

$$V_{ij}=\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_jV(\mathbf{r}_i-\mathbf{r}_j), \tag{11}$$

then

$$t_i^{(k')}=(4\pi)^{\frac{1}{2}}Y_i^{(k)}\sigma_i^{(1)}$$

($\boldsymbol{\sigma}$ is a tensor of rank one) is not an irreducible tensor, but it may be expressed as a sum of irreducible tensors of ranks:

$$k'=k-1,\ k,\ k+1.$$

It can be shown[2,6] that only terms for which k and also $k'+k+1$ are even numbers, contribute in the evaluation of the necessary matrix elements.

Thus for a $\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}$ force the tensors T_1 and T_2 in (7) are of odd rank only. Equation (9a) then tells us that

$$E_\sigma(j_1^{n_1}J_1=j_1,\ j_2^{n_2}J_2=j_2,\ J)=E_\sigma(j_1j_2J)\quad\text{for } \boldsymbol{\sigma}\cdot\boldsymbol{\sigma} \text{ force}. \tag{12}$$

But $E_\sigma(j_1j_2J)$ was just the term calculated by de-Shalit.[2] In the limit of zero range force,

$$\begin{aligned}E_\sigma(j_1^{n_1}J_1=j_1,\ j_2^{n_2}J_2=j_2,\ J)&=E_\sigma(j_1j_2J)\\&=-\tfrac{1}{2}F_0(2j_1+1)(2j_2+1)(j_1\tfrac{1}{2}j_2-\tfrac{1}{2}|j_1j_2J0)^2\frac{1}{2J+1}\\&\quad\times\left\{\frac{[(2j_1+1)+(-1)^{j_1+j_2+J}(2j_2+1)]^2}{4J(J+1)}-[1+2(-1)^{l_1+l_2+J}]\right\}.\end{aligned} \tag{13}$$

For Wigner force T_1 and T_2 are of even rank. Noting[5] that T_1^0 is the scalar n_1 and

$$(j_1^{n_1}J_1=j_1\|T_1^0\|j_1^{n_1}J_1=j_1)=n_1(2j_1+1)^{\frac{1}{2}},$$

and similarly for T_2^0, and also with

$$(-1)^{j_1+j_2-J}W(j_1j_2j_1j_2;J0)=[(2j_1+1)(2j_2+1)]^{-\frac{1}{2}},$$

we have finally

$$\begin{aligned}E_W(j_1^{n_1}J_1=j_1,\ j_2^{n_2}J_2=j_2,\ J)&=\frac{2j_1+1-2n_1}{2j_1-1}\,\frac{2j_2+1-2n_2}{2j_2-1}E_W(j_1j_2J)\\&\quad-F_0\left[n_1n_2-\frac{2j_1+1-2n_1}{2j_1-1}\,\frac{2j_2+1-2n_2}{2j_2-1}\right].\end{aligned} \tag{14}$$

[6] G. Racah and I. Talmi, Physica 18, 1097 (1952).
[7] G. Racah, Phys. Rev. 63, 367 (1943).
[8] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) A214, 515 (1952).
[9] G. Racah, *L. Farkas Memorial Volume.* (Research Council of Israel, Interscience Publishers, New York, Agents, 1952), p. 294.

TABLE I. $-E(j_1j_2J)/F_0$ for zero range forces.

j_1	1/2			3/2			5/2			7/2			9/2				
	W	σ^+	σ^-	W	σ^+	σ^-	W	σ^+	σ^-	W	σ^+	σ^-	W	σ^+	σ^-	$J-j_1$	j_2
	1	−3	1	1	1	−1.67	1	−1.40	1	1	1	−1.28	1	−1.22	1	−1/2	1/2
	1	1	−0.33	1	−0.60	1	1	1	−0.71	1	−0.78	1	1	1	−0.82	+1/2	
				2	−6	2	1.8	1.8	−3	1.72	−2.40	1.72	1.67	1.67	−2.14	−3/2	3/2
				1.20	1.20	0.67	0.89	0.20	0.89	0.76	0.76	0	0.70	−0.11	0.70	−1/2	
				0.40	−1.20	0.40	0.37	0.37	−1	0.38	−0.89	0.38	0.39	0.39	−0.82	+1/2	
				1.20	1.20	−0.86	1.29	−1	1.29	1.33	1.33	−1.09	1.36	−1.15	1.36	+3/2	
							3	−9	3	2.57	2.57	−4.29	2.38	−3.33	2.38	−5/2	5/2
							1.63	1.63	1.29	1.05	0.59	1.05	0.82	0.82	0.30	−3/2	
							0.69	−2.06	0.69	0.62	0.62	−1.67	0.62	−1.45	0.62	−1/2	
							0.91	0.91	0	0.90	−0.14	0.90	0.88	0.88	−0.24	+1/2	
							0.29	−0.86	0.29	0.27	0.27	−0.77	0.27	−0.71	0.27	+3/2	
							1.43	1.43	−1.17	1.52	−1.28	1.52	1.57	1.57	−1.36	+5/2	
										4	−12	4	3.33	3.33	−5.56	−7/2	7/2
										2.10	2.10	1.84	1.25	0.91	1.25	−5/2	
										0.95	−2.86	0.95	0.84	0.84	−2.27	−3/2	
										0.99	0.99	0.36	0.91	0.19	0.91	−1/2	
										0.47	−1.40	0.47	0.43	0.43	−1.24	+1/2	
										0.94	0.94	−0.26	0.95	−0.35	0.95	+3/2	
										0.23	−0.70	0.23	0.22	0.22	−0.65	+5/2	
										1.63	1.63	−1.42	1.71	−1.51	1.71	+7/2	

Again, for zero range forces,[2]

$$E_W(j_1j_2J) = -\tfrac{1}{2}F_0(2j_1+1)(2j_2+1)(j_1\tfrac{1}{2}j_2-\tfrac{1}{2}|j_1j_2J0)^2\frac{1}{2J+1} \times\left\{1+\frac{[(2j_1+1)+(-1)^{j_1+j_2+J}(2j_2+1)]^2}{4J(J+1)}\right\}. \quad (15)$$

In Table I are given values of $-E(j_1j_2J)/F_0$ computed from Eqs. (13), (15) with columns headed W for Wigner force and $\sigma\pm$ for spin force in states of positive and negative parity, respectively.

DISCUSSION

Wigner forces alone will not give the desired distinction between configurations of various l's, so we shall investigate the usefulness of a mixture of Wigner and Bartlett forces. If we take the interaction as

$$V_{ij} = -[(1-\alpha)+\alpha\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j]\delta(\mathbf{r}_1-\mathbf{r}_j), \quad (16)$$

it is interesting to plot against α the energy levels characterized by the different ways in which J_1 and J_2 can combine to the resultant J. In Fig. 1 are a typical set of such graphs for various values of n_1 and n_2 when we have chosen $j_1=5/2$, $j_2=7/2$ and l_1+l_2 is even. de-Shalit gives a number of such graphs for several values of j_1 and j_2 when $n_1=n_2=1$.

From Eqs. (12), (14) we can easily see how these graphs change as we put more particles in each shell:

For a pure $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ force, $\alpha=1$, the level structure is independent of n_1 and n_2.

for a pure Wigner force, $\alpha=0$, increasing n_1 and n_2 both lowers and compresses the level structure. The former effect, due to the second term in Eq. (14), and, in fact, any translation of the energy scale can be ignored here since it does not affect the level spacing at any mixture. As n_1 and n_2 increase through the first halves of their respective shells, the levels are bilinearly compressed. When either group is exactly half of a filled shell, all the levels (on the Wigner side) are degenerate. As one group only passes through the second half of its shell the levels are inverted and dilated: n_1 holes and n_2 particles give exactly the inverted level structure of n_1 particles and n_2 particles. n_1 holes and n_2 holes are again equivalent to n_1 particles and n_2 particles.

Surveying the data on odd-odd nuclei of Nordheim[10] and of Goldhaber and Hill,[11] and the recent note of

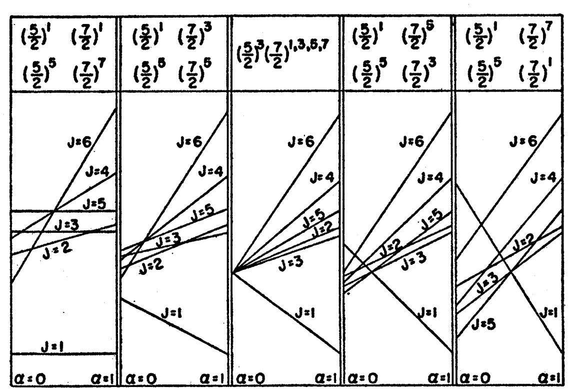

FIG. 1. Energy levels of configurations $(5/2)^{n_1}(7/2)^{n_2}$ with positive parity under interaction $V_{12}=-[(1-\alpha)+\alpha\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]\delta(\mathbf{r}_1-\mathbf{r}_2)$. Both subgroups have seniority one.

[10] L. W. Nordheim, Revs. Modern Phys. 23, 322 (1951).

[11] M. Goldhaber and R. D. Hill, Revs. Modern Phys. 24, 179 (1952).

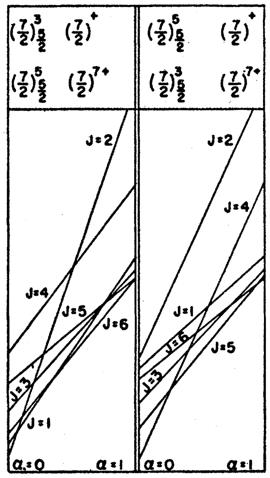

FIG. 2. Energy levels of configurations $(7/2)^{n_1}{}_{5/2}(7/2)^{n_2}{}_{7/2}$ with positive parity under interaction $V_{12}=-[(1-\alpha)+\alpha\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]\delta(\mathbf{r}_1-\mathbf{r}_2)$.

King and Peaslee,[12] we find that the above considerations furnish an excellent accounting of the measured and indicated ground state spins. Except for a few nuclei with the configurations $(3d_{5/2})^3$ and $(4f_{7/2})$,[3,5] there were no cases which could not be fitted to our graphs. However, it is known from the spins and magnetic moments of the odd-even nuclei Na^{23} and Mn^{55} that these exceptional configurations are probably not of seniority one and must be treated separately (see next section).

In many cases where the spin was known the configurations could be unambiguously assigned. In cases where measurements were incomplete, some certainty about the configurations involved allowed prediction of the most probable spin.

EXCEPTIONAL CONFIGURATIONS

In some cases it is desirable to calculate the level schemes involving configurations of seniority other than one. For such work we need a general relation between the perturbation energy in the state $(j_1^{n_1}J_1,j_2^{n_2}J_2,J)$ and the terms already calculated for the simple case

TABLE II. Effective n-p force mixtures indicated by observed spins

Nucleus	Configurations		Spin	α
Li^6	$(p_{3/2})^1{}_{3/2}$	$(p_{3/2})^1{}_{3/2}$	1	$\geqslant 0.10$
Li^8	$(p_{3/2})^1{}_{3/2}$	$(p_{3/2})^3{}_{3/2}$	2	$\leqslant 0.25$
B^8	$(p_{3/2})^3{}_{3/2}$	$(p_{3/2})^1{}_{3/2}$	2	$\leqslant 0.25$
B^{10}	$(p_{3/2})^3{}_{3/2}$	$(p_{3/2})^3{}_{3/2}$	3	$\geqslant 0.10$
Na^{24}	$(d_{5/2})^1{}_{5/2}$	$(d_{5/2})^5{}_{5/2}$	4	$\leqslant 0.42$
Cl^{36}	$(d_{3/2})^1{}_{3/2}$	$(d_{3/2})^3{}_{3/2}$	2	$\leqslant 0.25$
K^{40}	$(d_{3/2})^3{}_{3/2}$	$(f_{7/2})^1{}_{7/2}$	4	$\leqslant 0.50$
K^{42}	$(d_{3/2})^3{}_{3/2}$	$(f_{7/2})^3{}_{7/2}$	2	$\geqslant 0.17$
Sc^{46}	$(f_{7/2})^1{}_{7/2}$	$(f_{7/2})^5{}_{7/2}$	4	$\leqslant 0.30$
Co^{58}	$(f_{7/2})^7{}_{7/2}$	$(p_{3/2})^3{}_{3/2}$	2	$\leqslant 0.10$
Co^{60}	$(f_{7/2})^7{}_{7/2}$	$(p_{3/2})^3{}_{3/2}$	5	$\geqslant 0.10$
	or			
		$(f_{5/2})^1{}_{5/2}$		$\leqslant 0.50$
Rb^{88}	$(p_{3/2})^3{}_{3/2}$	$(d_{5/2})^1{}_{5/2}$	2	$\geqslant 0.17$; $\leqslant 0.50$
Sb^{122}	$(g_{7/2})^1{}_{7/2}$	$(h_{11/2})^7{}_{11/2}$	2	$\geqslant 0.10$

[12] R. W. King and D. C. Peaslee, Phys. Rev. **90**, 1001 (1953).

$n_1=n_2=1$. Such a relation is

$$E(j_1^{n_1}J_1,j_2^{n_2}J_2,J)=n\sum_{J_1'J'}(2J_1+1)(2J'+1)W^2(J_1'j_1JJ_2;J_1J')\times(j_1^{n_1-1}(J_1'),j_1J_1]\!\!]j_1^{n_1}J_1)^2E(j_1,j_2^{n_2}J_2,J'),$$

$$E(j_1,j_2^{n_2}J_2,J)=n_2\sum_{J_2'J''}(2J_2+1)(2J''+1)W^2(J_2'j_2J'j_1;J_2J'')\times(j_2^{n_2-1}(J_2'),j_2J_2]\!\!]j_2^{n_2}J_2)^2E(j_1j_2J''),$$

where $(j_1^{n_1-1}(J_1'),j_1,J_1]\!\!]j_1^{n_1}J_1)$ is the fractional parentage coefficient for the configuration $j_1^{n_1}J_1$ as computed by Edmonds and Flowers.[8] Of use in checking numerical work is the sum rule

$$\sum_J(2J+1)E(j_1^{n_1}J_1,j_2^{n_2}J_2,J)=-n_1n_2(2J_1+1)(2J_2+1)F_0. \quad (18)$$

Using Eq. (17) and Table I, we have computed the energy levels for the configuration $(7/2)^3{}_{5/2}(7/2)$ with positive parity under the perturbing potential (16). Since (8) still gives the correct relation between particles and holes for any seniority, we have the several cases shown in Fig. 2.

The spin 4 for Sc^{46} is well accounted for, and we would predict spin 6 for Mn^{52} and 4 or 5 for V^{50}. Recent work[13] gives spin 6 for V^{50}, which is best accounted for by assigning seniority one to the $(7/2)^3$ proton configuration.[14] For Sc^{44} the indicated spin 2 can be fitted only with an extremely small value of α. The correct configuration here is most likely a mixture of $(7/2)^3{}_{5/2}$ and $(7/2)^3{}_{7/2}$; this may also be the case in Mn^{52}.

For F^{20} the configuration $(s_{1/2})[(d_{5/2})^3{}_{5/2}]$ would have $J=3$ lowest whereas $(s_{1/2})[(d_{5/2})^3{}_{3/2}]$ would have $J=2$ lowest. The measurements do not yet indicate which of these two is correct.

Calculations for the nuclei Na^{22} and V^{48} with both neutron and proton configurations having seniority different from one require performing both sums of (17) and have not been carried out. However, for the latter nucleus, letting only one group have seniority one would lead to spin 4 or 5, which are the most probable indicated values.

CONCLUSIONS

In Table II are listed a number of nuclei from which we may infer something about the necessary amount of mixture of the two forces considered. It should be remarked that the two-body interaction taken as a perturbation here is not the total neutron-proton force but is supposed to be the residual of an averaging over all interparticle forces which gives the central potential well of the shell model. Especially since none

[13] Kikuchi, Sirvetz, and Cohen, Phys. Rev. **88**, 142 (1952).
[14] A. Hitchcock, Phys. Rev. **87**, 664 (1952).

but a scalar potential survives between outer particles and any subgroup coupled to zero total spin, there is no simple reason to expect the mixture of forces in the perturbation to be independent of A. However, with the exception of Co^{58} all of the nuclei can be fitted with a value of α between $\frac{1}{6}$ and $\frac{1}{4}$.

With regard to Nordheim's rules, we now see that their validity is not as general as was first indicated in the case of one odd proton and one odd neutron. Certainly for one less than a filled shell of both neutrons and protons they are equally valid; and the "strong" rule,

$$J=|j_p-j_n|, \quad \text{for } j_p+j_n+l_p+l_n \text{ even},$$

is still good when n_1 and n_2 are both more or both less than half of a filled shell. However, in general, the situation is more complicated, especially as it is unsafe to rely on the constancy of the mixture of forces.

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The Decay Scheme of Zr^{95}†*

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The decay scheme of Zr^{95} has been studied using the techniques of beta-ray spectroscopy and beta-gamma angular correlation. The decay of Zr^{95} is found to proceed by three beta gamma cascades. Two of the beta transitions are allowed and proceed to Nb^{95} levels at 722.0 kev and 754.4-kev. The third beta transition is to a 235-kev level in Nb^{95}. From K-conversion coefficient determinations and shell theory the 722-kev and 754.4-kev levels are both assigned even parity and a spin of 5/2 or 7/2.

THE radiations originating from the decay of Zr^{95} have been studied by a number of investigators with somewhat divergent results. A recent paper by Cork *et al.*,[1] published after the completion of this study, contains an excellent survey of the previous work.

In the present investigation, beta-ray spectrometer studies were made of the Zr^{95} spectra and gamma ray conversion lines. In addition, the angular correlation between the Zr^{95} beta and gamma rays in cascade to the ground state of Nb^{95} was measured.

BETA-RAY SPECTROMETER STUDIES

The beta-ray spectrometer used in this investigation was a double-focusing, high-resolution spectrometer patterned after that of Kurie, Slack, and Osaba.[2] In most of the runs reported here it was operated at a resolution of 0.2 percent. The electron detector used for studies of the shape of the beta spectrum was an end-window Geiger counter with a 0.6-mg/cm^2 rubber hydrochloride window. Later studies of conversion lines and of the high-energy end of the beta spectrum employed an end-window counter with a 3-mg/cm^2 mica window.

The magnetic field in the spectrometer was measured using a double-coil monitor with a temperature-stabilized permanent magnet supplying the reference field. The magnetic field could be kept constant to within 1 part in 20,000 for extended periods of time.

The source material used in all of the experiments consisted of a $Zr^{95}-Nb^{95}$ oxalate mixture obtained from Oak Ridge National Laboratory. The Oak Ridge analysis indicated that the radiochemical purity of the source material was better than 99 percent and that the source initially consisted of 40 percent Zr^{95} and 60 percent Nb^{95}.

The source for the spectrometer study of the spectrum shape was prepared by laying down the source material in the form of a line 3 mm wide by 1 inch high on a thin formvar film on which a film of gold had been vacuum evaporated. The areal density of the source backing was less than 125 μg/cm^2. The areal density of the source material was approximately 50 μg/cm^2.

Figure 1 shows the Kurie plot obtained with this source. It will be noted that the plot has an "allowed" shape from the end point of the Nb^{95} spectrum at 160 kev out to approximately 360 kev. At the high-energy end of the spectrum the 900-kev branch of the Zr^{95} decay is in evidence but the source was too weak to make significant measurements on it. The K conversion line resulting from the 235-kev isomeric transition in Nb^{95} is also in evidence in this figure.

The conversion lines from the gamma ray transitions in Nb^{95} and Mo^{95} were examined with a stronger source. This source was prepared by ruling a line 1 mm wide and

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[1] Cork, LeBlanc, Martin, Nester, and Brice, Phys. Rev. **90**, 579 (1953).
[2] Kurie, Osaba, and Slack, Rev. Sci. Instr. **19**, 771–6 (1948).