

TABLE I. Calculated and experimentally observed temperatures.

	Sn ¹¹⁸			Pt ¹⁹⁵ and Au ¹⁹⁷		
$E_{\max} - E$ (Mev)	10	12	24	10	12	23
T_{calc} (Mev)	0.9	0.8	1.6	0.7	0.6	1.2
T_{obs} (Mev)	1.0 ^a	0.6 ^b	~1.8 ^c	0.7 ^a	0.7 ^b	~1.7 ^c

^a Reference 11.^b Reference 10.^c Reference 12.

We would expect E_0 to decrease with A for large A , reflecting the lower binding energy of these nuclei; and following Hurwitz and Bethe² we might expect a very slight increase in E_0 for magic nuclei. At any rate, setting $E_0 = 8$ Mev, $E_1 = 20$ Mev (as is suggested by the experiments),^{4,5} and $K = \frac{1}{2}$ (as is suggested by Bethe and Bardeen's work),⁶ results for all nuclei in a considerably better fit to the experimental data¹⁰⁻¹² than the usual theory gives,^{9,13} as is shown in Table I. The calculated temperatures are all well within the estimated error of the observed temperatures.

In the analysis of this work we used a nuclear radius of $1.2 \times 10^{-13} A^{1/3}$ cm for proton emission,¹⁴ and we allowed for the emission of knock-on protons.¹⁵ Temperatures in this preliminary report were computed for energies of emitted particles such that the emitted particles all had approximately the same orbital angular momentum. If this were not done, it was feared that the large nuclear spin changes possible for the highest-energy emitted particles would be forbidden for other examples of lower energy. Comparison of residual nuclei level densities was sought for those cases where approximately the same fraction of the total number of levels was observed.

An attempt is being made to estimate shell energies and to employ the statistical method⁶⁻⁸ to compute $I(E)$ more precisely.

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Nuclear Shell Model ft Values for Intermediate Coupling

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USING the wave functions already calculated by one of us for various light nuclei of mass number from 6 to 15,¹ we have evaluated the beta-decay matrix elements, and thus the ft values, for several of the beta transitions among the light nuclei. These theoretical results agree satisfactorily with experiment,² and indicate that intermediate coupling is to be preferred as a nuclear model over strict LS or $j-j$ coupling.³

The present work depends upon the derivation of the nuclear wave functions, which used the shell model with intermediate coupling. Specific assumptions were:

(1) Two protons and two neutrons fill the $1s$ shell, forming an inert core. The remaining nucleons lie in the $1p$ shell, whose one-particle space wave functions, $(r/r_0) \exp[-\frac{1}{2}(r/r_0)^2] \times Y_{l,m}^{1,0,-1}$ (where $r_0 = 1.7 \times 10^{-13}$ cm and $Y_{l,m}^{1,0,-1}$ are the spherical harmonics),

TABLE I. Log ft values of various beta decays.

β transition	Experimental ^a	Log ft		
		LS coupling	$j-j$ coupling	Intermed. coupling
$n \rightarrow p + e^- + \nu^0$	3.13	3.13	3.13	3.13
$\text{He}^6 \rightarrow \text{Li}^6 + e^- + \nu$	2.94 ± 0.04	2.95	3.21	2.98
$\text{Be}^7 \rightarrow \text{Li}^7 + \nu$	3.37 ± 0.01	3.31	3.45	3.41
$\text{Be}^7 \rightarrow \text{Li}^{7*} + \nu$	3.53	3.61	4.08	3.58
$\text{C}^{10} \rightarrow \text{B}^{10*} + e^+ + \nu^0$	3.77 ± 0.20	3.43	3.43	3.43
$\text{N}^{13} \rightarrow \text{C}^{13} + e^+ + \nu$	3.67	3.61	3.61	3.69
$\text{C}^{14} \rightarrow \text{N}^{14} + e^- + \nu^0$	8.95	2.95	3.91	5.70
$\text{O}^{14} \rightarrow \text{N}^{14*} + e^+ + \nu^0$	3.52 ± 0.10	3.43	3.43	3.43
$\text{O}^{15} \rightarrow \text{N}^{15} + e^+ + \nu^0$	3.59 ± 0.03	3.61	3.61	3.61

^a These entries do not represent new work. The Fermi transitions are independent of nuclear structure, while the O^{15} beta-decay is completely described by $j-j$ coupling. (Also see reference 4).

^b L forbiddenness gives rise to a factor of ten in this matrix element, and a further partial cancellation of contributions occurs, as conjectured by Sachs [R. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Chicago, 1953), p. 347]. This is at variance with Inglis' finding (reference 3, p. 442), which is based on a different assumption for the nuclear interactions.

are coupled to the Pauli spin functions to form $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ one-particle wave functions.

(2) The interaction between all nucleon pairs is a four-forces mixture weighted 0.35 Wigner, 0.35 Majorana, 0.15 Bartlett, and 0.15 Heisenberg force, with a well depth of 30 Mev and a common radial dependence of $\exp[-(r/1.9 \times 10^{-13} \text{ cm})^2]$, and an additional spin-orbit force of

$$-2.8 \text{ Mev} [\mathbf{s}_1 \cdot (\mathbf{r}_{12} \times \mathbf{p}_1) + \mathbf{s}_2 \cdot (\mathbf{r}_{21} \times \mathbf{p}_2)] \frac{1}{\hbar^2} \exp\left(-\frac{r_{12}^2}{(1.9 \times 10^{-13} \text{ cm})^2}\right),$$

where 1 and 2 designate a pair of nucleons and the remaining symbols have their conventional significance. (The spin-other orbit terms of the usual interaction are neglected as being of secondary importance.) The tensor force is neglected.

(3) The wave functions contain no admixtures from higher shells—the energy is diagonalized wholly within the $1p$ (that is, $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$) shell.

The above assumptions led to wave functions yielding magnetic moments in agreement with experiment to the accuracy with which (3) can be expected to hold, (~ 5 percent). Using the same wave functions to calculate the matrix elements in $\log ft = C - \log(|\int \mathbf{1}|^2 + |\int \boldsymbol{\sigma}|^2)$, (where we take the Fermi and Gamow-Teller coupling constants equal, and fit the constant C to the neutron ft value), we find values which are presented in Table I and compared with those calculated on the basis of strict LS or $j-j$ coupling.

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Energy Spectra of Nucleons Evaporated from a Compound Nucleus*

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SEVERAL experiments appear to disagree with the idea that a compound nucleus de-excites itself by evaporating nucleons according to a Maxwellian energy distribution. A Maxwellian distribution follows from the assumption of the compound nucleus

model and the assumption that the density of levels, ω , of the residual nucleus at excitation E is given by $\omega = b \exp[(aE)^{1/2}]$.¹ Some of these experiments are: the inelastic scattering of 14-Mev neutrons² and 18-Mev protons³ from various elements; the inelastic scattering of 30-Mev protons from Al⁴; and the neutron spectra from nuclear reactions induced in various elements by 16-Mev protons.⁵ The results of these experiments have been conveniently compiled in a recent article by Cohen.⁶ All of these data indicate that there are more high-energy nucleons emitted, compared to the number of low-energy nucleons emitted, than can be explained by a Maxwellian distribution.

The following explanations of this discrepancy have been proposed: (a) at low excitation, the level density of the residual nucleus varies much more slowly than exponentially⁴; (b) the excitation energy of a nucleus should not be measured from the ground state but from a state which is above the ground state⁷; (c) there are selection rules in effect which make transition probabilities to low-lying levels in the residual nucleus very much larger than transition probabilities to highly excited levels⁸; and (d) the excess of high-energy nucleons is due to a direct interaction between the incoming particle and the nucleons forming the surface of the target nucleus.³

The purpose of this letter is to indicate that the last explanation is certainly adequate to account for the results obtained from the inelastic scattering of 30-Mev protons by Al, and is probably adequate to account for the results obtained from the lower energy (p, p'), (n, n'), and (p, n) experiments. The information required to prove this statement is obtained from recent experiments on the inelastic scattering of 31-Mev protons from several heavy elements.⁸ In these experiments it is found that the energy distributions of the inelastically-scattered protons bear no resemblance to Maxwellian distributions, except at a scattering angle of 135°. Further, it is found that the angular distributions for inelastic scattering are strongly peaked forward, and that the total cross sections for inelastic scattering are at least an order of magnitude larger than the compound nucleus cross section for evaporating protons through the Coulomb barrier of a heavy nucleus. These data definitely disagree with what would be expected in a compound nucleus process but can be explained qualitatively by assuming that, in heavy elements at 31 Mev, the inelastic scattering occurs when the incident proton collides with the rim of the target nucleus and makes a direct interaction with one or several nucleons.

In the 30-Mev (p, p') experiment on Al, the excess of protons, compared to a Maxwellian distribution, emitted in the energy range 15–25 Mev and at an angle of 90°, corresponds to an inelastic-scattering cross section $d^2\sigma/d\Omega dE$ of about 0.5 mb/sterad-Mev. In the 31-Mev experiment on the heavy elements, the cross section for producing inelastically-scattered protons by the direct interaction effect, in the energy range 15–25 Mev and at an angle of 90°, is about 1.0 mb/sterad-Mev. Furthermore, the direct interaction cross section at 90° shows only a very slow decrease with decreasing A in the range Pb to Sn. Thus the excess of high-energy protons emitted from Al can be accounted for by the direct interaction effect.

Now, the extrapolation of the direct interaction cross sections down to the lower energies is not known. However, some comparisons can be made. In the inelastic-scattering experiments involving 18-Mev incident protons, the difference between the 60° and 150° cross sections of Sn for the emission of 15-Mev protons, $d^2\sigma/d\Omega dE(60^\circ, 15 \text{ Mev}) - d^2\sigma/d\Omega dE(150^\circ, 15 \text{ Mev})$, is about 2 mb/sterad-Mev. At 31-Mev incident energy, the cross section of Sn for the emission of 15-Mev protons at 60° by the direct interaction effect was measured to be 1.5 mb/sterad-Mev. Thus, the direct interaction effect appears to be capable of accounting for the excess of high-energy protons emitted in the forward direction in the 18-Mev experiment. Numerical comparisons cannot be made in the experiments involving the emission of neutrons since absolute cross sections are not quoted. However, the cross sections for neutron emission would be expected to be similar to those for

proton emission at emission energies greater than the Coulomb barrier energy.

Finally it should be pointed out that, in the heavy elements at 31 Mev, the evidence indicates that even the inelastic scattering at 135° is predominantly a direct interaction effect. Although the energy distributions at this backward angle are of Maxwellian form, the differential cross sections are much larger than the predictions of compound nucleus theory and the temperatures required to fit the distributions are very high. For instance, the best fit to the Au energy distribution at 135° corresponds to a temperature of 7 Mev. If any thermodynamic significance is to be attributed to these high-temperature Maxwellian-like distributions, then the excitation of the compound nucleus must necessarily be confined to a very few nucleons (~ 3), since the total excitation energy is only about 40 Mev. This immediately leads back to the direct interaction picture. If the direct interaction cross sections in the backward directions extrapolate to lower energies as the forward cross sections seem to, then it would not be correct to assume that the energy distributions of evaporated nucleons, which are observed in the lower-energy experiments, are free from contamination due to the direct interactions effect, even when these energy distributions are measured in backward directions.

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Statistical Factors in Radiation Widths of Nuclear Energy Levels

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INASMUCH as theoretical evaluations of the radiative transition probabilities of nuclear energy levels are in the nature of rough estimates, statistical factors are usually omitted.¹ At the present time, however, the number of well-measured radiation widths is being increased rapidly by the results of high-resolution neutron spectroscopy,² which allows measurement of the parameters of the highly excited states of compound nuclei formed by neutron capture. The extent and accuracy of the radiation width measurements (of the order of 10–20 percent) justifies the use of statistical factors in the theoretical formulas and in particular makes possible an investigation of any variations in width, for a given multipole order, that might arise from statistical factors.

In the past few years some use of statistical factors has been made in comparison of radiative widths with theory. Thus Goldhaber and Sunyar³ multiplied measured transition probabilities of low-lying (isomeric) states by $(2J+1)$, with J the spin of the emitting state, in the comparison of these probabilities with Weisskopf's¹ theoretical single particle estimate. In this case, the transition probabilities used were those obtained from measured lifetimes. Radiation widths of highly excited states of light nuclei were compiled by Wilkinson⁴ in the form $(2J+1)\Gamma_\gamma$, although here the reason was that the experimental data (reaction cross sections) give this quantity rather than Γ_γ itself. Kinsey and Bartholomew,⁵ in comparing the measured widths of particular high-energy capture gamma transitions with theory, multiplied these widths by $(2J+1)$ in the process. The "measured width" is obtained from the total radiation width, Γ_γ , resulting from resonance analysis,² and measurement⁶ of the fraction of neutron captures resulting in occurrence of the particular gamma transition. In the light of these applications of statistical factors and the