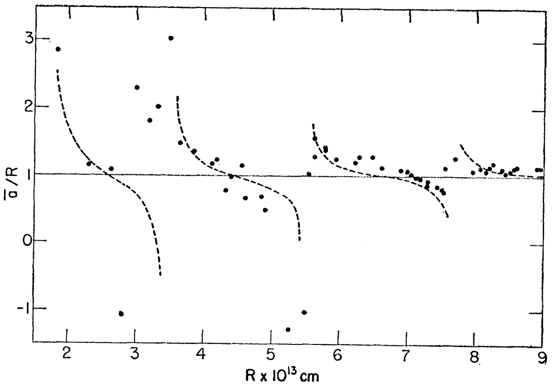

FIG. 1. The points show the variation of the average scattering length $\bar{a}$ divided by the nuclear radius R, as a function of R. The dashed curve represents the variation of a/R to be expected from a well with the parameters: $V=-42$ Mev, $0\leq r\leq R$; $V=0$, $r>R$; $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm.

where κ is the wave number associated with the potential well depth. If a well depth of 19 Mev is chosen, the resonance near $A=150$ will be the third S-wave resonance, and N will equal 2. However, the following examination of the low-energy neutron cross sections of lighter elements indicates that the resonance near $A=150$ is the fourth S-wave resonance, not the third, hence $N=3$ and the proper well depth must be about $[(3+\frac{1}{2})/(2+\frac{1}{2})]^2\times19$ Mev, or about 40 Mev.

Ford and Bohm[4] have previously discussed variations of the thermal neutron scattering length, a, with nuclear size, and concluded that a nuclear potential well of about 40-Mev depth was in agreement with the data. Since scattering lengths are strongly affected by individual, narrow, low-energy resonances, which the optical theory treats only on the average, it has not been clear that the deviations of the values of a from that expected from hard sphere scattering, for example, were significant of anything but the presence of nearby narrow energy levels.[5] Such fluctuations, which are not pertinent to the theory, can largely be eliminated by using for the value of the scattering length an average value defined as $\bar{a}=(\sigma_0/4\pi)^{\frac{1}{2}}$ where σ_0 represents the low-energy cross section averaged over narrow resonances. Values of $\bar{a}$, divided by R, are shown in Fig. 1, plotted against R, the nuclear radius. These values are taken primarily from total neutron cross-section measurements made at the University of Wisconsin by Barschall, Miller, Bockelman, and others.[6] Any estimate of this average cross section at zero energy is necessarily subjective but in only a few cases are the uncertainties in $\bar{a}$ greater than 5 percent, while the variations of interest are much larger than five percent. The sign of $\bar{a}$ is taken as negative when the measured coherent scattering lengths are negative.[7] Also plotted on Fig. 1 is the variation of a to be expected from a square well 42 Mev deep. The data show clearly that the number of resonances observed is that which one would expect from a well about 40 Mev deep, verifying the conclusions of Ford and Bohm.[4]

It seems likely that the increase of well depth from 19 Mev to about 40 Mev will not invalidate the general success of the Feshbach, Porter, and Weisskopf[1] formulation in predicting the variation of average neutron cross section with energy and nuclear radius near $A=150$, since application of the Sturm-Liouville theorem leads us to expect P- and D-wave resonances to bracket the S resonance, as a function of either neutron energy or nuclear radius, in much the same way, near $\kappa R=7\pi/2$, as near $\kappa R=5\pi/2$, independent of details of well shape. Preliminary calculations by Porter[8] seem to substantiate this view.

Conversations and correspondence with Dr. C. E. Porter, Dr. Ben Mottelson, and Dr. H. H. Barschall have contributed in an important way to this work.

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Level Densities in Heavy Nuclei of 10- to 20-Mev Excitation

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IT is quite evident that nuclei of 8- or 10-Mev excitation or less cannot be treated as excited Fermi gases of all the nucleons present. Hughes and his co-workers[1] have shown that capture cross sections for thermal and 1-Mev neutrons are a function of eveness or oddness of the atomic number of the target nucleus and a particularly pronounced function of the magic property of the target nucleus. The difference between even and odd Z, or between magic nuclei and their neighbors is entirely a matter of how the last few nucleons are bound. And if the behavior of two or three nucleons makes a great difference in the level densities of the compound nucleus as Hughes[1] and others[2] have shown to be the case, then a statistical theory cannot be valid. Moreover, it is clear that few of the total number of nuclear particles participate in the excitation.

Further evidence is supplied by Feshbach, Porter, and Weisskopf,[3] who show that a surprisingly good fit to Barschall's[4] total neutron scattering data from 1–3 Mev is obtained by scattering in a nuclear well in which the mean free path of a neutron in a 10-Mev excited nucleus is 20 nucleons long. This means that in nuclei of 100–200 atomic mass number only 5–10 nucleons can participate in the excitation. As Feshbach, Porter, and Weisskopf[3] have further observed, however, the data of Phillips, Davis, and Graves[5] on inelastic neutron cross sections at 14 Mev, in which the target nuclei appeared totally "black," indicate that all the nucleons in the target nuclei participate in a compound nucleus excitation of around 20 Mev.

We assume a general level-density formula whose energy dependence is common to many different nuclear models:[6–9]

$$\omega(E_{\max}-E)=\text{const}(E_{\max}-E)^{-1}\exp[KA'(E_{\max}-E)]^{\frac{1}{2}},\quad(1)$$

where $(E_{\max}-E)$ is the excitation energy of the nucleus, A' is the number of nucleons participating in the excitement, and K is an experimentally fitted parameter. We make a linear interpolation for A' between E_0, the excitation energy below which few nucleons are able to participate in the excitation, and E_1, the excitation energy above which all the nucleons can participate in the reaction. We then obtain for the spectrum of emitted particles from a nucleus left with an excitation energy less than E_1 and well above E_0

$$I(E)dE=\text{const}E\sigma_c(E)[E_{\max}-E-E_0]^{-1}\times\exp[KA(E_{\max}-E_0-E)^2/(E_1-E_0)]^{\frac{1}{2}}dE,\quad(2)$$

where A is the atomic mass number of the nucleus and $\sigma_c(E)$ is the capture cross section for the inverse event. Defining the temperature as

$$1/T=-\frac{d}{dE}\ln[I(E)/E\sigma_c(E)],\quad(3)$$

we obtain, using Eqs. (1) and (2),

$E_{\max}-E\leqslant E_1$:
$$1/T=[KA/(E_1-E_0)]^{\frac{1}{2}}-[E_{\max}-E_0-E]^{-1};\quad(4)$$

$E_{\max}-E\geqslant E_1$:
$$1/T=\tfrac{1}{2}[KA/(E_{\max}-E_0-E)]^{\frac{1}{2}}-[E_{\max}-E_0-E]^{-1}.\quad(5)$$

TABLE I. Calculated and experimentally observed temperatures.

	Sn^{118}			Pt^{195} and Au^{197}		
$E_{max}-E$(Mev)	10	12	24	10	12	23
T_{calc}(Mev)	0.9	0.8	1.6	0.7	0.6	1.2
T_{obs}(Mev)	1.0[a]	0.6[b]	~1.8[c]	0.7[a]	0.7[b]	~1.7[c]

[a] Reference 11.
[b] Reference 10.
[c] Reference 12.

We would expect E_0 to decrease with A for large A, reflecting the lower binding energy of these nuclei; and following Hurwitz and Bethe[2] we might expect a very slight increase in E_0 for magic nuclei. At any rate, setting $E_0=8$ Mev, $E_1=20$ Mev (as is suggested by the experiments),[4,5] and $K=\frac{1}{4}$ (as is suggested by Bethe and Bardeen's work),[6] results for all nuclei in a considerably better fit to the experimental data[10-12] than the usual theory gives,[9,13] as is shown in Table I. The calculated temperatures are all well within the estimated error of the observed temperatures.

In the analysis of this work we used a nuclear radius of $1.2\times10^{-13}A^{\frac{1}{3}}$ cm for proton emission,[14] and we allowed for the emission of knock-on protons.[15] Temperatures in this preliminary report were computed for energies of emitted particles such that the emitted particles all had approximately the same orbital angular momentum. If this were not done, it was feared that the large nuclear spin changes possible for the highest-energy emitted particles would be forbidden for other examples of lower energy. Comparison of residual nuclei level densities was sought for those cases where approximately the same fraction of the total number of levels was observed.

An attempt is being made to estimate shell energies and to employ the statistical method[6-8] to compute $I(E)$ more precisely.

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Nuclear Shell Model *ft* Values for Intermediate Coupling

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USING the wave functions already calculated by one of us for various light nuclei of mass number from 6 to 15,[1] we have evaluated the beta-decay matrix elements, and thus the *ft* values, for several of the beta transitions among the light nuclei. These theoretical results agree satisfactorily with experiment,[2] and indicate that intermediate coupling is to be preferred as a nuclear model over strict *LS* or *j-j* coupling.[3]

The present work depends upon the derivation of the nuclear wave functions, which used the shell model with intermediate coupling. Specific assumptions were:

(1) Two protons and two neutrons fill the 1*s* shell, forming an inert core. The remaining nucleons lie in the 1*p* shell, whose one-particle space wave functions, $(r/r_0)\exp[-\frac{1}{2}(r/r_0)^2]\times Y_1^{1,0,-1}$ (where $r_0=1.7\times10^{-13}$ cm and $Y_1^{1,0,-1}$ are the spherical harmonics), are coupled to the Pauli spin functions to form $p_{\frac{3}{2}}$ and $p_{\frac{1}{2}}$ one-particle wave functions.

(2) The interaction between all nucleon pairs is a four-forces mixture weighted 0.35 Wigner, 0.35 Majorana, 0.15 Bartlett, and 0.15 Heisenberg force, with a well depth of 30 Mev and a common radial dependence of $\exp[-(r/1.9\times10^{-13}\text{ cm})^2]$, and an additional spin-orbit force of

$$-2.8\text{ Mev }[\mathbf{s}_1\cdot(\mathbf{r}_{12}\times\mathbf{p}_1)+\mathbf{s}_2\cdot(\mathbf{r}_{21}\times\mathbf{p}_2)]\frac{1}{\hbar^2}\exp\left(-\frac{r_{12}^2}{(1.9\times10^{-13}\text{ cm})^2}\right),$$

where 1 and 2 designate a pair of nucleons and the remaining symbols have their conventional significance. (The spin-other orbit terms of the usual interaction are neglected as being of secondary importance.) The tensor force is neglected.

(3) The wave functions contain no admixtures from higher shells—the energy is diagonalized wholly within the 1*p* (that is, $p_{\frac{3}{2}}$ and $p_{\frac{1}{2}}$) shell.

TABLE I. Log *ft* values of various beta decays.

β transition	Log *ft* Experimental[a]	*LS* coupling	*j-j* coupling	Intermed. coupling
$n\rightarrow p+e^-+\nu$[a]	3.13	3.13	3.13	3.13
$He^6\rightarrow Li^6+e^-+\nu$	2.94±0.04	2.95	3.21	2.98
$Be^7\rightarrow Li^7+\nu$	3.37±0.01	3.31	3.45	3.41
$Be^7\rightarrow Li^{7*}+\nu$	3.53	3.61	4.08	3.58
$C^{10}\rightarrow B^{10*}+e^++\nu$[a]	3.77±0.20	3.43	3.43	3.43
$N^{13}\rightarrow C^{13}+e^++\nu$	3.67	3.61	3.61	3.69
$C^{14}\rightarrow N^{14}+e^-+\nu$[b]	8.95	2.95	3.91	5.70
$O^{14}\rightarrow N^{14*}+e^++\nu$[a]	3.52±0.10	3.43	3.43	3.43
$O^{15}\rightarrow N^{15}+e^++\nu$[a]	3.59±0.03	3.61	3.61	3.61

[a] These entries do not represent new work. The Fermi transitions are independent of nuclear structure, while the O^{15} beta-decay is completely described by *j-j* coupling. (Also see reference 4).
[b] *L* forbiddenness gives rise to a factor of ten in this matrix element, and a further partial cancellation of contributions occurs, as conjectured by Sachs [R. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Chicago, 1953), p. 347]. This is at variance with Inglis' finding (reference 3, p. 442), which is based on a different assumption for the nuclear interactions.

The above assumptions led to wave functions yielding magnetic moments in agreement with experiment to the accuracy with which (3) can be expected to hold, (~5 percent). Using the same wave functions to calculate the matrix elements in $\log ft=C-\log(|\int 1|^2+|\int \boldsymbol{\sigma}|^2)$, (where we take the Fermi and Gamow-Teller coupling constants equal, and fit the constant C to the neutron *ft* value), we find values which are presented in Table I and compared with those calculated on the basis of strict *LS* or *j-j* coupling.

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Energy Spectra of Nucleons Evaporated from a Compound Nucleus*

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SEVERAL experiments appear to disagree with the idea that a compound nucleus de-excites itself by evaporating nucleons according to a Maxwellian energy distribution. A Maxwellian distribution follows from the assumption of the compound nucleus