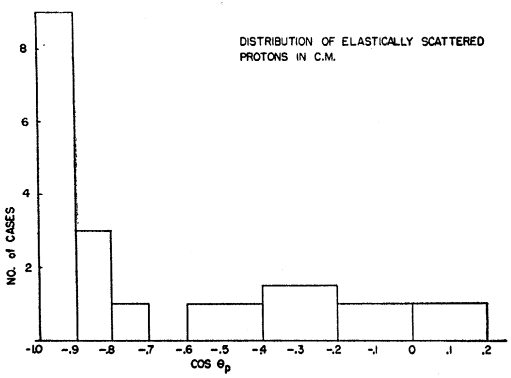

FIG. 2. Angular distribution in the c. m. system of 22 elastically scattered protons.

$$\text{(c)}\quad \pi^-+p\rightarrow\pi^++\pi^-+n \text{ — 20 events,}$$

$$\text{(d)}\quad \pi^-+p\rightarrow \left.\begin{array}{l}\pi^0+n\\ 2\pi^0+n\\ \Lambda^0+\theta^0\end{array}\right\}\text{ — 22 events,}$$
$$\text{etc. . . .}$$

$$\text{(e)}\quad \pi^-+p\rightarrow\pi^++2\pi^-+p \text{ — 1 event.}$$

It is possible that some of the cases which appear to be π^0 production are cases of elastic collisions in which one of the products has struck a neutron. However, it seems likely that this does not happen often for the following reasons: (1) Frequent secondary interactions would tend to scatter protons to large angles which in turn should result in many rejections by criterion (3). (The inelastic protons actually seem to come out very much in the forward direction in the lab system.) (2) In a three-particle reaction the energy of one of the particles with its angle and the angle of one other track uniquely specifies the reaction. In 12 cases good momentum measurements on both charged particles could be made, and on this basis 2 were rejected.

Some of the cases classed as single π^0 production might be double π^0 production. There are not thought to be many of these since there are so few cases of multiple production of charged π's. By momentum and energy balance one can occasionally check for multiple π^0 production.

If a collision appears to be elastic within a momentum discrepancy of 200 Mev/c it is classed as an elastic collision. Figure 1 shows an example of π^0 production in a $\pi^- - p$ collision. Figure 2 shows the angular distributions of the protons in elastic scattering. Figures 3 and 4 show the angular distributions of the protons, π^- and π^0, and angles between π^- and π^0 in the production cases.

The angular distributions show that in both the elastic and π^0 production cases the protons tend to go backward and the π^-

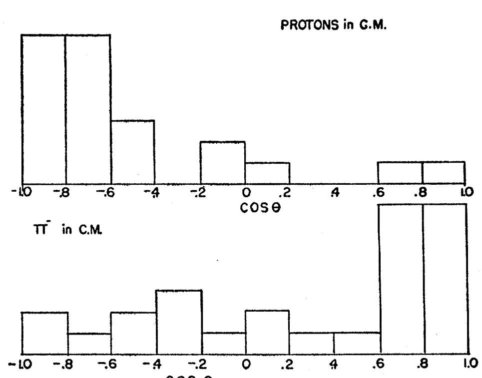

FIG. 3. Angular distribution of p and π^- from the reaction $\pi^-+p\rightarrow\pi^-+p+\pi^0$.

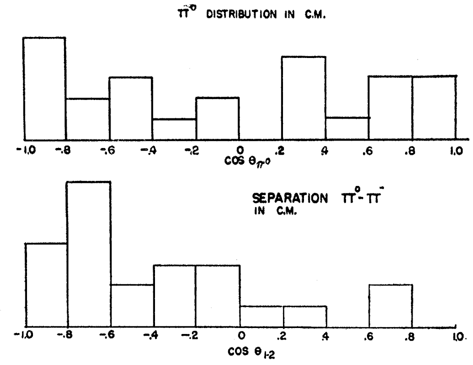

FIG. 4. Angular distribution of π^0 in the c. m. system and the distribution of π^0 with respect to the π^- in the c. m. system.

forward in the center-of-mass system. The elastic scatterings seem to break into two categories: a diffraction-type scattering which gives deflections of 10°–30° in the center-of-mass system, and a low-angular-momentum interaction which gives a big deflection and change of momentum in the center-of-mass system. The π^- and π^0 have rather different angular distributions. The π^- and π^0 seem to have a tendency to come apart with a rather large angular separation. This angular separation between mesons seems to be characteristic of the cases in which $\pi^++\pi^-+n$ are produced. In the center-of-mass system the π^0, π^-, and proton have on the average about equal momenta.

If the angular distributions of the π^0 and π^- are different, it seems to rule out the possibility of a statistical-type theory of these interactions.

The authors wish to thank Dr. Cool, Dr. Piccioni, and Dr. Madansky for their aid in making the first exposures and also Dr. Hill and Dr. Widgoff for making the later exposures. The authors wish to acknowledge the scanning aid given by Miss P. Griesbach, Mrs. E. Awschalom, H. Fisher, B. Munir, and J. Klose. It is a real pleasure to acknowledge the cooperation and hospitality of Dr. G. B. Collins and Dr. E. O. Salant.

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Nuclear Potential Well Depth*

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FESHBACH, Porter, and Weisskopf[1] have shown that the pattern[2] of variation of the total cross sections of heavy nuclei with neutron energy and nuclear size can be explained by representing the neutron-nucleus interaction as that of a square well with absorption

$$\begin{array}{ll} V(r)=-19\text{ Mev}(1+i0.05), & 0\leq r\leq R\equiv 1.45A^{\frac{1}{3}}\times 10^{-13}\text{ cm};\\ V(r)=0, & r>R. \end{array}$$

Bohr and Mottelson[3] have pointed out that the value of 19 Mev used for the real part of the potential is considerably smaller than that necessary to account for the bound levels of heavy nuclei as described by the shell model. It is the purpose of this note to show that a well depth of about 40 Mev, consistent with the shell model, is indicated by the scattering data.

A most striking characteristic of the total cross-section pattern is the variation of the average low-energy neutron-nucleus cross section as the value of A, the number of nucleons in the nucleus, increases from about 100 to about 200. A characteristic well-type S-wave resonance behavior occurs at about $A=150$. Using a square-well shape, and neglecting absorptive effects for simplicity, it is clear that κr will at resonance be equal to $(N+\frac{1}{2})$ for $r=R$,

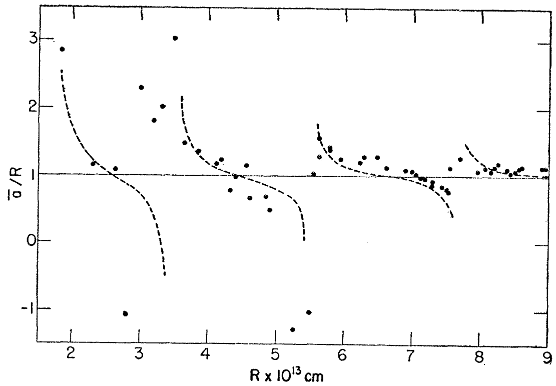

FIG. 1. The points show the variation of the average scattering length $\bar{a}$ divided by the nuclear radius R, as a function of R. The dashed curve represents the variation of a/R to be expected from a well with the parameters: $V=-42$ Mev, $0\leq r\leq R$; $V=0$, $r>R$; $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm.

where κ is the wave number associated with the potential well depth. If a well depth of 19 Mev is chosen, the resonance near $A=150$ will be the third S-wave resonance, and N will equal 2. However, the following examination of the low-energy neutron cross sections of lighter elements indicates that the resonance near $A=150$ is the fourth S-wave resonance, not the third, hence $N=3$ and the proper well depth must be about $[(3+\frac{1}{2})/(2+\frac{1}{2})]^2\times19$ Mev, or about 40 Mev.

Ford and Bohm[4] have previously discussed variations of the thermal neutron scattering length, a, with nuclear size, and concluded that a nuclear potential well of about 40-Mev depth was in agreement with the data. Since scattering lengths are strongly affected by individual, narrow, low-energy resonances, which the optical theory treats only on the average, it has not been clear that the deviations of the values of a from that expected from hard sphere scattering, for example, were significant of anything but the presence of nearby narrow energy levels.[5] Such fluctuations, which are not pertinent to the theory, can largely be eliminated by using for the value of the scattering length an average value defined as $\bar{a}=(\sigma_0/4\pi)^{\frac{1}{2}}$ where σ_0 represents the low-energy cross section averaged over narrow resonances. Values of $\bar{a}$, divided by R, are shown in Fig. 1, plotted against R, the nuclear radius. These values are taken primarily from total neutron cross-section measurements made at the University of Wisconsin by Barschall, Miller, Bockelman, and others.[6] Any estimate of this average cross section at zero energy is necessarily subjective but in only a few cases are the uncertainties in $\bar{a}$ greater than 5 percent, while the variations of interest are much larger than five percent. The sign of $\bar{a}$ is taken as negative when the measured coherent scattering lengths are negative.[7] Also plotted on Fig. 1 is the variation of a to be expected from a square well 42 Mev deep. The data show clearly that the number of resonances observed is that which one would expect from a well about 40 Mev deep, verifying the conclusions of Ford and Bohm.[4]

It seems likely that the increase of well depth from 19 Mev to about 40 Mev will not invalidate the general success of the Feshbach, Porter, and Weisskopf[1] formulation in predicting the variation of average neutron cross section with energy and nuclear radius near $A=150$, since application of the Sturm-Liouville theorem leads us to expect P- and D-wave resonances to bracket the S resonance, as a function of either neutron energy or nuclear radius, in much the same way, near $\kappa R=7\pi/2$, as near $\kappa R=5\pi/2$, independent of details of well shape. Preliminary calculations by Porter[8] seem to substantiate this view.

Conversations and correspondence with Dr. C. E. Porter, Dr. Ben Mottelson, and Dr. H. H. Barschall have contributed in an important way to this work.

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Level Densities in Heavy Nuclei of 10- to 20-Mev Excitation

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It is quite evident that nuclei of 8- or 10-Mev excitation or less cannot be treated as excited Fermi gases of all the nucleons present. Hughes and his co-workers[1] have shown that capture cross sections for thermal and 1-Mev neutrons are a function of eveness or oddness of the atomic number of the target nucleus and a particularly pronounced function of the magic property of the target nucleus. The difference between even and odd Z, or between magic nuclei and their neighbors is entirely a matter of how the last few nucleons are bound. And if the behavior of two or three nucleons makes a great difference in the level densities of the compound nucleus as Hughes[1] and others[2] have shown to be the case, then a statistical theory cannot be valid. Moreover, it is clear that few of the total number of nuclear particles participate in the excitation.

Further evidence is supplied by Feshbach, Porter, and Weisskopf,[3] who show that a surprisingly good fit to Barschall's[4] total neutron scattering data from 1–3 Mev is obtained by scattering in a nuclear well in which the mean free path of a neutron in a 10-Mev excited nucleus is 20 nucleons long. This means that in nuclei of 100–200 atomic mass number only 5–10 nucleons can participate in the excitation. As Feshbach, Porter, and Weisskopf[3] have further observed, however, the data of Phillips, Davis, and Graves[5] on inelastic neutron cross sections at 14 Mev, in which the target nuclei appeared totally "black," indicate that all the nucleons in the target nuclei participate in a compound nucleus excitation of around 20 Mev.

We assume a general level-density formula whose energy dependence is common to many different nuclear models:[6–9]

$$\omega(E_{\max}-E)=\mathrm{const}(E_{\max}-E)^{-1}\exp[KA'(E_{\max}-E)]^{\frac{1}{2}},\quad(1)$$

where $(E_{\max}-E)$ is the excitation energy of the nucleus, A' is the number of nucleons participating in the excitement, and K is an experimentally fitted parameter. We make a linear interpolation for A' between E_0, the excitation energy below which few nucleons are able to participate in the excitation, and E_1, the excitation energy above which all the nucleons can participate in the reaction. We then obtain for the spectrum of emitted particles from a nucleus left with an excitation energy less than E_1 and well above E_0

$$I(E)dE=\mathrm{const}E\sigma_c(E)[E_{\max}-E-E_0]^{-1}\times\exp[KA(E_{\max}-E_0-E)^2/(E_1-E_0)]^{\frac{1}{2}}dE,\quad(2)$$

where A is the atomic mass number of the nucleus and $\sigma_c(E)$ is the capture cross section for the inverse event. Defining the temperature as

$$1/T=-\frac{d}{dE}\ln[I(E)/E\sigma_c(E)],\quad(3)$$

we obtain, using Eqs. (1) and (2),

$E_{\max}-E\leqslant E_1$:
$$1/T=[KA/(E_1-E_0)]^{\frac{1}{2}}-[E_{\max}-E_0-E]^{-1};\quad(4)$$

$E_{\max}-E\geqslant E_1$:
$$1/T=\tfrac{1}{2}[KA/(E_{\max}-E_0-E)]^{\frac{1}{2}}-[E_{\max}-E_0-E]^{-1}.\quad(5)$$