Rearrangement Collisions

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THE theory of exchange and rearrangement collisions has been discussed recently, in *The Physical Review*, by several authors.[1–4] The scattering amplitude determining the cross section for a transition between stationary states i and j, wave functions ϕ_i and ϕ_j, may be written, to Born's approximation, in either of the forms $\langle\phi_i|U|\phi_j\rangle$, $\langle\phi_i|V|\phi_j\rangle$, U and V being the interaction potentials between the separated systems before and after the rearrangement, respectively. It is well known that these expressions are identical[5] provided ϕ_i, ϕ_j are the exact unperturbed wave functions, but that they may differ seriously when approximate wave functions are employed. Under these circumstances, it has been claimed[1,2] that the scattering amplitude containing the *prior* interaction U is "correct" and is to be preferred to that containing the *post* interaction V.

The Born scattering amplitude satisfies the requirement of the reciprocity theorem[6,7] that the scattering amplitudes, for the transition $i\rightarrow j$ and for the time-reversed transition $j\rightarrow i$, shall be identical. However, for the time-reversed collision V is the *prior* and U the *post* interaction, so that it is *a priori* meaningless to assert the superiority of either the post or the prior forms of interaction, when used in conjunction with approximate unperturbed wave functions.

It may be remarked, that if approximate cross sections are required that satisfy the reciprocity theorem, it has been the reasonable practice to take either the geometric[8] or the arithmetic[9] means of the post and prior amplitudes, although no theoretical justification can be given for this procedure.

1 S. Altshuler, Phys. Rev. **91**, 1167 (1953).
2 S. Altshuler, Phys. Rev. **92**, 1157 (1953).
3 S. Borowitz and B. Friedman, Phys. Rev. **89**, 441 (1953).
4 A. Dalgarno, Phys. Rev. **91**, 198 (1953).
5 L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 230.
6 Bates, Fundaminsky, Leech, and Massey, Trans. Roy. Soc. (London) **243**, 93 (1950).
7 B. A. Lippman and J. Schwinger, Phys. Rev. **79**, 469 (1950).
8 M. H. Hebb and D. H. Menzel, Astrophys. J. **92**, 408 (1940).
9 R. S. Christian and J. L. Gammel, Phys. Rev. **91**, 100 (1953).

Effect of Deuteron Formation on Multiple Meson Production*

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IN a statistical theory of multiple meson production such as that proposed by Fermi,[1] it is supposed that the probabilities of various competing final states are determined by statistical considerations. This is made reasonable by the argument that in a high-energy collision, the energy available for the reaction is momentarily concentrated in a small volume of very high excitation in which a thermal equilibrium is very rapidly established. Even if this situation actually exists, however, it is easy to see that an equilibrium established in this region does not simply determine the weighting of the final states. One can suppose that in the region of high temperature, very strong interactions rapidly bring about equilibrium, but as the particles leave this volume, the less strong but longer range interactions become predominant. These serve to perturb the wave functions of the outgoing particles in such a way as to enhance the transitions into a final state in which the amplitude at small distances is large relative to the asymptotic value. These "final-state interaction" effects are very familiar in low-energy phenomena where, for example,[2–4] the final state 2-nucleon interaction increases single meson production by 2 or 3 orders of magnitude. Three types of such final state interactions can be expected to be appreciable in multiple meson production, namely, the nucleon-nucleon, meson-nucleon, and meson-meson interactions. It is the purpose of this note to point out the importance of the first of these, which manifests itself most simply through deuteron formation.[5]

Let us consider the volume in phase space for the production in a two-nucleon collision of n mesons and two nucleons. Treating the mesons and final nucleons nonrelativistically, we have

$$\rho_F=(2\pi)^{-3(n+1)}\int d\mathbf{q}_1\cdots\int d\mathbf{q}_n d\mathbf{p}\,\delta\!\left(T_0-\frac{p^2}{M}-T_1-\cdots-T_n\right),\quad (1)$$

where T_i is the kinetic energy of the ith meson, $T_0=E_0-n\mu$ is the total available kinetic energy, and $\mathbf{p}$ is the relative nucleon momentum. The phase space for the production of the same number of mesons and a deuteron is obtained from this by the simple modification,[2,3]

$$(\rho_F)_D=(2\pi)^{-3n}\int d\mathbf{q}_1\cdots\int d\mathbf{q}_n\delta(T_0-T_1-\cdots-T_n)\,|\psi_D(0)|^2,\quad (2)$$

where $\psi_D(0)$ is the deuteron wave function in coordinate space evaluated at the origin. Let us for simplicity consider only the ratio of these two processes. The integrals of Eqs. (1) and (2) are easily carried out to give the simple result:

$$\sigma(n\pi+D)/\sigma(n\pi+N+P)=(\rho_F)_D/\rho_F=\frac{8\pi^{\frac{3}{2}}|\psi_D(0)|^2\Gamma(\frac{3}{2}n+\frac{3}{2})}{M^{\frac{3}{2}}(E_0-n\mu)^{\frac{3}{2}}\Gamma(\frac{3}{2}n)},\quad (3)$$

where Γ is the gamma function. In evaluating this ratio, we shall use the Hulthén wave function. It is interesting to note that for large n this ratio takes on the simple form:

$$\sigma(n\pi+D)/\sigma(n\pi+N+P)\underset{n\gg 1}{=}\frac{8\pi^{\frac{3}{2}}|\psi_D(0)|^2}{M^{\frac{3}{2}}(E_0-n\mu)^{\frac{3}{2}}}\left(\frac{3n}{2}\right)^{\frac{3}{2}},\quad (4)$$

so that the importance of the effect increases rapidly for large multiplicities.

The result of Eq. (3) is given in Fig. 1 for various meson multiplicities and for various values of $E_0-n\mu$ (E_0 the total energy) above threshold. The very large effect near threshold is apparent, as is also the strong dependence on multiplicity.

For comparison with this result, we have also evaluated this ratio for the case in which the matrix elements depend linearly on

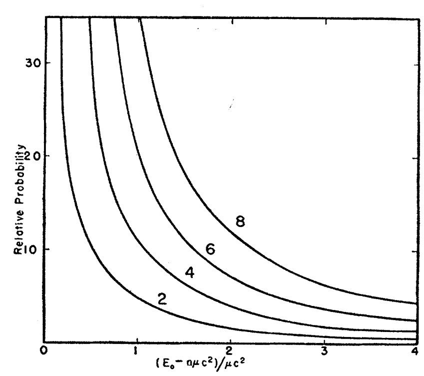

Fig. 1. Probability of multiple meson production with deuteron formation, relative to the statistical result neglecting neutron-proton interaction The curves are labeled by multiplicity of mesons produced.

the meson momentum, as might be expected in pseudoscalar meson theory. The result is very easily evaluated as in the simple case of Eqs. (3) and (4), to give

$$\sigma(n\pi+D)/\sigma(n\pi+N+P)=\frac{8\pi^{\frac{3}{2}}|\psi_D(0)|^2\Gamma(5n/2+\frac{3}{2})}{M^{\frac{3}{2}}(E_0-n\mu)^{\frac{3}{2}}\Gamma(5n/2)}.\quad (5)$$

The effect is similar, except that the enhancement of the deuteron cross section is larger (by a factor of 2.5 to 3.0) than for the case of constant matrix elements.

These results are only valid in the nonrelativistic limit for all of the final particles involved; it is, however, easy to verify that the