An Experimental Measurement of the Gyromagnetic Ratio of the Free Electron*

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The gyromagnetic ratio of the free electron is measured by a method which is an extension of the classical double-scattering experiment. A magnetic field is interposed between the first and second scattering foils, whose direction is parallel to the path followed by the electrons. The electron spins precess in the magnetic field, resulting in a rotation of the plane of maximum asymmetry, as observed after the second scattering event. In the experiment reported, the rotation is approximately 1800 degrees. In the motion of the electron between the two scatterers the small lateral component of velocity gives rise to a "cyclotron" motion whose frequency is, theoretically, the same as the spin precession frequency to within about one part in a thousand. The cyclotron motion, therefore, furnishes a convenient reference frequency, but it also introduces problems in that it causes the asymmetries which have their origin in geometrical misalignment, finite aperture, etc., to follow the rotation of the spin asymmetry. By comparing all measurements made with the foils of high atomic number with measurements made with an aluminum foil of equal scattering power, and by further precautionary procedures and cross checks, the spin asymmetry is separated from asymmetries of other origin. The result, for 420-kev electrons and gold scatterers, is $g=2.00\pm0.01$. Plans for a more precise measurement are mentioned.

INTRODUCTION

WHILE many experiments of great precision are available for the determination of the magnetic moment and the gyromagnetic ratio for the electron when bound in an atom, no such fortunate situation exists in regard to the measurement of these properties for the free electron. In fact, it has been only within recent years that the existence of a magnetic moment in the free electron has been demonstrated experimentally, although, of course, there were very strong theoretical reasons for believing that it was present. No experiment, prior to the one to be reported in this paper, has yielded a quantitative value for the moment, except in order of magnitude. Over the past half-dozen years a new interest in the problem of making a precise measurement of the magnetic moment of the free electron has developed, because of the discovery that in the case of bound electrons, the magnetic moment differs from the Bohr magneton by about one part in a thousand. The anomaly, which first made its appearance as a slight discrepancy between experimental and theoretical values in certain energy levels in atoms,[1–3] is now firmly and accurately established.[4–6] The theory, which connects the discrepancy with a correction to the magnetic moment of the electron,[7] has been worked out in detail.[8–10] While it is expected that the same correction will be found to apply to the magnetic moment of the free electron, it is by no means to be considered as a foregone conclusion, and therefore there is interest in the development of experimental techniques which will be capable of the required precision. All of the existing methods and proposals for measuring the magnetic moment or the g factor for the free electron should, therefore, be reappraised as to whether or not they have the inherent capability of attaining a precision of at least one part in a thousand, since this has become the important region, theoretically. In this paper we shall make this examination briefly, and shall then present the results of an experiment which, although it does not attain the accuracy mentioned, at least constitutes an opening wedge and furnishes possibilities for more precise experiments.

EXPERIMENTAL METHODS AND PROPOSALS

A type of experiment for measuring the magnetic moment of the free electron which has always had prominence in the literature, but in a negative fashion, is the one which is analogous to the Stern-Gerlach experiment. Bohr[11] pointed out that since in such an experiment a knowledge of both the moment and the classical trajectory of the electron would be implied, the scheme would encounter difficulties for reasons expressed by the uncertainty principle. Subsequent writers presented this argument as being more sweeping in character than was, perhaps, justified. It was taken to mean that no form of the Stern-Gerlach experiment for free electrons, was possible.[12] It is realized now that, while the uncertainty principle argument does preclude the simultaneous measurement of moment and trajectory for an individual electron, there remains the possibility of doing the experiment in a statistical fashion,

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[11] See W. Pauli, *Handbuch der Physik* (J. Springer, Berlin, 1933), Vol. 24, Part 1, p. 242.
[12] For example, see N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, London, 1949), pp. 65, 69, 74.

that is, by sending a large number of electrons through the apparatus and by attempting to use the detailed line shape to reveal the effects of the magnetic moment. The thing which the uncertainty principle argument does show to be inescapable is that the displacement, or splitting, of the line that one would be seeking to find would be smaller than the line breadth. While the extraction of information from the shape of a line has been done successfully in many instances, such an attack appears particularly unpromising in connection with the electron moment problem. Its solution could at most give an indication of the presence of the moment, and it would certainly not give the 0.1 percent accuracy which is now sought, for comparison with the recent theory.

The other experiment which is well known, and which is the only one to date which has given positive results, is the double-scattering experiment. This method depends upon the fact, shown theoretically by Mott[13] in 1929, that a beam of electrons undergoing single nuclear scattering is partially polarized in a direction normal to the plane defined by the incident beam and the scattered direction. A second scattering process exhibits an aximuthal asymmetry in intensity, if measured in the same plane. It is not possible to state here the set of conditions for which the asymmetry is expected to be a maximum, because calculations have been made only for one angle of scattering: 90 degrees. The asymmetry increases monotonically with Z but has a maximum with respect to energy. For 90-degree scattering and for $Z=80$ the optimum energy is about 150 kev, and the asymmetry 13 percent. Attempts to observe this asymmetry experimentally have been made over a long period of time,[14–19] and the results of the later experiments[17–19] show beyond doubt that an asymmetry of about the predicted magnitude exists. They are not quantitative experiments, however, in the sense of measuring the magnetic moment.

Another line of attack is based upon the magnetic resonance method and has given rise to proposals for experiments in two somewhat different forms. In both forms polarized electrons are trapped in stable orbits in a magnetic field. A radio-frequency perturbing field is then applied, and the frequency which destroys the polarization is determined. From the frequency which destroys the polarization, and the strength of the magnetic field, the value of the gyromagnetic ratio is obtained. The two methods differ in the way in which the electrons are polarized, prior to trapping, and in the way in which the presence or absence of polarization is determined after the application of the perturbing field. In the first method, electrons with an extremely sharp low-energy cutoff are sorted as to spin state by being allowed to encounter a very carefully adjusted potential hill, in the process of entering the part of the field in which they are to be trapped. Ideally, those with one spin orientation will pass and be trapped, while those with the opposite spin orientation will be rejected. Analysis, after the application of the perturbing rf, consists, again, in the sorting action of the potential hill in the process of letting the electrons escape from the trapping field. Since the sorting depends upon the exceedingly small difference which results from the spin interacting with the magnetic field, the experiment demands carefully determined electron energies and precise control of fields and potentials. An experiment along these lines was tried, without positive results, by Dicke,[20] and another one, differing in details, is now being prepared by Bloch.[21] The other form of the reso. nance method, proposed by Tolhoek,[22] but so far not tried, uses high-energy electrons, and employs double scattering as the means of polarization and analysis- The trapping procedure, with the application of the perturbing rf field, takes place between the first and second scattering events.

It should be remarked, in connection with both the proposals which involve resonance, that since the required perturbing frequency is almost identical to the cyclotron frequency, the strong coupling to the cyclotron motion might introduce serious difficulties. In order to achieve the accuracy of about one part in 10^5, which seems to be the ultimate goal of all such experiments, the motion would have to persist for the order of 10^5 cyclotron revolutions, and during this time considerable energy might be transferred from the perturbing field to the cyclotron motion. Nevertheless, the magnetic resonance methods just described are the only ones, with the exception of the proposed extension[23] of our own experiment, which have the inherent possibility of giving really quantitative results, that is, results of sufficient accuracy to reveal the correction to the electron moment.

A number of schemes have been suggested or tried which are based in one way or another upon use of aligned atoms. A very early attempt, by Myers and Cox,[24] to demonstrate polarization in an electron beam was a double-scattering experiment using magnetized iron foils as the scatterers. Fues and Hellman[25] proposed an experiment in which the polarized electrons were to be obtained by letting ultraviolet light act upon the beam of neutral, aligned atoms produced by a Stern-Gerlach apparatus. Analysis was to be accomplished by allowing the electrons to be recaptured by oriented ions, similarly derived from a Stern-Gerlach beam. The same

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authors[25] have suggested that photoelectrons, ejected from the aligned atoms of a magnetized iron foil, might be polarized. The analysis of polarization through the measurement of the transmission or scattering of the electron beam by magnetized iron foil has been proposed, and was, in fact, used, in the experiment of Myers and Cox.

Finally, there are several suggestions for means of obtaining polarized electrons which have not been developed at all, but which might be worthy of further consideration. It appears that it is feasible to align nuclei in a strong magnetic field at near absolute zero temperature. Beta rays emitted from nuclei so aligned might be polarized,[26] provided the kind of nuclear transition were properly chosen. Also in connection with beta disintegration, there is a possibility which does not require the prealignment of the nuclei. Where a beta ray is followed by a gamma ray, the spin orientation of the beta ray could be expected to be correlated with the directions of emission of the beta ray and gamma ray, provided, again, that a nucleus having favorable type of transition were selected for observation. The possibility of finding cases in the photoelectric effect in which the polarization of the photoelectron is correlated with the polarization of the incident light should be worth investigating.

ROTATION OF THE PLANE OF POLARIZATION IN THE DOUBLE-SCATTERING EXPERIMENT

The extension of the double-scattering experiment which we have introduced[23,27] is to interpose between the first and second scatterers a magnetic field, parallel to the path between the scatterers. This causes the electron to precess, and rotates the plane of polarization so that the plane of maximum asymmetry after the second scattering no longer coincides with the plane of the first scattering. By measuring the angle of rotation and knowing the magnetic field, the electron energy, and the distance, the gyromagnetic ratio for the electron may be found. If the magnetic field is intense enough and the path long enough to produce many revolutions of the plane of polarization, the measurement may be quite precise. The precision with which the plane of polarization may be ascertained is independent of the number of revolutions it has made between the scatterers. Therefore, the relative precision improves in direct proportion to the number of revolutions.

In the experiment to be reported here, five revolutions of the plane of polarization were produced, and the probable error in the resulting value for the g factor was $\pm\frac{1}{2}$ percent. Had it been possible to have more revolutions, the g factor could have been determined to a correspondingly greater accuracy, without the

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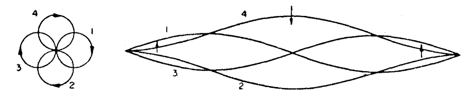

FIG. 1. The "cyclotron" or "beta-ray spectrometer" motion, shown for one focal length and for four initial directions of the lateral component of velocity. Left: looking along the magnetic field direction. Right: side views of the same paths. The arrows on one of the paths indicate the precession of the spin direction.

necessity of any greater absolute precision in the determination of the plane of polarization.

A fact which has a dominating influence upon the design of the experiment is that the orbital rotation frequency or "cyclotron frequency" of the electron in the magnetic field differs from the frequency of precession of the spin direction by the higher-order correction terms only, or by about one part in a thousand. The two frequencies are

$$\nu_c = eH/2\pi mc,$$
$$\nu_s = (eH/4\pi mc)g,$$

where

$$g = 2(1+\alpha/2\pi+\cdots).$$

The relativistic change in mass enters in the same way in the two equations. It has been shown that the above equation for the spin precession frequency (with g set equal to 2 exactly) results alike from semiclassical considerations and from Dirac theory.[28] Since one part in a thousand lies beyond the degree of precision of the present experiment, we must proceed on the assumption that the cyclotron and spin rotations are not separable, and we may use semiclassical concepts to describe the motion.

The cyclotron motion results only from the lateral component of momentum of the electron relative to the solenoid axis and so depends upon the finite angular spread which is admitted by the system of diaphragms. We are familiar with this motion, of course, in the solenoid type of beta-ray spectrometer. A diagram showing the motion, with the spin direction indicated by small arrows on one of the paths, and with the cyclotron motion exaggerated, is given in Fig. 1. The first consequence of the motion described, as far as this experiment is concerned, is the fact that all asymmetries in the beam, whether they are associated with the spin or not, rotate around together. Therefore, the mere fact that an asymmetry is observed which has the expected angle of rotation does not in itself prove that a polarization effect is being observed. The fact that it is a rotation of polarization that is being observed has, then, to be proved in other ways.

The second consequence of the type of motion we are concerned with is the fact that the beam comes to a focus at the end of each length corresponding to a cyclotron revolution. This means that the size and structure of the spot incident upon the analyzer foil,

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or its defining diaphragm, changes periodically, as the total angle of rotation is increased. This raises problems in comparing directly the results for different total angles of precession, unless they differ by an integral number of revolutions.

Neither of the above two consequences of the cyclotron motion is in any way fatal to the method, but nevertheless, the precautions and cross checks which have to be made on their account constitute a major part of the effort of the experiment.

All of the results of the cyclotron motion are not bad. The beta-ray spectrometer type focusing, which comes with the cyclotron motion, is of vital importance in preserving the intensity of the beam. The entire beam which is admitted to the system by the diaphragms at the polarizer end of the solenoid is transmitted to the analyzer, regardless of the distance. Without this advantage, the experiment would not have been possible. Also, the fact that the spin precession and cyclotron frequencies are so nearly the same contains an advantage in that, experimentally, the one can be measured directly with reference to the other, and the resulting value of the g factor, therefore, does not depend upon accurate measurements of the magnetic field and the electron energy.

APPARATUS

The general layout of the experiment is shown by the schematic diagram in Fig. 2. With this as a guide, the various components of the apparatus will be described, and the important problems in the design of the system will be discussed.

Electron Accelerator

The electron beam was obtained from the injector system of the Michigan synchrotron. This was a 3-gap accelerator tube, powered by a Cockroft Walton type condenser-rectifier voltage multiplier. The electron beam from this apparatus had 420-kev energy, regulated to ± 0.8 kev. The electron moment experiment was set up alongside the synchrotron, and the electron beam was borrowed nightly. The change-over was made simply by deflecting the beam, by means of a small magnet, into a side pipe which led to the electron moment apparatus, and by making a minor circuit change to render the beam continuous rather than pulsed. The necessity of using the synchrotron injector in the form in which it existed, with a minimum of disturbance to the synchrotron program, is what dictated the value of the electron energy, 420 kev, which was used in the electron moment experiment. We call attention to the reason for the choice because, as is well known, 420 kev is considerably above the energy which should theoretically give the maximum asymmetry in the double-scattering experiment.

Solenoid

The magnetic field between the scatterers was provided by solenoid consisting of a single layer of $\frac{1}{4}$-in. copper tubing wound on a 6-in. brass tube, 30 ft long. The current was furnished by a motor generator and was maintained constant to 0.1 percent by means of an electronic regulator circuit. The resistance of the coil was about 1 ohm, and the current used in the experiments to be reported was about 58 amp. Cooling was

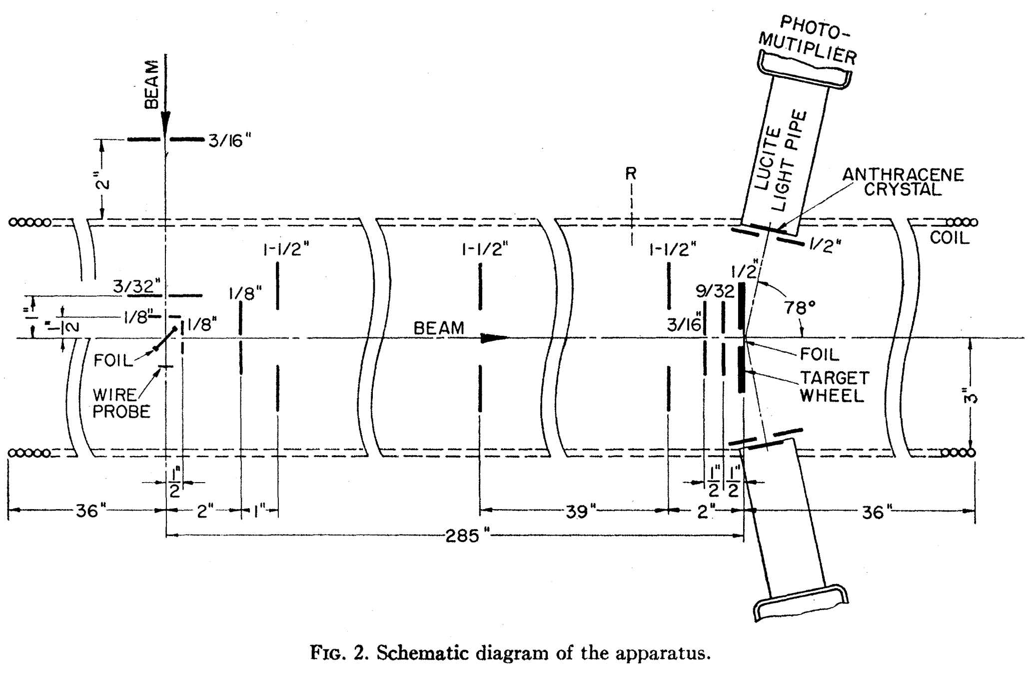

FIG. 2. Schematic diagram of the apparatus.

accomplished by water flowing through the copper tubing.

Earth's Magnetic Field Compensating Coils

The earth's magnetic field was compensated by a set of four rectangular coils, each 1 ft×30 ft, mounted outside the solenoid in such a way that the wires made the edges of a 1-×1-×30-ft rectangular parallelopiped. Each coil contained 20 turns and the currents used were the order of 1 amp, supplied from storage batteries.

Scattering Foils

The first scattering foil (polarizer) was supported in a ring of ½-in. inside diameter. Its plane was at 45 degrees to both the incident beam and the solenoid axis, and it was in the transmission position. Gold foil (0.135 mg/cm^2) was always used for the polarizer. At times it was removed so that measurements of the intensity of scattering from the edges of the ring could be made. The analyzer foil was mounted so that the beam was incident normally upon it. The foil holder in this case was multiple, consisting of a disk with four apertures, one open and the others covered by foils of gold, 0.135 mg/cm^2, silver, 0.23 mg/cm^2, and aluminum, 0.67 mg/cm^2. The disk could be rotated from outside the vacuum, so that rapid changes from one to another could be made for intercomparison.

There can be no doubt that the foils were thin enough so that true single scattering was obtained. For the gold foil, considering normal incidence, the angle for which $\pi p^2nt=1$ is approximately 2 degrees. (p is the classical impact radius, n is the number of atoms per cc, and t is the thickness of the foil.) According to Wenzel's criterion, it is only necessary that the scattering angle (90° and 78° in our case) be several times the angle for which $\pi p^2nt=1$. The silver and aluminum foils were about the same, with respect to the criterion. While the foils were much thinner than necessary from the standpoint of insuring single scattering, they were thick enough to give an adequate counting rate after the two scattering processes, and the thicknesses were chosen mainly for the latter consideration. The counting rate at each of the scintillation counters was about 100/sec. This was obtained with a current of approximately one microampere incident upon the polarizer foil. The ratio is consistent with the value obtained using single-scattering theory and the apertures in the apparatus.

Counters

Two scintillation counters were used to detect the electrons scattered from the analyzer. Each consisted of a No. 5819 photomultiplier, a Lucite light pipe 4 in. long, and an anthracene crystal. The anthracene crystal, 5 mm thick, was about flush with the inner surface of the solenoid coil so the window of the No. 5819 was about 4 in. outside the coil. The problem here, of course, was to reduce the magnetic field in the photomultiplier, by a combination of magnetic shielding and separation by the light pipe, to a value small enough so that it would not influence the measurements. Each photomultiplier was enclosed in an iron pipe of ⅜-in. wall thickness. In addition a laminated iron shield of 10 layers of 0.014-in. sheet iron was built up around the iron pipe. It was found that this amount of shielding, plus the separation provided by the light pipe, gave the required isolation. The test applied was to make the scintillation counter count by means of a gamma-ray source, and then to compare its counting rates with and without current in the solenoid.

The circuits consisted of two parallel channels, each comprising a linear amplifier, pulse-height discriminator (integral type), and scaler.

Rotation of the Counter Assembly

The entire analyzer end of the apparatus, consisting of about the last 3½ ft of the solenoid, and including the analyzer foil assembly, diaphragms, and the two opposed scintillation counters, could be rotated about the axis of the solenoid to allow the measurement of the asymmetry in the scattered intensity throughout the 360 degrees of angle about the solenoid axis. The location of the rotating joint is indicated at R in Fig. 2.

Diaphragm System

The important geometrical features of the apparatus, with the locations and diameters of the diaphragms, are shown in Fig. 2. The diaphragm outside the solenoid, which defines the entering beam, is actually above the plane of the drawing, since the entering beam bends downward in going from the edge of the solenoid to the center, across the magnetic field. The systems of small diaphragms close to the polarizer and analyzer foils serve to define a small area in the center of each foil which electrons can strike, and to prevent electrons from being scattered from the foil holders. The real aperture of the system, which we call γ, the maximum angle an electron path can make with the solenoid axis, is fixed by a 1½-in. diameter diaphragm located at the last antinode (½ focal length) before the analyzer foil. When the magnetic field is set so as to give five focal lengths between polarizer and analyzer, γ is 2.25 degrees. For a given value of γ the required size and position of the diaphragm are, clearly, dependent upon the magnetic field. However, in the experiments that were carried out the field was varied through only a small range (cyclotron rotations of 1781, 1814, and 1847 degrees). A single compromise position for the diaphragm was used which maintained γ constant to within 10 percent.

Shielding

The fact that over 24 ft of distance separated the analyzer foil and counters from the polarizer and the electron source greatly simplified the shielding problem. The two ends of the solenoid were in different rooms, separated by a concrete wall 3 ft thick. In addition, a patch of lead $\frac{1}{2}$ in. thick was placed on the wall to shadow the counters from the radiation originating in the electron accelerator. The signal-to-background ratio, which will be explained later, was approximately 10 to 1.

ALIGNMENT, ADJUSTMENT, AND PRELIMINARY TESTS

Alignment

The diaphragms defining the beam incident upon the polarizing foil were aligned by using the wire probe, shown in Fig. 2. By this means the transmitted beam was located and made to fall upon the center of the foil. Because of the bending of the beam by the magnetic field, the alignment had to be adjusted for each value of the field. The diaphragms between foils were aligned by optical means. A filament was placed at the position of the polarizer foil, and the holes in the diaphragms were reduced to pinholes by means of annular fillers. The actual alignment was then done by eye.

The preliminary alignment of the magnetic field was done as follows. A small electron gun, giving a beam of about 1000-ev energy was placed on the solenoid axis at the position of the polarizing foil and a fluorescent screen was placed at the position of the analyzer foil. At this low electron energy the spot on the fluorescent screen was essentially the same size as the source, and the beam followed quite exactly the magnetic lines of force. The currents in the four correcting coils were simply adjusted until the spot fell in the center of the analyzing foil. This test showed that the centers of the two foils were on the same line of force; it did not show that the line of force was a straight line. The experiment was not highly sensitive to the latter consideration, since the defining diaphragms were located near the two ends only. The solenoid and correcting coils were

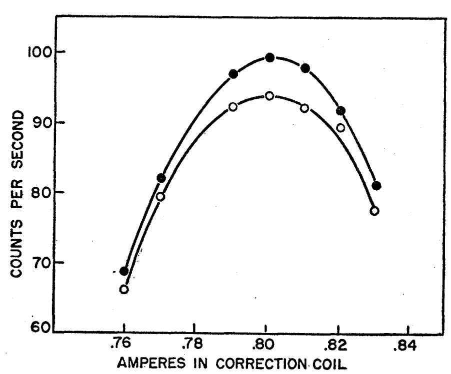

FIG. 3. Typical plot used for centering the beam on the analyzer foil. Circles and dots refer to the two counters. Cyclotron rotation, 1814 degrees.

mechanically straight, and it was assumed that the field was sufficiently straight.

The final alignment was made by using the 420-kev doubly scattered beam itself. By making a plot of the counting rates in the two counters while the beam was slowly moved by means of one of the pairs of earth's field correcting coils, the setting giving the maximum current at the analyzer foil could easily be found. Individual plots of this kind were made for the vertical and horizontal directions, after the apparatus had been otherwise aligned for each value of magnetic field. One of the plots is shown in Fig. 3.

As expected, the maxima were sharper when the focal spot was small (near a focal condition) than when it was off focus. There was, however, no difficulty in setting the correction coils for the maxima at any of the three values of magnetic field used.

Preliminary Tests of the Counting System

An important method by which background was reduced was the use of a pulse-height discriminator, which, as far as possible, admitted only those pulses due to electrons which had not suffered energy losses. To determine how well this could be done, two tests of the system were made. In the first, the 0.66-Mev internal conversion line of Cs^{137} was resolved. The Cs source was mounted on a thin Zapon film which was then coated with a thin aluminum layer. Runs were made with one counter, at a series of discriminator biases, and a differential curve obtained by applying the subtraction process to the data. The other test was the resolution of the 420-kev peak due to the electrons from the accelerator, elastically scattered. In obtaining the curves one counting channel was kept at fixed gain and discriminator setting in order to take account of variations in beam current, while a bias *vs* counting rate curve was run on the other channel. The experiment was then repeated with the channels interchanged. The results, for the Cs^{137} and the 420-kev electrons, are shown in Fig. 4. The discriminator bias for each channel was set to operate, during the electron polarization experiments, at a value corresponding to the point of maximum slope on the left side of the peak in the figure.

Magnetic Field Calibrations

As mentioned earlier, the value of the g factor may be obtained from a direct comparison of the rotation of the plane of polarization and the cyclotron rotation, without the necessity for precise measurements of the magnetic field (which would have to be averaged over the path) or of the energy of the electron beam. This method was adopted, and a means had to be devised for measuring the number of cyclotron revolutions as a function of the current in the solenoid.

The intensity of the beam reaching the analyzer end of the solenoid was large enough so that a spot was easily observed visually on a fluorescent screen. The

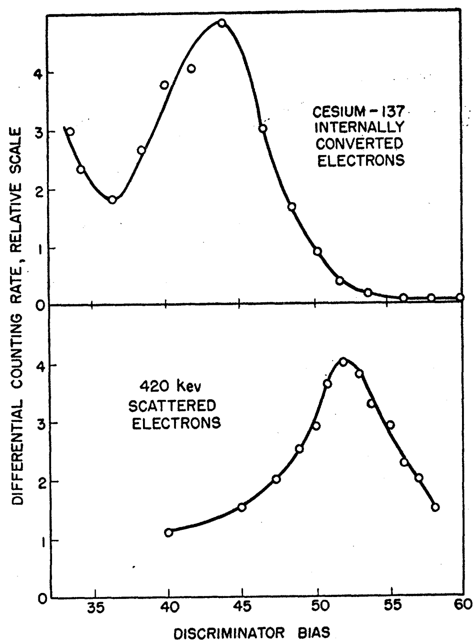

FIG. 4. Discriminator curves after differentiation. The abscissa scales of the two curves are not directly comparable, because the amplifier gains were not the same.

analyzer foil holder and the small defining diaphragms were removed, and the fluorescent screen was placed on the solenoid axis at a position $3\frac{1}{16}$ in. from the analyzer foil position toward the polarizer. When focused, the image spot was not sensibly larger than the object. The object in this case was the actual source of scattered electrons, which was (when projected onto the plane normal to the solenoid axis) a spot $\frac{3}{32}$ in. in diameter. The accuracy of the method may be estimated from the geometry of the system. The observations were made with the fifth focus on the fluorescent screen, and with an aperture angle γ of 2.25 degrees. Under these conditions it can be shown by simple geometry that the spot diameter changes by approximately $\frac{1}{4}$ in., or about twice the diameter of the focused image, for each 1 percent change in solenoid current. The measurements made were therefore estimated to be accurate ± 0.5 percent. The result of a number of determinations of the focal condition, after correction for the length factor mentioned above, was that 57.45 ± 0.30 amp corresponded to five cyclotron revolutions between the two scattering foils.

Bending at Entry and Exit

The beam from the accelerator, before striking the polarizer, has to travel at right angles to the magnetic field in the solenoid, a distance equal to the radius of the solenoid. This produces a change in direction of about 17 degrees. There is a more gradual bending of the beam outside the solenoid, and in the opposite sense. The direction of incidence of the beam at the polarizer target was measured by means of the slit system and the wire probe. A series of measurements gave a mean of 12.5 ± 1 deg with respect to the horizontal, at 57.9 amperes in the solenoid.

The bending of the electron paths between the analyzer foil and the counter windows takes place entirely within the uniform field of the solenoid so the angle can be calculated more accurately than it can be measured. The central path is a helical arc whose chord extends from the center of the analyzer foil to the center of the counter window, the latter being $2\frac{1}{2}$ in. from the solenoid axis. The angle between the counter axis and the direction of emergence of this path from the analyzer foil is 8.7 degrees at 57.9-amp current in the solenoid.

ASYMMETRIES NOT ASSOCIATED WITH SPIN AND THEIR ELIMINATION FROM THE DATA

As mentioned earlier, certain sources of asymmetry having nothing to do with the polarization effect are inherent in the experiment, and present a problem especially because they follow the polarization asymmetry as it rotates around. In this section the origins of these asymmetries will be pointed out, and the methods of dealing with them will be described.

There is only one cause of asymmetry, other than polarization, which does not have its origin in instrumental alignment or adjustment. It is the one which is the result of the nonlinearity in the relation between the scattering cross section and angle, and the finite aperture. Consider the typical double-scattering geometry sketched in Fig. 5. Four paths, AC', AD', BC, and BD lead to the counters. Because of the small differences in scattering angles, due to the finite aperture, BD contributes the most and AD' the least. AC' and BC are intermediate and contribute equally, because they differ only by the order in which the two scattering angles occur. However, the contribution of BD and AD' is not equal to that of AC plus BC because of the nonlinearity of the relation between scattering cross section and angle. Thus an asymmetry is produced which depends upon the angle γ. An integration over a circular aperture gives an asymmetry (ratio of intensities at C and D) of $1+2\gamma^2$. For $\gamma=2.25°$, which is the value it had in the experiment, the asymmetry is 0.3 percent. The sense of the asymmetry is opposite to that produced by polarization.

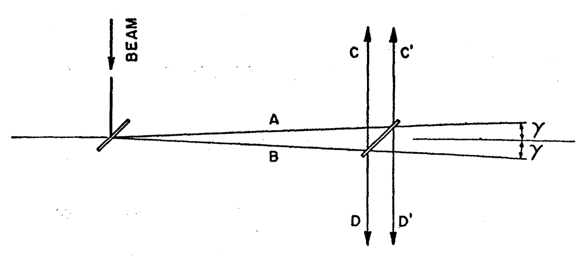

FIG. 5. The introduction of a nonspin asymmetry in the double-scattering experiment by a finite aperture.

A somewhat related way in which asymmetry may be produced has to do with misalignment of the diaphragm system. Applying the argument of the above paragraph it is clear that if there is any misalignment which makes the maximum angle admitted greater for, say, the A path than for the B path, an asymmetry at the counters will be produced. The more holes the beam has to pass through, the greater is the chance for such an effect. For this reason, it was considered important to avoid overdetermining the aperture by using more diaphragms than necessary, and to locate the diaphragms only near the two ends of the system.

A strong asymmetry is expected to come from the difference in response of the two counter channels. A somewhat different aperture at the scintillation crystal may be responsible for a small part of this, but in the main it is due to differences in the photomultiplier tubes, amplifiers, and pulse-height discriminators. Before measurements were made, the counting rates in the two channels were balanced to within about 5 percent by the use of the Cs^{137} source, and by means of the scattered electrons themselves.

The multiplicity of ways in which spurious asymmetries enter into the experiment would render extremely tedious, as well as risky, any attempt to keep the apparatus in delicate enough adjustment so that the spin asymmetry would stand out above the other asymmetries, in an absolute sense. A far more reliable way of dealing with the spurious asymmetries is to obtain and use, as far as possible, only ratios and not

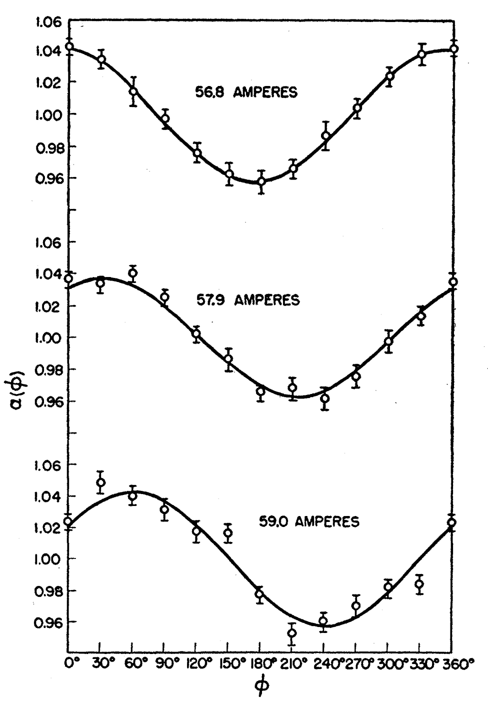

FIG. 6. Asymmetry in intensity as a function of direction, after scattering by a gold polarizer and a gold analyzer.

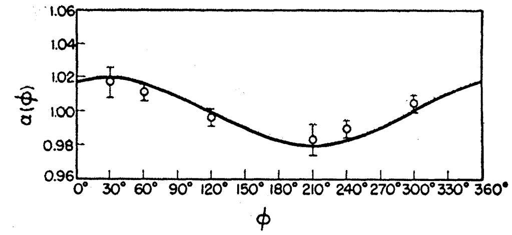

FIG. 7. Asymmetry in intensity as a function of direction, after scattering by a gold polarizer and a silver analyzer.

absolute quantities. Such a scheme was worked out for the present experiments, and it will be best understood by going through the procedure step by step.

(1) The measurement at each angle setting of the counting head was made with and without the analyzer foil in the beam. This was done by rotating the foil holder wheel back and forth between two positions. The counting rate recorded for each channel was the difference between the rates obtained with and without the foil in place. The net rate for channel 1 was $R_1(\phi)-r_1(\phi)$, and for channel 2, which was diametrically opposite, it was $R_2(\phi+\pi)-r_2(\phi+\pi)$, R and r standing for the rates with and without the foil, and the subscript identifying the channel. Changes in background associated with the position of the head were in this way eliminated. The counting rates without the foil were approximately 10 percent of those with the foil.

(2) The pair of measurements (with and without foil) at each setting of the head was repeated, with the head rotated 180 degrees. Two asymmetries were thus obtained:

$$K_1=\frac{R_1(\phi)-r_1(\phi)}{R_2(\phi+\pi)-r_2(\phi+\pi)},$$

and

$$K_2=\frac{R_2(\phi)-r_2(\phi)}{R_1(\phi+\pi)-r_1(\phi+\pi)},$$

in which the background count was eliminated. These two measurements allowed the counter asymmetry to be eliminated, giving

$$N(\phi)=[K_1(\phi)K_2(\phi)]^{\frac{1}{2}}$$

as the apparent asymmetry. The validity of this way of eliminating the counter channel asymmetry was checked by evaluating $[K_1(\phi)/K_2(\phi)]^{\frac{1}{2}}$, the counter asymmetry, for all angles. It was found to be independent of angle to within the expected statistical fluctuation.

(3) The procedures described in 1 and 2 above were repeated, substituting an aluminum foil of equivalent scattering power for the gold foil, in the analyzer position. (Gold was used as the polarizer in all cases.) The aluminum analyzer was expected to give the asymmetries associated with diaphragm and beam misalignment and finite aperture, in approximately the same degree as they were given by the gold, but it was

TABLE I. Experimental results for the three different values of the magnetic field. All angles are given in degrees. ϕ_c is the cyclotron rotation between scatterers, ϕ_d is the sum of the angles of deflection at entry and exit to the solenoid field, B and β are the amplitude and phase constants, respectively, in the cosine wave which was fitted to the data by least squares, ϕ_s is the angle through which the asymmetry was rotated relative to the direction of the beam before entry into the solenoid field, and g is the gyromagnetic ratio, which is $2(\phi_s-\phi_d)/\phi_c$.

ϕ_c	ϕ_d	B	β	ϕ_s	g
1781 ± 9	20.8 ± 2	0.041	7 ± 4	1807 ± 4	2.00 ± 0.01
1814 ± 9	21.2 ± 2	0.037	33 ± 5	1833 ± 5	2.00 ± 0.01
1847 ± 9	21.6 ± 2	0.042	60 ± 4	1860 ± 4	1.99 ± 0.01

expected to give only about one-tenth of the polarization asymmetry given by the gold. The use of aluminum as a reference was the way in which effects of these asymmetries were eliminated. The value for the true asymmetry due to polarization was then obtained as

$$\alpha(\phi)=N_{\mathrm{Au}}(\phi)/N_{\mathrm{Al}}(\phi).$$

To indicate the magnitude of the correction introduced by normalizing to $N_{\mathrm{Al}}(\phi)$ it may be stated that the amplitude of $N_{\mathrm{Al}}(\phi)$ was about the same as that of $N_{\mathrm{Au}}(\phi)$ and that its phase was different. An actual plot of $N_{\mathrm{Al}}(\phi)$ will be given in the section on results.

(4) Finally, a set of measurements was made with a foil of intermediate atomic number, to determine whether the asymmetry attributed to polarization changed in the expected way with the atomic number.

MEASUREMENTS AND RESULTS

The program of measurement had to be limited to a small number of different sets of conditions for two reasons: (1) The amount of work involved in lining up the beam and making check runs was considerable, and had to be repeated for each new set of conditions. (2) The beta-ray-spectrometer type of focusing caused the size of the spot at the analyzing end, and consequently the current, which passed through the defining diaphragm, to vary rapidly with magnetic field. For these reasons the experimental effort was concentrated on just three settings, one being approximately at the focus corresponding to five cyclotron revolutions (1814°) and a setting on either side of the focus (1781° and 1847°). In all, approximately 1000 runs were made, each consisting of about 4000 counts in each channel, and each for a particular foil, angle, and field. In all of the measurements the polarizer foil was gold. Measurements with gold and with silver as the analyzer foil were made. In all measurements the analyzer foil was alternated with the aluminum foil and the blank hole for comparison and elimination of background count, as described earlier.

The results for the gold analyzer are plotted in Fig. 6. In Table I the angles of rotation due to the Lorentz forces—the cyclotron rotation and the deflections at entry and exit to the solenoid—as experimentally determined at the three values of field are given as ϕ_c and ϕ_d. A wave of the form $[1-B\cos(\phi-\beta)]$ representing the best fit by the least-squares method was determined. B and β are given in the table. The total observed angle of rotation of the polarization direction, ϕ_s, and finally the value of the gyromagnetic ratio, g, which is merely $2(\phi_s-\phi_d)/\phi_c$, are given. The consistency of the result over the range of approximately 60 degrees in angle indicates that the number of whole revolutions, i.e., 5, was the correct number.

The results for the silver analyzer are shown in Fig. 7. Since the purpose of this part of the experiment was only to check the dependence of magnitude of the asymmetry upon atomic number, measurements were made at fewer angles than in the case of the gold analyzer, and the results were not used in the determination of the g factor. The magnitude of the asymmetry was approximately one-half that obtained with the gold analyzer, a ratio consistent with the theoretical expectation, which is $Z_{\mathrm{Ag}}/Z_{\mathrm{Au}}$ or 47/79.

As a check on possible asymmetry due to geometrical misalignment or to the finite aperture, a series of measurements was made using a defining diaphragm one-half the diameter of the one used in the main series of measurements. The analyzer was gold, and all conditions other than the aperture were the same as those applying to the middle curve in Fig. 6. The reduction of the aperture produced no effect, other than a reduction in counting rate. The results are plotted in Fig. 8, and the points from the middle curve of Fig. 6 are included for comparison.

In Fig. 9 are plotted (open circles) the values of $N_{\mathrm{Al}}(\phi)$ which were obtained in the experiment on gold at 57.9 amp and which were used for the normalization of the values given in the middle curve of Fig. 6. The solid dots give the values of $N_{\mathrm{Al}}(\phi)$ obtained with the reduced aperture ($=1.12°$). The plots of $N_{\mathrm{Al}}(\phi)$ are presented because of their important bearing upon the reliability of the final results.

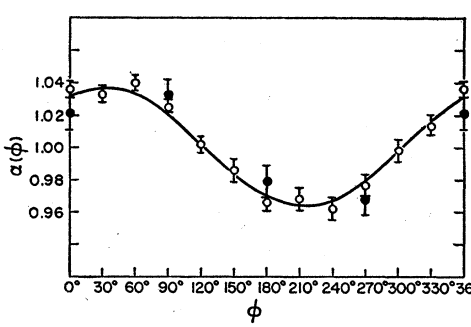

FIG. 8. Results obtained with reduced aperture (solid dots) compared with those obtained with full aperture (open circles).

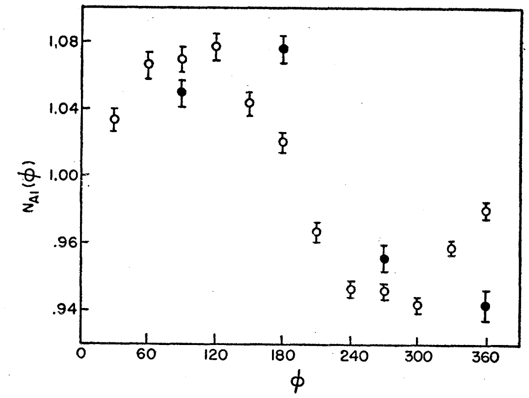

FIG. 9. The asymmetry obtained with the aluminum analyzer, i.e., the normalizing factor. Open circles: values used in the computation of the middle curve of Fig. 6. Solid dots: values used in the computation of the reduced aperture asymmetry in Fig. 8.

DISCUSSION OF RESULTS

The results presented in the foregoing section establish beyond doubt the practicality of observing and measuring the precession of the spin of the free electron in a magnetic field. The design of the experiment is such that the recognized asymmetries which are not associated with the spin precession are in principle eliminated, and the results of the two auxiliary experiments which it was possible to perform (change of Z and change of aperture) confirmed the belief that such asymmetries were in fact eliminated. The standard deviation given with the experimental value for g was composed of the standard deviations estimated for (a) the determination of the phase of the asymmetry β, (b) the measurement of the number of cyclotron revolutions, and (c) the angles of bending at entry and exit to the solenoid. It was not possible to include an estimate of the error introduced by a residual nonspin asymmetry, if it existed, in spite of the methods used to eliminate it. Such a residual would introduce into the asymmetry an additional cosine term of a different phase and amplitude. Since there is no formal way in which such a term can be estimated, the conviction that it does not exist in an important degree can be gotten only from a critical consideration of the design of the experiment, and the auxiliary checks which it was possible to make.

The checks whose results support the conclusion that the observed asymmetry was due to the spin of the electron, and that the errors in the measurement were small, may be listed, as follows.

(1) Both the magnitude of the observed asymmetry and the value of the g factor found were in good agreement with those predicted by theory. This double agreement constituted, in a sense, a cross check.

(2) When an analyzer foil of intermediate atomic number was substituted for the gold foil, the magnitude of the asymmetry decreased by the expected amount.

(3) A change in the aperture of the diaphragm system produced no essential change in either the amplitude of the asymmetry or the value of g.

(4) Measurements taken on different days, after realignment and adjustment of the apparatus gave results which agreed with one another, within the expected statistical limits.

(5) The individual asymmetries obtained for the gold analyzer and for the aluminum analyzer were quite far wrong in both phase and amplitude (see Fig. 9), but when they were divided, one by the other, the result was very close to the theoretical expectation, both in phase and amplitude. This greatly decreased the likelihood that the observed "spin" asymmetry was just a manifestation of the cyclotron rotation.

The amplitude of the asymmetry in the double-scattering experiment is defined by Mott as δ, where the ratio of the intensities in opposite directions in the line of maximum asymmetry is $(1+\delta)/(1-\delta)$. Calculations which give δ as a function of energy have been made by Mott and others[29] but none are available for angles of scattering other than 90°. For gold polarizer and analyzer, at 420 kev and for both scattering angles 90°, δ is expected to be 0.05. The experimental value of B in Table I is the asymmetry for gold-gold normalized to gold-aluminum. Since, theoretically, the latter is about 10 percent of the former, we may estimate δ_{Au} as 1.1 B or 0.044. The scattering angles used in the experiment were 90° and 78°, which would be expected to give a value for δ somewhat smaller than 0.05. The agreement is, therefore, well within the limits of the available theoretical predictions.

The precision of which the present method is capable is not sufficient to reveal the correction to the g factor, which is about one part in a thousand. An extension of the experiment, in which the electrons will be trapped in a magnetic field and in which spin will precess through at least 10^4 revolutions, is now under way. This should make possible a determination of the g factor to one part in about 10^5.

The authors are indebted to Professor K. M. Case for many discussions of the theoretical aspects of the problem, to Mr. H. A. Westrick and Mr. O. E. Haas for the instrument work, and to Mr. T. LaRocca for assistance in operating the apparatus.

[29] For curves of δ *vs* energy, see H. A. Tolhoek and S. R. DeGroot, Physica 17, 1 (1951).