Intermediate State Coupling and the Interpretation of High-Energy Nucleon-Nucleon Scattering*

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The effect on the scattering of protons by protons of the ${}^3P_2-{}^3F_2$ coupling due to an intermediate state of the two-nucleon system is investigated. The problem is treated in a scattering matrix formalism, with total angular momentum $j=2$, orbital angular momentum $L=j\pm1$, and partial half-width Γ_L. Only the ${}^1S_0$, ${}^1D_2$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2-{}^3F_2$ scattering anomalies are considered. Fits to the experimental data at 240 Mev are studied as typical. For ${}^1K_0\approx31°$, ${}^1K_2>0$ it is found that the data may be reasonably well represented provided $(\Gamma_1+\Gamma_3)/(E_R-E)\lesssim0.3$. It has been found possible to fit data on n-p and p-p scattering employing phase shifts agreeing with the hypothesis of charge independence of nuclear forces. Reasonable agreement with experiment is obtained employing either the regular or the inverted order of ${}^3P_{0,1,2}$ levels.

I. INTRODUCTION

THE early work on the interpretation of nucleon-nucleon scattering was in terms of potential energies which were taken to be velocity-independent for each of the states with definite L and J. Generalizations of this description have been made through the introduction of other forms of terms in a two-particle Hamiltonian as in the case of the tensor force[1] or in the consideration of a slow variation of the magnitude of the effective potential[2] with energy for a state with definite J and L. In connection with attempts to fit data by means of meson theoretic potentials, an attempt[3] has been made more recently to estimate the approximate magnitude of the velocity dependence by ascribing it to the formation of an intermediate state. An effect of the order of 10 percent in the effective meson mass was found on the assumption that the two-nucleon system may become excited to an isobaric state at an energy equal approximately to the rest mass energy of the pion. An excitation energy in this general range of values can hardly be excluded on account of the well-known difficulty of employing weak coupling meson theory. The question arises therefore as to the possibility of modifications in the static potentials by the temporary formation of intermediate states with higher L than that of the ${}^1S_0$ state in the work referred to.[3] The velocity-dependent features of such coupling are difficult to ascertain since the assignment of the partial cross sections to different L is clear only at low energies. It would be premature to attempt such a test at this time. On the other hand, the question arises as to whether the experimental data admit of such coupling and as to the approximate magnitude which may be assigned to it. For these reasons it appeared desirable to investigate a case in which the intermediate state formation has other marked features than the velocity dependence. It accordingly appeared of interest to try out the possibility in the case of scattering at $\sim$250 Mev paying attention mainly to the observed angular distributions and to the plausible requirement of charge independence of nuclear forces. This type of calculation has an additional interest also from a more general viewpoint,[4] according to which one can justify the employment of phase shifts from considerations of fundamental symmetries such as symmetry of space to rotations and independently of the applicability of special models or Hamiltonians. In this view, at sufficiently large distances the wave function may be expressed as a sum of parts, each of which has a definite real phase shift and corresponds to a definite value of the total angular momentum. There are in general, however, several states for a given J, each with a phase shift of its own. These states were distinguished from each other[4] by the subscript α.

The possibility of producing special angular effects by coupling of a state of higher L through the intermediate state to a state of lower L appeared especially intriguing because for an assumed interaction potential one expects phase shifts for sufficiently high L to become negligible and the mechanism under consideration is able to cause the appearance of effects of high L at lower energies. For p-p scattering the first possibility of this type occurs for the coupling of ${}^3P_2$ to ${}^3F_2$. The numerical work in the present paper is concerned

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[1] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941); see L. E. Eisenbud and E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **27**, 281 (1941) for classification of possible forms of generalization.

[2] Breit, Condon, and Present, Phys. Rev. **50**, 825 (1936). This reference is quoted as BCP. For Eq. (3.2) of present note Eqs. (5.1) . . . (6.7) of BCP are of interest. Breit, Thaxton, and Eisenbud, Phys. Rev. **55**, 1018 (1939). This paper is referred to as BTE in the text. The velocity dependence is being discussed in connection with Table XXIII.

[3] G. Breit and M. C. Yovits, Phys. Rev. **81**, 416 (1951). Related ideas have been expressed by R. B. Raphael and J. Schwinger, Phys. Rev. **90**, 373 (1953). Preliminary publication of material in the present paper is found in: Thaler, Bengston, and Breit, Phys. Rev. **91**, 454 (1953); Phys. Rev. **93**, 644 (1954).

[4] G. Breit, in *University of Pennsylvania Bicentennial Conference* (University of Pennsylvania Press, Philadelphia, 1941).

exclusively with this possibility and makes use of values obtained for representation of the data of the preceding paper[5] by means of s and p waves alone as a starting point. In the work reported on here, it is considered that the state $^3P_2-{}^3F_2$ exhibits a resonance for some energy greater than the region of $\sim$300 Mev. Such a resonance could perhaps be interpreted as due to an isobaric state of the two-proton system with an energy level approximately at the resonance energy. In preliminary publications, the state through which coupling takes place has been referred to as the isobaric state. In the present note the word "isobaric" is replaced by "intermediate" so as not to overemphasize the feature of metastability which is not directly used. The employment of resonance theory parameters is of course also not needed from a formal viewpoint, the specification of the scattering matrix sufficing for all practical purposes. It is used below mainly in order to facilitate visualization of effects by means of convenient quantities such as the resonance energy and the partial widths. The problem is formulated in terms of a one-level treatment employing a scattering matrix.[6] In addition to the contribution to the cross section caused by the coupling of $^3P_2$ to $^3F_2$, the effects of the five phase shifts for $^1S_0$, $^1D_2$, $^3P_{0,1,2}$ are included as non-vanishing. Since no specific nuclear interaction is assumed, the five phase shifts plus two parameters specifying the coupling give a total of seven free parameters which can be adjusted to fit experiment. Fits to data at 240 Mev[2,3] are used as typical throughout this work. It is found that the data do not exclude the possibility of intermediate state coupling, but rather indicate that such a state may prove helpful in fitting experiment. In particular they suggest that if coupling of the type investigated should be taking place, appreciable changes of values of phase shifts of states not participating in the coupling must also be made. In view of the many attempts to fit high-energy scattering data by means of phenomenologic potentials, it appears to be of interest to point out that conclusions regarding the effective potential in the ^1S state or in $^3P_0$, $^3P_1$ are not altogether independent of unknown couplings of states with different L. Only the simplest example of these has been considered below. It will be noted, however, that the effects on the phase shift without coupling are quite large.

[5] R. M. Thaler and J. Bengston, preceding paper [Phys. Rev **94**, 679 (1954)].

[6] The forms used in the calculations are related to those used in G. Breit, Phys. Rev. **58**, 1068 (1940) and especially to Eqs. (6.3), (7.7) in G. Breit, Phys. Rev. **69**, 472 (1946). While the present work was in progress, there appeared related considerations regarding the parametrization of the scattering matrix by J. M. Blatt and L. C. Biedenharn, Phys. Rev. **86**, 399 (1952). These are in part based on the use of eigenstates of the scattering matrix credited by them to J. Schwinger's 1947 unpublished lectures. The classification according to eigenstates of the scattering matrix is essentially the same as the classification according to the index α in a previously quoted reference (see reference 4). The parametrization of Blatt and Biedenharn, which employs the explicit specification of eigenstates and a total of three independent parameters, has advantages if an exhaustive enumeration is desired. On the other hand, for the purpose of qualitative consideration with unknown eigenstates the method used here is convenient.

Notation and Symbols

E = kinetic energy of incident nucleon in the laboratory system.

E_R = resonance energy as defined by Eqs. (1).

Γ_L = partial resonance width for orbital angular momentum L. In numerical applications Γ_L is used for total angular momentum $j=2$ and is expressed in terms of E.

$T=\tan^{-1}[(\Gamma_1+\Gamma_3)/(E_R-E)]$.

$y=\Gamma_3/(\Gamma_1+\Gamma_3)$.

L = orbital angular momentum about center of mass in units of $\hbar=1$.

F_L, G_L = respectively the regular and irregular solutions of the differential equation for $r\times$radial function normalized so as to be asymptotic at $r=\infty$ to $\sin\varphi$ and $\cos\varphi$ with $\varphi=\rho-\frac{1}{2}L\pi-\eta\ln 2\rho+\sigma_L$.

$\rho=kr$, $k=2\pi/(\text{de Broglie wavelength})$, $\eta=e^2/\hbar v$.

^1K_L, ^3K_L = singlet and triplet phase shifts, respectively.

$\delta_{L,j}$ = triplet phase shifts for orbital angular momentum L in the absence of intermediate state coupling; in some cases $\delta_{J-1,J}$ is called δ_- and $\delta_{J+1,J}$ is called δ_+; in most cases the comma is omitted and the symbol is written as δ_{Lj}.

$s_{L,j}=\sin\delta_{L,j}$.

σ_L = Coulomb phase shift $=\arg\Gamma(L+1+i\eta)$.

χ_{0s}, χ_μ = singlet and triplet spin functions, respectively.

P_L = Legendre function, $Y_M{}^L$ = spherical harmonic $=(-1)^m(2^l l!)^{-1}[(2l+1)/4\pi]^{\frac{1}{2}}$ $\times([l-m]!/[l+m]!)^{\frac{1}{2}}\exp(im\varphi)\sin^m\theta$ $\times(d/d\cos\theta)^{l+m}(\cos^2\theta-1)^l$.

θ = scattering angle in the center-of-mass system.

$\psi^{L,j}{}_\mu$ = spin-angle factor of eigenfunction for orbital angular momentum L, total angular momentum j and projection μ.

$\begin{pmatrix} L, j \\ \mu-M, M \end{pmatrix}$ = transformation coefficient defined by Eq. (2.2).

$\mathbf{P}$ = proton-proton scattering cross section per unit solid angle in the center-of-mass system.

$\mathbf{P}_M$, $\mathbf{P}_{NUC}$, $\mathbf{P}_{INT}$ are the Mott, the specific nuclear, and the Coulomb interference parts of $\mathbf{P}$, respectively.

$\sigma^{NP}(\theta)$ = differential scattering cross section for np scattering taken per unit solid angle in the center-of-mass system at colatitude angle θ.

$\mathbf{c}$, $\mathbf{s}$ = used as in preceding paper.

II. EFFECT OF INTERMEDIATE STATE

The interaction between two levels with different values of orbital angular momentum may be represented[6] by means of the matrix

$$(C)=e^{iT}\sin T\begin{pmatrix}(1-y)\exp[2i\delta_-], & [y(1-y)]^{\frac{1}{2}}\exp[i(\delta_-+\delta_+)]\\ [y(1-y)]^{\frac{1}{2}}\exp[i(\delta_-+\delta_+)], & y\exp[2i\delta_+]\end{pmatrix}, \tag{1}$$

where the parameters are related to the resonance energy E_R and the half-widths Γ_-, Γ_+ of the disintegration channels by

$$y=\Gamma_+/(\Gamma_-+\Gamma_+),\quad \tan T=(\Gamma_-+\Gamma_+)/(E_R-E). \tag{1.1}$$

Here Γ_-, Γ_+ refer, respectively, to the levels with $L=j-1$ and $L=j+1$ and a similar convention is being used for δ_-, δ_+. The convention regarding C is that the regular functions F_-, F_+ are changed by the nuclear interaction to

$$\begin{pmatrix}F_-\\ F_+\end{pmatrix}+\left\{\begin{pmatrix}Q(\delta_-), & 0\\ 0, & Q(\delta_+)\end{pmatrix}+\begin{pmatrix}C_{--}, & C_{-+}\\ C_{+-}, & C_{++}\end{pmatrix}\right\}\begin{pmatrix}H_-\\ H_+\end{pmatrix}, \tag{1.2}$$

with

$$H=G+iF, \tag{1.3}$$

$$Q(\delta)=e^{i\delta}\sin\delta. \tag{1.4}$$

The scattering matrix in this convention has the form

$$\|S_{ij}\|=\begin{pmatrix}\tau_- & 0\\ 0 & \tau_+\end{pmatrix}\begin{pmatrix}y+(1-y)e^{2iT}, & [y(1-y)]^{\frac{1}{2}}(e^{2iT}-1)\\ [y(1-y)]^{\frac{1}{2}}(e^{2iT}-1), & 1-y+ye^{2iT}\end{pmatrix}\begin{pmatrix}\tau_- & 0\\ 0 & \tau_+\end{pmatrix}, \tag{1.5}$$

where

$$\tau=e^{i\delta}. \tag{1.6}$$

The scattering matrix contains in this form 4 parameters δ_-, δ_+, y, T. Since there are[6] only three independent parameters in the scattering matrix the results for some independently specified values of the four parameters are the same. The result of specifying two phase shifts δ_-, δ_+, which are then modified through coupling by means of an intermediate state, is not necessarily different, therefore, from the scattering obtained by means of another pair of δ_-, δ_+. It is thus not possible to distinguish uniquely between effects of phase shifts δ_-, δ_+ which would be there for $y=0$ and effects caused by the coupling through the intermediate state. In the calculations reported on below, no attempt is made to cover all possibilities systematically and the question of exhaustive and complete parametric representation is therefore not of immediate importance, the primary object being a preliminary survey of the type of effect caused by the coupling of states with different L. In the numerical work the phase shift $\delta_+=0$ and the matrix $\|S_{ij}\|$ is expressed in terms of three parameters. In this representation the diagonal element referring to the higher L is $1+y(e^{2iT}-1)$, so that there is scattering of this wave into its own channel as long as y and T do not vanish. At the same time, for such values of y and T the nondiagonal element of $\|S_{ij}\|$ is not zero. The cases covered are therefore distinct from those obtainable by employing phase shifts δ_+ and δ_- but with $y=0$.

The two nucleons are supposed to be colliding with random directions of the spins. A statistical mixture of incident plane waves with equal probabilities for each of the four mutually orthogonal spin states is used and the calculation is otherwise standard. Thus the effect of the matrix (C) and of phase shifts outside (C) is to change the part of the wave function having the smaller orbital angular momentum L, spin function χ_μ for spin projection μ, to the value

$$i^L[4\pi(2L+1)]^{\frac{1}{2}}(1/\rho)\exp(i\sigma_L)\times\left\{\sum_j\begin{pmatrix}L,j\\ 0,\mu\end{pmatrix}\psi^{L,j}{}_\mu[F_L+Q(\delta_{L,j})H_L]+\begin{pmatrix}L,J\\ 0,\mu\end{pmatrix}[\psi^{L,J}{}_\mu C_{L,L}H_L+\psi^{L+2,J}{}_\mu C_{L,L+2}H_{L+2}]\right\}, \tag{2}$$

where

$$J=L+1, \tag{2.1}$$

and the summation is taken over $j=L-1$, L, $L+1$. Here the composition of angular momenta to form a resultant function with total angular momentum J is represented by

$$\psi^{L,J}{}_\mu=\sum_M\begin{pmatrix}L, J\\ \mu-M, M\end{pmatrix}Y^L{}_{\mu-M}\chi_M. \tag{2.2}$$

Similarly, for the larger orbital angular momentum $L+2$ and incident spin function χ_μ, the effect of the phase shifts and of the matrix (C) is to change the term originally present to

$$i^{L+2}[4\pi(2L+5)]^{\frac{1}{2}}(1/\rho)\exp(i\sigma_{L+2})\times\left\{\sum_j\begin{pmatrix}L+2, j\\ 0, \mu\end{pmatrix}\psi^{L+2,j}{}_\mu[F_{L+2}+Q(\delta_{L+2,j})H_{L+2}]+\begin{pmatrix}L+2, J\\ 0, \mu\end{pmatrix}[\psi^{L+2,J}{}_\mu C_{L+2,L+2}H_{L+2}+\psi^{L,J}{}_\mu C_{L+2,L}H_L]\right\}. \tag{2.3}$$

The procedure used is closely similar to that of Kittel and Breit[7] and of Breit, Kittel, and Thaxton.[7] The formulas employed so far are valid for any L but become specialized to $L=1$ beginning with Eq. (3.3). On summing intensities for different spin orientations the cross section in the system of the center of mass is readily found. It is expressible in the form

$$\mathbf{P}=\mathbf{P}_M+\mathbf{P}_{NUC}+\mathbf{P}_{INT}, \tag{3}$$

where designations M, NUC, INT stand for "Mott," "nuclear," and "interference," respectively. The values of the three contributions to the cross section are as follows. The Mott part is

$$\mathbf{P}_M=(e^2/2\mu v^2)^2\{\mathbf{s}^{-4}+\mathbf{c}^{-4}-\mathbf{s}^{-2}\mathbf{c}^{-2}\cos[\eta\log(\mathbf{s}^2\mathbf{c}^{-2})]\}, \tag{3.1}$$

as is well known. The interference term P_{INT} may be written reasonably compactly in the notation of BCP[2] and its immediate extensions summarized by

$$\begin{aligned}\varphi_L&=K_L+2(\sigma_L-\sigma_0);\quad \varphi_L{}^\sigma=\varphi_L+\eta\ln\sigma^2,\quad (\sigma=\mathbf{c},\,\mathbf{s}),\\ \varphi_{Lj}&=\delta_{Lj}+2(\sigma_L-\sigma_0),\quad \varphi_{Lj}{}^\sigma=\varphi_{Lj}+\eta\ln\sigma^2,\\ \varphi_L{}^\sigma(T)&=T+\eta\ln\sigma^2+2(\sigma_L-\sigma_0),\end{aligned} \tag{3.2}$$

and is expressible in the form

$$\begin{aligned}-(\eta/2)(2\mu v^2/e^2)^2P_{INT}={}&\mathbf{s}^{-2}\cos\varphi_0{}^s+\mathbf{c}^{-2}\cos\varphi_0{}^c\\ &+\{\textstyle\sum_j(2j+1)\sin\delta_{1j}[\mathbf{s}^{-2}\cos\varphi_{1j}{}^s+\mathbf{c}^{-2}\cos\varphi_{1j}{}^c]\\ &+5(1-y)[\mathbf{s}^{-2}\cos(T+\delta_{12}+\varphi_{1,2}{}^s)\\ &-\mathbf{c}^{-2}\cos(T+\delta_{1,2}+\varphi_{1,2}{}^c)]\sin T\}P_1\\ &+5\sin K_2[\mathbf{s}^{-2}\cos\varphi_2{}^s+\mathbf{c}^{-2}\cos\varphi_2{}^c]P_2\\ &+5y\sin T[\mathbf{s}^{-2}\cos\varphi_3{}^s(T)-\mathbf{c}^{-2}\cos\varphi_3{}^c(T)]P_3.\end{aligned} \tag{3.3}$$

The nuclear scattering term $\mathbf{P}_{NUC}$, is obtainable from

$$k^2\mathbf{P}_{NUC}={}^3\alpha_0P_0+{}^3\alpha_2P_2+{}^3\alpha_4P_4, \tag{4}$$

with

$$\begin{aligned}{}^3\alpha_0={}&\sin^2K_0+\textstyle\sum_j(2j+1)(s_{1j})^2+5\sin^2T\\ &+10(1-y)s_{12}\sin T\cos(T+\delta_{12})+5\sin^2K_2,\end{aligned} \tag{4.1}$$

where

$$s_{Lj}=\sin\delta_{Lj}, \tag{4.2}$$

and with

$$\mathbf{C}=y^{\frac{1}{2}},\quad \mathbf{S}=(1-y)^{\frac{1}{2}}, \tag{4.3}$$

$$\begin{aligned}{}^3\alpha_2={}&\tfrac{3}{2}s_{11}{}^2+(7/2)s_{12}{}^2+4s_{10}\cos(\delta_{10}-\delta_{12})\\ &+9s_{11}s_{12}\cos(\delta_{12}-\delta_{11})+\sin^2T[(7/2)\mathbf{S}^4\\ &+8\mathbf{C}^2\mathbf{S}^2+(32/7)\mathbf{C}^4+2(6)^{\frac{1}{2}}(1+(\mathbf{C}^2/7))\mathbf{CS}\cos B\\ &+(12/7)\mathbf{C}^2\mathbf{S}^2\cos^2B]\\ &+s_{10}\sin T[4\mathbf{S}^2\cos(T+2\delta_{12}-\delta_{10})\\ &+4(6)^{\frac{1}{2}}\mathbf{CS}\cos(T+\sigma_3-\sigma_1+\delta_{12}-\delta_{10})\\ &+6\mathbf{C}^2\cos(A+\delta_{12}-\delta_{10})]\\ &+s_{11}\sin T[9\mathbf{S}^2\cos(T+2\delta_{12}-\delta_{11})\\ &-6^{\frac{1}{2}}\mathbf{CS}\cos(\sigma_3-\sigma_1+T+\delta_{12}-\delta_{10})\\ &+6y\cos(A+\delta_{12}-\delta_{11})]+s_{12}\sin T[7\mathbf{S}^2\cos(T+\delta_{12})\\ &+2(6)^{\frac{1}{2}}\mathbf{CS}\cos(T+\sigma_3-\sigma_1)+(6/7)\mathbf{C}^2\cos A]\\ &+(50/7)\sin^2K_2+10\sin K_0\sin K_2\\ &\times\cos(K_2-K_0+2\sigma_2-2\sigma_0),\end{aligned} \tag{4.4}$$

where

$$A=T+2\sigma_3-2\sigma_1-\delta_{12},\quad B=\sigma_3-\sigma_1-\delta_{12}, \tag{4.5}$$

while

$$\begin{aligned}{}^3\alpha_4={}&\{\sin^2T[10\mathbf{C}^4+(40)6^{\frac{1}{2}}\mathbf{CS}\cos B+(240)\mathbf{C}^2\mathbf{S}^2\cos^2B]\\ &+(120)\mathbf{C}^2s_{12}\sin T\cos A+90\sin^2K_2\}/7.\end{aligned} \tag{4.6}$$

In the above expressions the f-wave phase shifts for no coupling $\delta_{3,j}$ were taken as zero, and partial waves for $L>3$ were assumed to make no contribution to the scattering except through the Coulomb interaction.

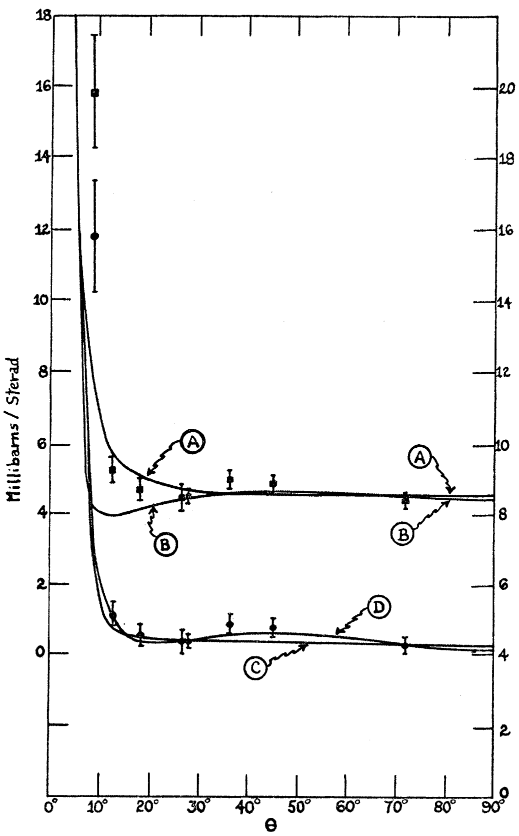

FIG. 1. Differential cross section for proton-proton scattering at $E=240$ Mev plotted against scattering angle θ in the center-of-mass system. Values of phase shifts and other parameters for the four curves are as follows (all phase shift values are in degrees):

Curve	${}^1K_0$	${}^1K_2$	δ_{10}	δ_{11}	δ_{12}	T	y
A	31.3	3.5	45.4	10.0	−20.0	5.7	0.3
B	31.3	3.5	−45.0	20.0	−10.0	5.7	0.7
C	31.3	0	−81.2	−3.0	−2.5	2.9	0.7
D	31.3	0	−50.1	10.0	−15.3	5.7	0.7

The left-hand scale is for curves A and B, the right-hand scale for C and D. Experimental points (see references 8 and 9) are indicated by circles and squares for right- and left-hand scales, respectively.

[7] C. Kittel and G. Breit, Phys. Rev. **56**, 744 (1939); Breit, Kittel, and Thaxton, Phys. Rev. **57**, 255 (1940). The four equations (unnumbered) in the middle of page 256 show the simple procedure.

III. *P-P* SCATTERING

Substitution of numerical values for the phase shifts ${}^1K_0$, ${}^1K_2$, $\delta_{1,0}$, $\delta_{1,1}$, $\delta_{1,2}$, T, and the coupling constant y into the above expression enables one to construct a good fit to the 240-Mev *p-p* scattering data.[2,3] In fact, it has been shown in the preceding paper[5] that the choice ${}^1K_2=T=0$ is consistent with the data. The question as to whether the data are in contradiction to the hypothesis of appreciable coupling between the ${}^3P_2$ and ${}^3F_2$ states is discussed below.

The experimental cross section[8,9] is represented in Fig. 1. The Coulomb contribution, $\mathbf{P}_M$, being negligibly small for $\theta \gtrsim 30°$, it is evident that for $\theta \gtrsim 30°$, the Coulomb and interference parts of the cross section contribute negligibly in comparison with the nuclear part $\mathbf{P}_{NUC}$. Since for $\theta \gtrsim 30°$ the cross section is approximately independent of angle, the coefficients α_2 and α_4 of Eq. (4) have been taken to be small as compared with α_0. Sets of values of the scattering parameters ${}^1K_0$, ${}^1K_2$, $\delta_{1,0}$, $\delta_{1,1}$, $\delta_{1,2}$, T, and y which satisfy, to within the experimental uncertainty, the equations

$$k^{-2}\alpha_0 = 4.5 \text{ mb}, \quad \alpha_0 = 1.30, \tag{5}$$

$$\alpha_2 = 0, \tag{5.1}$$

$$\alpha_4 = 0 \tag{5.2}$$

may be seen to fit the data for $\theta \geq 30°$. In addition, certain of these sets of values of the scattering parameters, when substituted into the complete expression for the scattering cross section, have been found to yield a reasonably good fit to the data over the entire angular range.

The *s*-wave phase shift ${}^1K_0$ was taken to be ${}^1K_0 = 31.3°$, as computed in first Born approximation from the meson well

$$V = -C\exp(-r/a)/(r/a), \tag{6}$$

with $C=89.8\ mc^2$ and $a=0.42\ e^2/mc^2$, which is consistent with the low-energy data.[10] The employment of this potential well is clearly open to serious criticism. Since the same holds for practically any other choice of ${}^1K_0$, the computation was nevertheless carried through employing the procedure just described so as to avoid unnecessary arbitrariness. The ${}^3P_2-{}^3F_2$ anomaly is defined by the three parameters $\delta_{1,2}$, T, y, the ${}^3F_2$ contributions not arising from the coupling to ${}^3P_2$ having been taken to be zero. It is easily seen that solutions of Eqs. (5) can exist only for a limited range of these parameters. By definition $0 \leq y \leq 1$, and T is restricted by $0 < T < \pi/2$.[11] From Eqs. (5) and (5.2) there follow

TABLE I. Values of ${}^3\alpha_4$ and of γ for various T, $\delta_{1,2}$, and y. Criteria of acceptability are ${}^3\alpha_4 \leqq 0$, $\gamma \leqq 1$.

	$\sin T=0.1$			$\sin T=0.2$			$\sin T=0.3$		
	$\delta_{1,2}$	γ	${}^3\alpha_4$	$\delta_{1,2}$	γ	${}^3\alpha_4$	$\delta_{1,2}$	γ	${}^3\alpha_4$
$y=0.1$	−40°	1.64	−0.05	−40°	1.25	−0.04	−40°	0.91	+0.04
	−30°	0.89	−0.04	−30°	0.60	−0.02	−30°	0.38	+0.07
	−20°	0.34	−0.02	−20°	0.18	−0.03	−20°	0.18	+0.14
	−10°	0.05	+0.00	−10°	0.04	+0.08			
	0	0.05	+0.04	0	0.20	+0.14			
	+10°	0.35	+0.06						
	+20°	0.91	+0.09						
$y=0.3$	−40°	1.74	−0.17	−40°	1.48	−0.16	−40°	1.27	+0.04
	−30°	0.98	−0.14	−30°	0.79	−0.10	−30°	0.68	+0.13
	−20°	0.40	−0.08	−20°	0.31	+0.03			
	−10°	0.32	+0.06						
	0	0.05	+0.09						
	+10°	0.32	+0.18						
$y=0.5$				−40°	1.70	−0.34	−40°	1.63	−0.10
	−30°	1.07	−0.25	−30°	0.98	−0.25	−30°	0.97	+0.01
	−20°	0.47	−0.15	−20°	0.45	−0.05	−20°	0.52	+0.31
	−10°	0.11	−0.02	−10°	0.18	+0.21			
	0	0.05	+0.12						
$y=0.7$	−30°	1.16	−0.39	−30°	1.17	−0.50	−30°	1.26	−0.32
	−20°	0.54	−0.26	−20°	0.58	−0.31	−20°	0.73	−0.04
	−10°	0.15	−0.08	−10°	0.25	+0.10	−10°	0.45	+0.54
	0	0.05	+0.12						
$y=0.9$	−30°	1.25	−0.56	−30°	1.36	−0.89	−30°	1.55	−0.96
	−20°	0.60	−0.40	−20°	0.72	−0.60	−20°	0.93	−0.59
	−10°	0.18	−0.18	−10°	0.32	−0.18	−10°	0.55	+0.00
	0	0.05	+0.08	0	0.20	+0.32	0	0.45	+0.72

the inequalities:

$$1.30-\sin^2 K_0 = 1.30-0.27 = 1.03 \leq 5s_{12}{}^2 + 5\sin^2 T$$
$$+10(1-y)s_{12}\sin T\cos(T+\delta_{1,2}) \equiv \gamma(\delta_{1,2},y,T), \tag{7}$$

$${}^3\alpha_4(\delta_{1,2},y,T) \geq 0, \tag{7.1}$$

where ${}^3\alpha_4$ is as in Eq. (4.6) and its functional dependence on δ_{12}, y, and T is now indicated for emphasis. The values of $\gamma(\delta_{1,2},y,T)$ and ${}^3\alpha_4(\delta_{1,2},y,T)$ which appear in Table I illustrate the permissible range of the parameters $\delta_{1,2}$, y, T. It is seen from Table I that solutions for Eqs. (5) will probably arise only for $\delta_{1,2}<0$ and $T<20°$. If a set of values of $(\delta_{1,2},y,T)$ are chosen such that $\gamma(\delta_{1,2},y,T)<1$ and ${}^3\alpha_4(\delta_{1,2},y,T)<0$, it is usually possible to find values of $(\delta_{1,0},\delta_{1,1},{}^1K_2)$ such that Eqs. (5) are approximately satisfied. If, furthermore, these values satisfy the inequality

$$\sum_{j=0}^{2}(2j+1)\sin\delta_{1,j}\cos\delta_{1,j} + 5\sin T\cos T$$
$$+5\sin K_2\cos K_2 < 0, \tag{8}$$

then the data will be approximately represented by substituting these values into the formula for the cross section, Eq. (3); for the inequality, Eq. (8), assures that the cross section will not have a minimum in the angular region where the Coulomb interference term is important.

Several typical fits to the data, obtained by the procedure outlined above, are plotted in Fig. 1. The cross-section dependence plotted in Fig. 1B is a typical one for which Eqs. (5) are approximately obeyed, but which does not obey the inequality of Eq. (8), and so does not fit the data at small angles. If a fit to the data

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[8] C. L. Oxley and R. D. Shamberger, Phys. Rev. **83**, 274 (1951).

[9] O. A. Towler, Jr., Phys. Rev. **85**, 1024 (1952).

[10] Yovits, Smith, Hull, Bengston, and Breit, Phys. Rev. **85**, 540 (1952).

[11] The quantity T is used with the restriction $|T|<\pi/2$, no resonance having been observed in the experimental region. The additional restriction $T>0$ is being made, there being no evidence for $E_R<0$.

also obeys the condition

$$\delta_{1,2}<\delta_{1,0},\ \delta_{1,1}, \tag{9}$$

then it will be said that such a fit corresponds to level inversion. The particular criterion used is not uniquely related to inversion of levels in nuclear spectroscopy. It is nevertheless of interest to employ a classification which has a qualitative relationship to the empirical studies of nuclear shell structure. In Fig. 1C is plotted a case which corresponds to this inversion.

It should be noted that Eqs. (5) serve as an approximate guide only. Equally good and better fits to the data can be obtained provided that $|\alpha_2/\alpha_0|\lesssim 0.2$ and $|\alpha_4/\alpha_0|\lesssim 0.2$. For example, the cross section plotted in Fig. 1D for ${}^1K_0=31.3°$, ${}^1K_2=0$, $\delta_{1,0}=-50.1°$, $\delta_{1,1}=10°$, $\delta_{1,2}=-15.3°$, $y=0.7$, $T=5.7°$ is one for which $\alpha_0=1.27$, $\alpha_2=0.00$, $\alpha_4=-0.17$.

IV. *N-P* SCATTERING

In Fig. 2 is a typical fit to the 260-Mev *n-p* data[12] which is consistent, in the sense of "charge independence," with the fit to the 240-Mev *p-p* data[8,9] plotted in Fig. 1A. The phase shifts ${}^1K_{0,1}$ and ${}^3K_{0,2}$ are calculated for 260 Mev from singlet and triplet meson wells, respectively,[13] the quantities ${}^1K_2$, $\delta_{1,0}$, $\delta_{1,1}$, $\delta_{1,2}$, y, T are as in Fig. 1A. This fit can be improved by suitable adjustment of ${}^1K_1$, ${}^3K_{0,2}$. Furthermore "inverted" fits to the 240-Mev *p-p* data, for which $\delta_{1,2}>\delta_{1,1}>\delta_{1,0}$, have been found which are likewise consistent with charge independence.

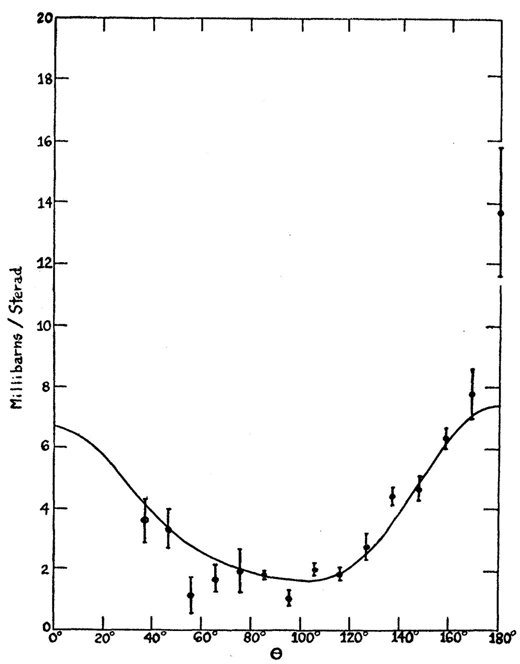

FIG. 2. Differential cross section for neutron-proton scattering at $E=260$ Mev for ${}^1K_0=30.9°$, ${}^1K_1=13.3°$, ${}^1K_2=3.5°$, ${}^3K_0=50.8°$, ${}^3K_2=9.9°$, $\delta_{10}=45.4°$, $\delta_{11}=10.0°$, $\delta_{12}=-20.0°$, $T=5.7°$, $y=0.3$. These values agree with parameters used for curve *A* of Fig. 1 in the sense of charge indepencence. Data are due to Kelly *et al.* (see reference 12).

For charge-independent nuclear forces, the neutron-proton scattering cross section corresponding to the assumption of intermediate state formation for $j=2$ in the triplet system, Eq. (7), may be written as

$$\begin{aligned}\sigma^{NP}(\theta)=(2k)^{-2}\{&k^2\mathbf{P}_{NUC}(\theta)+6P_1\sin({}^1K_0)\\&\times\sin({}^1K_1)\cos({}^1K_0-{}^1K_1)\\&+30P_1P_2\sin({}^1K_1)\sin({}^1K_2)\cos({}^1K_1-{}^1K_2)\\&+9(P_1)^2\sin^2({}^1K_1)+3\sin^2({}^3K_0)\\&+30P_2\sin({}^3K_0)\sin({}^3K_2)\cos({}^3K_0-{}^3K_2)\\&+2P_1\sin({}^3K_0)[\textstyle\sum_{j=0}^2(2j+1)\sin\delta_{1,j}\\&\times\cos(\delta_{1,j}-{}^3K_0)+5(1-y)\sin T\\&\times\cos(2\delta_{1,2}+T-{}^3K_0)]+10P_1P_2\sin({}^3K_2)\\&\times[\textstyle\sum_{j=0}^2(2j+1)\sin\delta_{1,j}\cos(\delta_{1,j}-{}^3K_2)\\&+5(1-y)\sin T\cos(2\delta_{1,2}+T-{}^3K_2)]\\&+10P_3\sin({}^3K_0)y\sin T\cos(T-{}^3K_0)\\&+50P_2P_3\sin({}^3K_2)y\sin T\cos(T-{}^3K_2)\\&+75(P_2)^2\sin^2({}^3K_2)\},\end{aligned} \tag{10}$$

where the quantities ${}^1K_{0,2}$, $\delta_{1,0}$, $\delta_{1,1}$, $\delta_{1,2}$, y, T are as in $\mathbf{P}_{NUC}(\theta)$ and where partial waves corresponding to $j>2$ have been excluded. The quantities in the above expression which may be adjusted to fit the *n-p* data are ${}^3K_{0,2}$, ${}^1K_1$.

V. CONCLUSION AND DISCUSSION

The calculations show that data on the *p-p* scattering cross section at 240 Mev admit an interpretation in which an appreciable part of the cross section is attributed to intermediate state formation resulting in the coupling of ${}^3P_2$ to ${}^3F_2$. If it is assumed on the other hand that nuclear force effects are confined to phase shifts for *s*, *p*, ${}^1D_2$ states and to ${}^3P_2-{}^3F_2$ coupling, then it is possible to exclude large values of the coupling parameters. Within these restricting assumptions regarding nuclear force effects, it proved possible to remain in agreement with the hypothesis of charge independence of nuclear forces. The *p*-wave phase shifts of the charge-independent fits may be made to correspond either to the regular or inverted level order of the two-nucleon system; there is thus no obvious difficulty in reconciling the high-energy scattering data with the shell theory of nuclear structure.

The possible influence of coupling on the determination of phase shifts not directly concerned with the intermediate state has been discussed in the introduction, and the effect on the interpretation of phase shifts participating in the coupling may be seen in

[12] Kelly, Leith, Segrè, and Wiegand, Phys. Rev. **79**, 96 (1950).
[13] Equation (11) of preceding paper.

Table I. It has a bearing on indications of the existence of semistable states in pion-nucleon scattering[14] and on attempts to use isobaric state models for the theory of the deuteron.[15] The present work was in fact begun under the influence of similar views tentatively considered at this laboratory. The continued lack of success and the possibility of omission of essential elements[14] in the development of fundamental nuclear force theory would make a claim of a connection with the isobaric state of pion-nucleon phenomena rather speculative, especially since there is serious doubt[16] regarding the justification of such an interpretation of the meson data.

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[15] H. A. Bethe and N. Austern, Phys. Rev. **86**, 121 (1952); N. Austern, Phys. Rev. **87**, 208 (1952).

[16] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952); H. L. Anderson and E. Fermi, Phys. Rev. **86**, 794 (1952); Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953); E. Fermi and N. Metropolis, Los Alamos unclassified report LA-1492, 1952 (unpublished).

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Interactions of Negative Pions in Carbon and Lead*

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A cloud chamber investigation of the nuclear interactions of 125-Mev negative pi mesons has been carried out. Carbon and lead were employed as the scattering material. The experimental arrangement was designed to observe the charge-exchange process:

$$\pi^- + \mathrm{C} \rightarrow \mathrm{B}^{(\mathrm{exc})} + [\pi^0 \rightarrow 2\gamma].$$

High-energy photons, probably attributable to charge exchange, are found to arise from nuclear interaction, with a small but finite cross section. Total and differential cross sections for elastic and inelastic scattering, as well as total nuclear reaction cross sections, are presented. A square well (optical model) analysis fits the data with a potential whose real part is attractive and 30 Mev deep, and whose imaginary part corresponds to a mean free path in nuclear matter of 3×10^{-13} cm. The optical model parameters are found to be energy dependent, the well depth increasing and the mean free path decreasing as the energy is raised.

I. INTRODUCTION

A CONSIDERABLE body of experimental fact, relevant to the interaction of pions in complex nuclei, is already established. Nuclear emulsion experiments have demonstrated the principal mechanisms of the interactions,[1] and given information regarding their energy dependence.[2] More recent experiments with counters,[3] cloud chambers,[4] and emulsions,[5] have continued these studies, and have more clearly shown the dependence of these processes on nuclear size.

The following over-all picture presents itself to us: at low energies (0–40 Mev) the dominant process is mesonic absorption with subsequent nuclear excitation resulting in star formation. The star cross section is about half geometric in the 40-Mev region. Only a small elastic scattering cross section is observed. As the bombarding energy increases, both star and elastic cross sections rise. At 60-Mev inelastic scattering begins to occur with an appreciable probability which increases rapidly with energy. The total reaction cross section is nearly geometric at 80 Mev for all complex nuclei. In some cases it seems to rise slowly as higher energies of bombardment are employed.[6]

The picture is completed by consideration of the charge exchange reaction, which is found to be of great

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† Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy. Now at RCA Laboratories, Princeton, New Jersey.

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