Phase Shift Analysis of High-Energy Nucleon-Nucleon Scattering Experiments*

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The high-energy proton-proton scattering data are analyzed in terms of s-wave and p-wave anomalies. Different phase shifts are used for the three p waves with total angular momentum $j=0, 1, 2$. It is found that the data can be fitted without violating charge independence of nuclear forces. These high energy fits vary smoothly with energy and are not in contradiction to the requirements of the intermediate energy (10–100 Mev) data. Inversion of nuclear levels suggested by the shell theory of nuclear forces can also be accounted for.

I. INTRODUCTION

SCATTERING of protons by protons at bombarding energies between 100 and 400 Mev gives[1–5] a roughly isotropic intensity distribution in the system of the center of mass of the two protons. As is well known, the large value of the cross section rules out an explanation of isotropy in terms of a pure s-wave scattering anomaly, and makes it necessary to include states of higher angular momentum in the analysis. While most of the current theories of p-p scattering use some form of phenomenologic potential energy as a starting point, their practical accomplishment is that of yielding proper combinations of phase shifts, especially those of the three p waves and of the d wave. On the other hand the most general velocity-dependent interaction, satisfying general requirements of invariance, can be described[6] through an arbitrary assignment of phase shifts. It was thought of interest therefore to make an analysis entirely on the basis of phase shifts and without any preconceived potential. The present note contains a brief account of reasonably successful fits made by employing only four phase shifts, *viz.*, one for the s wave and three for the p waves.[7]

SYMBOLS AND NOTATION

E	energy of the bombarding particle.
$\mathbf{P}$	proton-proton scattering cross section per unit solid angle in the center-of-mass system.
$\mathbf{P}_{NUC}$	that part of $\mathbf{P}$ specifically due only to the nuclear interaction as distinguished from the Coulomb interaction.
$\mathbf{P}_M$	that part of $\mathbf{P}$ specifically due only to the Coulomb interaction.
$\mathbf{P}_{INT}$	that part of $\mathbf{P}$ due to interference between the outgoing nuclear and Coulomb waves.
$\Lambda=2\pi/k$	wavelength of relative motion.
P_L	Legendere function of order L.
θ	scattering angle in the center-of-mass system.
${}^1K_0, {}^1K_1$	nuclear phase shift for singlet state for $L=0, 1$.
${}^3K_0, {}^3K_2$	nuclear phase shifts for triplet state for $L=0, 2$.
L	orbital angular momentum.
j	total angular momentum.
$\delta_0, \delta_1, \delta_2$	nuclear phase shifts for triplet p state for $j=0, 1, 2$.
$\eta=e^2/\hbar v$.	
σ^{NP}	neutron-proton scattering cross section per unit solid angle in the center-of-mass system.

$$\langle \mathbf{P}_{NUC}\rangle=\tfrac{1}{2}\int_{-1}^{1}\mathbf{P}_{NUC}d(\cos\theta).$$

$\mathbf{c}=\cos\theta/2$, $\mathbf{s}=\sin\theta/2$.

$\mathbf{X}_0, \mathbf{X}_1, \mathbf{Y}_0, \mathbf{Y}_1$ quantities similar to $\mathbf{X}$, $\mathbf{Y}$ of Breit, Thaxton, and Eisenbud[8] defined by Eqs. (5–6).

$\alpha_L=\eta\log\mathbf{s}^2+2\Sigma_{n=0}{}^L\tan^{-1}(\eta/n)$.

$\beta_L=\eta\log\mathbf{c}^2+2\Sigma_{n=0}{}^L\tan^{-1}(\eta/n)$.

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7 Preliminary accounts of the present work were given at the 1952 Rochester Conference by G. Breit, and in Phys. Rev. **91**, 454 (1953); Phys. Rev. **93**, 643, 644 (1954). A related investigation has been published by A. Garren, Phys. Rev. **92**, 213 (1953).

8 This has been discussed in detail in Breit, Condon, and Present, Phys. Rev. **50**, 825 (1936); Breit, Thaxton, and Eisenbud, Phys. Rev. **55**, 1018 (1939); and Breit, Kittel and Thaxton, Phys. Rev. **57**, 255 (1940).

Equation (1) of Breit, Kittel, and Thaxton contains the effects studied in the present paper. It gives the differential cross section in units $(e^2/2\mu v^2)^2$. By combining their term in $(108P_1{}^2/\eta^2)\Sigma g_i\sin^2\delta_i$, and the term in $(12/\eta^2)(3\cos^2\theta-1)$, and multiplying by $(e^2/2\mu v^2)^2$, there results Eq. (2) of the present paper. The interference terms of Eq. (4) of the text similarly correspond to the terms in $(18P_1/\eta)$ of Eq. (1) of BKT. The latter formula is arranged so as to collect in one place all effects which vanish when $\delta_0=\delta_1=\delta_2$. Since in the present work the values of the δ_i are often very different from each other the rearrangement used here proved convenient. Equation (10) of the present paper contains the terms in Eq. (15) of C. Kittel and G. Breit, Phys. Rev. **56**, 744 (1939) and in addition the effect of ${}^3K_2$.

II. P-P SCATTERING

The proton-proton scattering cross section in the center-of-mass system may be written as

$$\mathbf{P}=\mathbf{P}_{NUC}+\mathbf{P}_{M}+\mathbf{P}_{INT}, \tag{1}$$

where

$$\mathbf{P}_{NUC}=(\Lambda/2\pi)^2\{\sin^2 {}^1K_0+\Sigma_0{}^2(2j+1)\sin^2\delta_j +P_2(\cos\theta)[(3/2)\sin^2\delta_1+(7/2)\sin^2\delta_2 +4\sin\delta_0\sin\delta_2\cos(\delta_0-\delta_2) +9\sin\delta_1\sin\delta_2\cos(\delta_1-\delta_2)]\}, \tag{2}$$

$$\mathbf{P}_M=(\Lambda/2\pi)^2\{(\eta^2/4)[\mathbf{s}^{-4}+\mathbf{c}^{-4} -\mathbf{s}^{-2}\mathbf{c}^{-2}\cos(\eta\log\mathbf{s}^2\mathbf{c}^{-2})]\}, \tag{3}$$

$$\mathbf{P}_{INT}=(\Lambda/2\pi)^2\{-(\eta/2)\mathbf{X}_0\sin{}^1K_0\cos{}^1K_0 +(\eta/2)\mathbf{Y}_0\sin^2{}^1K_0+P_1(\cos\theta) \times[-(\eta/2)\mathbf{X}_1\Sigma_0{}^2(2j+1)\sin\delta_j\cos\delta_j +(\eta/2)\mathbf{Y}_1\Sigma_0{}^2(2j+1)\sin^2\delta_j]\}, \tag{4}$$

and

$$\mathbf{X}_L=\mathbf{s}^{-2}\cos\alpha_L+(-1)^L\mathbf{c}^{-2}\cos\beta_L, \tag{5}$$

$$\mathbf{Y}_L=\mathbf{s}^{-2}\sin\alpha_L+(-1)^L\mathbf{c}^{-2}\sin\beta_L. \tag{6}$$

The choice of phase shifts is appreciably simplified by the fact that in the angular range $30°<\theta<90°$ the terms $\mathbf{P}_M$ and $\mathbf{P}_{INT}$ are practically constant. Since in the same angular range the observed $\mathbf{P}$ is practically independent of θ, the coefficient of $P_2(\cos\theta)$ in the formula for $\mathbf{P}_{NUC}$ must be approximately zero. Various combinations of the triplet p phase shifts for which the coefficient of $P_2(\cos\theta)$ in $\mathbf{P}_{NUC}$ vanishes, i.e., for which

$$(3/2)\sin^2\delta_1+(7/2)\sin^2\delta_2+4\sin\delta_0\sin\delta_2\cos(\delta_0-\delta_2) +9\sin\delta_1\sin\delta_2\cos(\delta_1-\delta_2)=0, \tag{7}$$

are plotted in Fig. 1. It is seen from Eq. (2) that as a consequence of Eq. (7) the quantity

$$\alpha\equiv\Sigma_0{}^2(2j+1)\sin^2\delta_j, \tag{8}$$

together with ${}^1K_0$, determine uniquely the quantity $\mathbf{P}_{NUC}/(\Lambda/2\pi)^2$. Thus the fits under consideration are such that for an assigned value of ${}^1K_0$ the observed angle independent σ_Ω determines one of the contours in Fig. 1. An estimate of the s-wave phase shift ${}^1K_0$ as calculated in first Born approximation for the meson well

$$V=-\mathcal{C}'(mc^2)[r(e^2/mc^2)a']^{-1}\exp\{-[r/(e^2/mc^2)a']\},$$

with $\mathcal{C}'=89.8$, and $a'=0.42$, is used. As an example, for $E=100$ Mev and ${}^1K_0=35.0°$, numerical substitution shows that for $\langle\mathbf{P}\rangle=4.5$ mb one must have

$$\alpha=0.21,$$

defining a contour located approximately in the center of Fig. 1. It will be noted that in the same figure for $\alpha=1.30$ there appear on the edges of the diagram small subsidiary curves unconnected with the major portions of the contours. Since these subsidiary curves only occur for large values of α, they can probably be ruled out of consideration on the grounds that the solutions represented by them appear to correspond to a discontinuity with respect to energy.

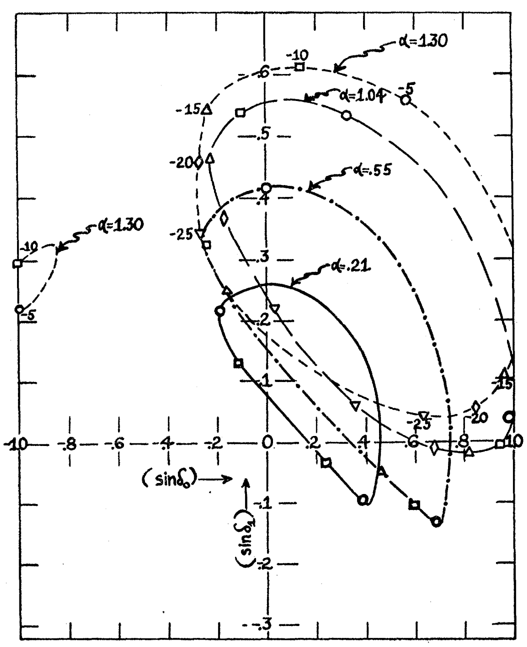

FIG. 1. Family of p-phase shift curves for which the triplet part of $\mathbf{P}_{NUC}$ is angle-independent. Each contour is labeled by a value of $\alpha=\Sigma_{j=0}{}^2(2j+1)\sin^2\delta_j$. Values of δ_2 in degrees are directly indicated along some of the contours. In other cases the values of $\delta_2=-5°$, $-10°$, $-15°$, $-20°$, $-25°$ are indicated by the point designations ○, □, △, ◇, ▽, respectively. A second family of contours is obtained by changing all δ_j to $-\delta_j$.

TABLE I. Values of ${}^1K_0$ and α for which $\mathbf{P}(\theta=90°)=4.5$ millibarn/sterad ±20 percent at 100 Mev and 240 Mev. Second line is for meson well with parameters $\mathcal{C}=89.8mc^2$ and $a=0.42e^2/mc^2$.

$E=100$ Mev		$E=240$ Mev	
${}^1K_0$	α	${}^1K_0$	α
47.4°±6.3°	0	90°	0.30±0.26
35.0°	0.21±0.11	31.3°	1.04±0.26
0	0.54±0.11	0	1.30±0.26

Under the assumptions that the experimental value of the cross section at 90° is known to within 20 percent, and that ${}^1K_0$ is not known, there is considerable latitude in the choice of α as is illustrated numerically in Table I. Nevertheless, for the sake of definiteness, in the work that follows for $E=240$ Mev, ${}^1K_0$ is taken to be ${}^1K_0=31.3°$ (see Fig. 2), and $\langle\mathbf{P}_{NUC}\rangle$ is taken to be $\langle\mathbf{P}_{NUC}\rangle=4.5$ mb, corresponding to $\alpha=1.04$. The mathematical condition for flatness, i.e., for $\mathbf{P}_{NUC}(\theta)=\text{const}$, used in determining the plots for Fig. 1 is more stringent than the experimental data require. To fit the data it is necessary merely that $|1-\mathbf{P}_{NUC}(\theta)/\mathbf{P}(\theta=90°)|$ be small. In Fig. 2 there has been plotted the family of solutions for which $|1-\mathbf{P}_{NUC}(\theta)/\mathbf{P}(\theta=90°)|\lesssim0.1$ at $E=240$ Mev with $\langle\mathbf{P}_{NUC}\rangle=4.5$ mb and $\alpha=1.04$. The small subsidiary regions on the edges of Fig. 2 correspond to the subsidiary curve which appears in Fig. 1 for $\alpha = 1.30$. There are no mathematically flat subsidiary solutions, however, for $\alpha = 1.04$.

The 240-Mev Rochester[3,4] data (see Fig. 3) indicate a large increase in the cross section for small angles. At these angles, however $\mathbf{P}_{INT}$ and $\mathbf{P}_M$ are appreciable, so that some of the solutions plotted in Fig. 1 for $\alpha = 1.04$ may correspond to a cross section similar to the experimentally determined cross section at all angles. The only quantity in Eqs. (1–4) which differs from one solution to another is the quantity

$$\beta \equiv \Sigma_0{}^2 (2j+1) \sin\delta_j \cos\delta_j, \tag{9}$$

which appears in Eq. (4). Several typical cross sections have been plotted in Fig. 3 to indicate the dependence of the cross section on the value of the quantity β. Solutions for which β is positive correspond to a dip in the cross section in the angular region $10° < \theta < 20°$ and are excluded as not fitting the data. The regions for which β is negative are indicated on Fig. 2 by cross-hatching. It may be noted that the cross-hatched region in the lower left hand corner of Fig. 2 corresponds to the well known inversion of the nuclear ${}^3P$ levels.

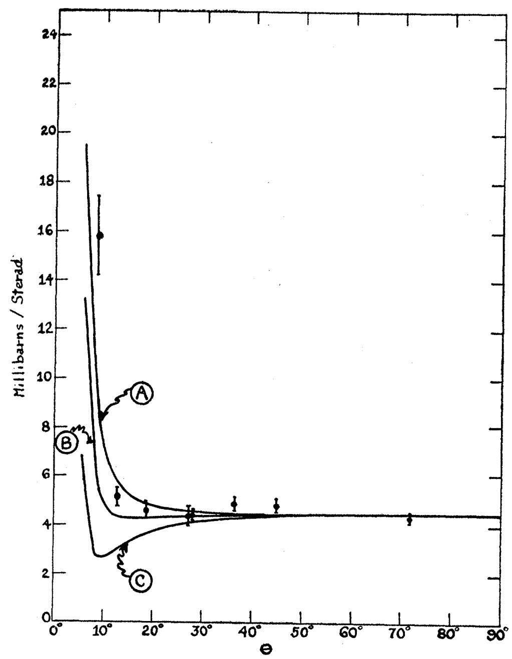

FIG. 3. Proton-proton scattering cross section at 240 Mev. For case A, $\beta = -1.0$; for case B, $\beta = 0$; for case C, $\beta = +1.0$. Data due to Rochester group (see references 3 and 4).

III. N-P SCATTERING

In order to determine whether the results obtained are in conflict with the hypothesis of charge independence of nuclear forces, consideration was given to the possibility of obtaining a fit to the experimental neutron-proton scattering data at 260 Mev.[9] For proton-proton scattering at 260 Mev, ${}^1K_0$ was taken to be ${}^1K_0 = 30.9°$, as calculated from the meson well above, giving $\alpha = 1.14$ for $\langle \mathbf{P}_{NUC} \rangle = 4.5$ mb. From the curve for $\alpha = 1.14$ in Fig. 1 may be read the values of the p-wave phase shifts, δ_j, which satisfy Eq. (7). By analogy with the result at 240 Mev, those values of the δ_j which satisfy Eq. (7), but for which $\beta > 0$ are considered to be unacceptable. If after the value ${}^1K_0 = 30.9°$ and after a set of values for the triplet p phase shifts, δ_j, taken from Fig. 1 for $\alpha = 1.14$, are substituted into the expression for the neutron-proton scattering cross section with all other singlet phase shifts for even L and all other triplet phase shifts for odd L being taken to be zero, it is possible to fit the 260-Mev data by adjustment of the remaining phase shifts, such a quartet of values of ${}^1K_0$ and δ_j will be said to represent a solution which does not violate charge independence of nuclear forces.

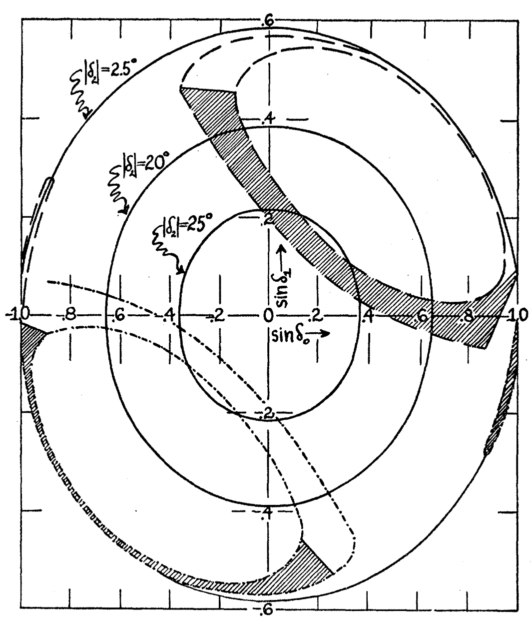

FIG. 2. Possible p-phase shifts for $E = 240$ Mev. The ellipses give values of $|\delta_2|$ corresponding to $\alpha = 1.04$ and to the assumed cross section. The contours in upper right-hand corner dashed lines enclose a region within which $\delta_2 < 0$ and $\mathbf{P}_{NUC}$ differs from its value at $\theta = 90°$ by less than 10 percent, while ${}^1K_0 = 31.3°$. The region enclosed by contours in lower right-hand corner (dash-dotted lines) is for $\delta_2 > 0$; in other respects it satisfies conditions described for the dashed line contours. Cross hatches indicate regions for which β of Eq. (9) satisfies $\beta < 0$. The cross-hatched region in the lower left-hand corner corresponds to inversion of the ${}^3P$ levels.

The neutron-proton scattering cross section may be written as

$$\begin{aligned}\sigma^{NP} = \tfrac{1}{4}\mathbf{P}_{NUC} + \tfrac{1}{4}(\Lambda/2\pi)^2\{&3\sin^2({}^3K_0)\\ &+[6\sin({}^1K_0)\sin({}^1K_1)\cos({}^1K_0 - {}^1K_1)\\ &+2\beta\sin({}^3K_0)\cos({}^3K_0) + 2\alpha\sin^2({}^3K_0)]P_1(\cos\theta)\\ &+9\sin^2({}^1K_1)P_1{}^2(\cos\theta) + [10\beta\sin({}^3K_2)\cos({}^3K_2)\\ &+10\alpha\sin^2({}^3K_2)]P_1(\cos\theta)P_2(\cos\theta)\\ &\qquad +75\sin^2({}^3K_2)P_2{}^2(\cos\theta)\}.\end{aligned} \tag{10}$$

[9] Kelly, Leith, Segrè, and Wiegand, Phys. Rev. **79**, 96 (1950).

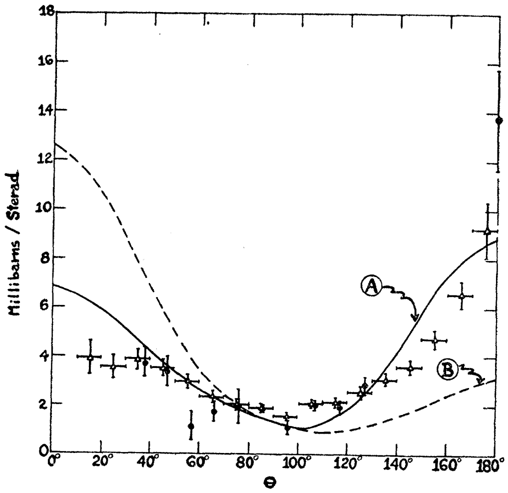

FIG. 4. Neutron-proton scattering cross section at 260 Mev ${}^1K_0=30.9°$, ${}^1K_1=13.3°$; calculated from meson well with parameters $a=0.42e^2/mc^2$, $\mathcal{C}=89.8mc^2$. ${}^3K_0=50.8°$, ${}^3K_2=9.9°$; calculated from meson well with parameters $a=0.42e^2/mc^2$, $\mathcal{C}=132.4mc^2$. For case A, $\beta=-1.4$; for case B, $\beta=1.4$. Data indicated by solid circles are for 260 Mev and are due to Kelley *et al.* (reference 9). Data indicated by triangles are high-energy cloud chamber measurements of J. De Panghen, Jr. [University of California Radiation Laboratory Report UCRL-2153, March, 1953 (unpublished)], and show the probable behavior of σ^{NP} for small angles.

All phase shifts which do not appear in the above expression have been taken to be equal to zero. The phase shifts in Eq. (10) which are now undetermined are ${}^3K_0$, ${}^3K_2$ and ${}^1K_1$.

The experimental 260-Mev neutron-proton cross section[9] (see Fig. 4) is asymmetric about $\theta=90°$, being smaller near $\theta=0$, than near $\theta=180°$. It is clear from Eq. (10) that for ${}^3K_0$, ${}^3K_2$ and ${}^1K_1$ each positive and less than 90° such an asymmetry cannot be reproduced for $\beta>0$. The condition that β cannot be positive is just the condition imposed on the δ_j by the small angle proton-proton scattering data.[4] If ${}^3K_0$, ${}^3K_2$ are calculated from the low energy triplet meson well with $\mathcal{C}'=132.4$, $a'=0.42$, and ${}^1K_0$, ${}^1K_1$ are calculated from the low energy triplet meson well with $\mathcal{C}'=89.8$, $a'=0.42$, the values obtained are

$$ {}^3K_0=50.8°;\quad {}^3K_2=9.9°;\quad {}^1K_0=30.9°;\quad {}^1K_1=13.3°. \tag{11}$$

For Eq. (11), ${}^3K_0$ was calculated by means of a numerical integration employing Hartree's procedure, the other phase shifts were used in first Born approximation. The values of the phase shifts ${}^3K_0$, ${}^3K_2$, ${}^1K_0$, ${}^1K_1$ in Eq. (11) together with values for δ_j taken from Fig. 1 for $\alpha=1.14$ may be substituted into Eq. (10), and the resulting cross section compared with experiment. The range of β is $-1.5\lesssim\beta\lesssim 1.5$. The best fit to the data occurs for β a large negative number. The large distinction between positive and negative β is illustrated in Fig. 4. It is of interest to note that the subsidiary solutions appearing on the edges of the diagram in Fig. 1 and Fig. 2, which have been ruled out on the basis of continuity with respect to energy, correspond to $-0.1\lesssim\beta\lesssim 0.1$. For $|\beta|\lesssim 0.1$, the condition $\sigma^{NP}(\theta=180°)/\sigma^{NP}(\theta=0)>1$ cannot be reproduced with the values of the phase shifts of Eq. (11).

CONCLUSION

It is possible to fit the high-energy proton-proton data with s waves and p waves alone in a manner such that these fits vary smoothly and continuously with energy without violating either charge independence or the inversion of nuclear levels.

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