

Effect of Elastic Strain on the Electrical Resistance of Metals

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(Received November 16, 1953)

The strain coefficients of electrical resistivity of some eighteen metals and alloys have been determined experimentally. It has been shown that for most metals the free electron theory accounts qualitatively for the observed effects. In the case of bismuth and antimony, the discussion of the possible deformation of the Brillouin zones of these metals with strain seems to explain their abnormal behavior.

THE phenomenon of the change of the electric resistance of metallic wires with elastic strain was first investigated by Lord Kelvin¹ in 1856 and his pupil Tomlinson.² Their research was followed by that of Donaldson and Wilson,³ Smith,⁴ Bridgman,⁵⁻⁷ Rolnick,⁸ Allen,⁹ and others. Most recently an excellent study was published by Druyvesteyn,¹⁰ who measured the variation of electrical resistivity of metals with elastic deformation in different crystallographic directions. The phenomenon is today utilized in the operation of electrical-resistance strain gauges.¹¹

The change in electrical resistance due to strain is partly due to the geometrical changes taking place in the deformed body and partly due to physical changes within the metal itself. In the case of a metallic wire stretched longitudinally, these two factors enter the following equation:

$$\frac{dR}{Rd\epsilon} = (1 + 2\mu) + \frac{d\rho}{\rho d\epsilon}, \quad (1)$$

where R is the total resistance of the wire, ρ its specific resistivity, ϵ the strain, and μ Poisson's ratio. $dR/Rd\epsilon$ could be called the coefficient of strain sensitivity of resistance, or gauge factor, and $d\rho/\rho d\epsilon$ the strain coefficient of specific resistivity. The first term on the right side of Eq. (1) represents the purely geometrical effect of the deformation and the second, the physical one. In this paper we shall discuss only the effect of elastic strains upon the specific resistivity of metals and alloys.

EXPERIMENTAL RESULTS

A number of wires of pure metals and alloys were tested in the apparatus specially built for the purpose and described elsewhere.¹² The wire was stretched by

the lever action of a beam and the increments of length were measured with a cathetometer. The electrical resistance was measured with a carefully calibrated Wheatstone bridge.

The strain sensitivity coefficients of electrical resistivity were measured for the following metals and alloys: copper, nickel, copper-nickel alloys, platinum, aluminum, tungsten, molybdenum, tantalum, tin, iron, cobalt, and bismuth. The results are listed in Table I. Data on bismuth-lead, bismuth-tin, and bismuth-selenium alloys have been published previously.¹³

TABLE I. Strain coefficients of resistivity of metallic wires of various diameters.

Material	Wire diameter (cm)	$\frac{d\rho}{\rho d\epsilon}$	Remarks	
Cu	25.80 × 10 ⁻³	1.60	Annealed	
	12.85	1.55	Annealed	
	5.48	1.20	Annealed	
	2.61	0.90	Annealed	
	1.06	0.90	Annealed	
Ni	12.06	- 8.82 to - 9.82	Grade A ann.	
	2.22	- 3.95 to - 7.12	Grade A ann.	
	23.80	- 2.95	As drawn	
	12.00	- 3.22 to - 5.62	As drawn	
	2.51	- 2.12 to - 2.92	As drawn	
	25.30	-11.62 to -12.62	Mond nickel	
	2.69	- 8.82	Annealed	
Al	25.00	1.40	Annealed	
Pt	12.67	2.60	Annealed	
	2.41	2.22	Annealed	
Fe	22.62	0.44	Annealed	
Co	14.50	0.84	Annealed	
Mo	12.45	0.80	As received	
Ta	12.42	1.30	As received	
W	12.95	1.16	As received	
	1.42	1.16	As received	
	31.80	3.34	Extruded	
Bi	29.20	-11.36	Extr. and ann.	
	7.70	-11.16	Extr. and ann.	
Cu-Ni (Atom % Ni)	1.08	25.30	1.54	Annealed
		2.54	0.43	Annealed
	11.50	25.80	0.70	Annealed
		2.69	0.80	Annealed
	45.00	25.60	0.30	Annealed
	72.00	25.60	- 0.19	Annealed
		2.64	- 0.19	Annealed
	96.00	25.30	-10.82 to -11.62	Annealed
		98.30	-13.06	Annealed
		99.50	-13.62	Annealed
	2.64	- 8.12	Annealed	

¹ W. Thompson, *Trans. Roy. Soc. (London)* **146**, 649 (1856).

² H. Tomlinson, *Trans. Roy. Soc. (London)* **174**, 1 (1883).

³ J. A. Donaldson and R. Wilson, *Proc. Roy. Soc. (London)* **27**, 18 (1907).

⁴ N. F. Smith, *Phys. Rev.* **28**, 107, 429 (1909).

⁵ P. W. Bridgman, *Proc. Am. Acad. Arts Sci.* **57**, 39 (1922).

⁶ P. W. Bridgman, *Proc. Am. Acad. Arts Sci.* **59**, 117 (1923).

⁷ P. W. Bridgman, *Proc. Am. Acad. Arts Sci.* **60**, 423 (1925).

⁸ H. Rolnick, *Phys. Rev.* **36**, 506 (1930).

⁹ M. Allen, *Phys. Rev.* **42**, 848 (1942).

¹⁰ M. J. Druyvesteyn, *Physica* **17**, 748 (1951).

¹¹ A. V. deForest, *Instruments* **15**, 112 (1942).

¹² J. T. Norton and G. C. Kuczynski, *Instruments* **21**, 328 (1948).

¹³ G. C. Kuczynski and J. T. Norton, *J. Appl. Phys.* **19**, 683 (1948).

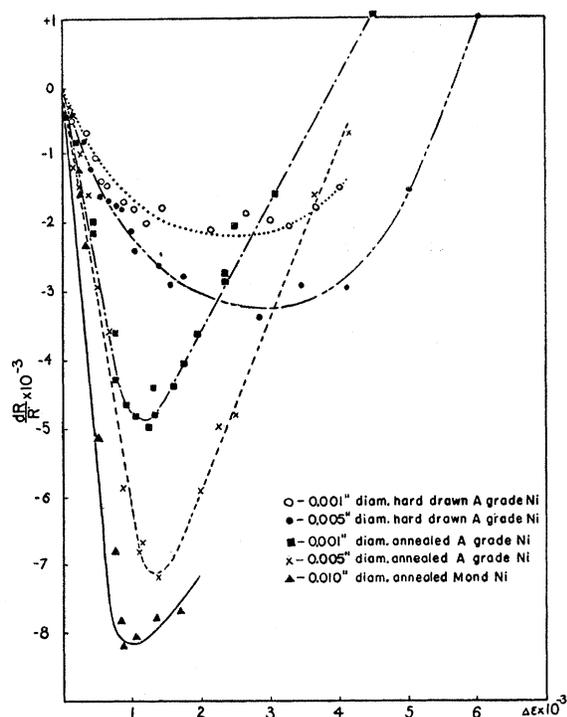


FIG. 1. Fractional change of electrical resistance for various nickel samples as a function of the strain increment $\Delta\epsilon$.

A study of the Table I reveals that: (a) Most metals and alloys investigated have rather low and positive strain coefficients of electrical resistivity. (b) Notable exceptions to this rule are Ni and Bi and some Ni and Bi alloys.¹³ (c) The strain sensitivity decreases with the wire diameter. It should be noted that this effect is not geometrical but due to the crystallographic anisotropy of the factor $d\rho/\rho d\epsilon$.¹⁰ (d) The results listed in Table I are in fair agreement with those obtained by other investigators, as can be seen in Table II.

In addition it has been observed that, beyond the yield point, the values of $d\rho/\rho d\epsilon$ decreased greatly for all metals. In the case of Ni, even within the elastic limit, a notable change of $d\rho/\rho d\epsilon$ has been observed. In Fig. 1, dR/R for Ni is plotted against increments of strain $\Delta\epsilon$. The curves seem to reach a minimum at a

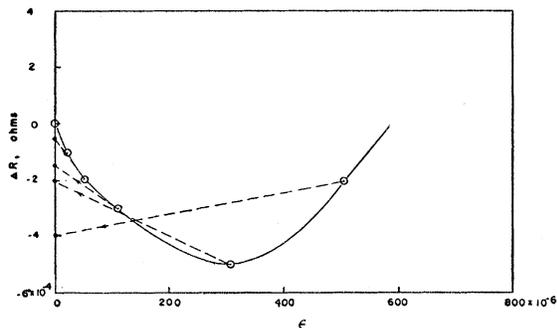


FIG. 2. Change in electrical resistance versus strain ϵ , for nickel, illustrating hysteresis.

certain value of $\Delta\epsilon$ and subsequently the coefficient becomes positive with an ultimate value of about 3.0. Similar behavior was observed by McKeehan.¹⁴ Also a hysteresis effect below the elastic limit of Ni has been observed and is shown in Fig. 2.

The variation of the strain coefficient of electrical resistivity with the concentration of Ni in Cu-Ni alloys is represented in Fig. 3. In these alloys the absolute value of strain sensitivity decreases with increasing alloy content. The same observation has been made recently by Linde *et al.*^{15,16} in the case of copper-silicon, copper-nickel, silver-tin, silver-manganese, and gold-chromium alloys.

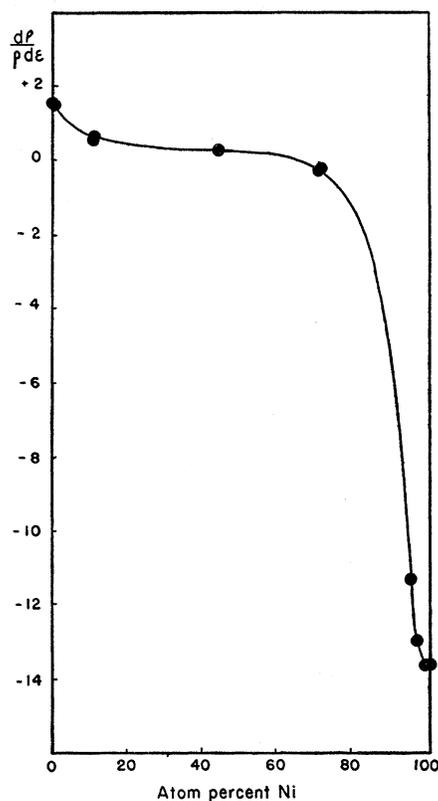


FIG. 3. Strain coefficient of resistivity of Ni-Cu alloys as a function of atomic concentration of nickel.

DISCUSSION OF RESULTS

Many properties of certain groups of metals such as copper, gold, and silver can be semiquantitatively treated as the properties of a hypothetical degenerate electron gas. Thus, in discussing metallic properties, one often assumes that the electrons in a metal can be treated as noninteracting particles moving in some kind of average potential of the periodicity of the lattice. This potential can be considered as being weak in the sense that the distribution of energy levels is

¹⁴ L. W. McKeehan, Phys. Rev. 36, 948 (1930).

¹⁵ Linde, von Heijne, and von Sabsay, Arkiv Fysik 2, 81 (1950).

¹⁶ J. O. Linde and C. H. Stade, Arkiv Fysik 2, 99 (1950).

determined largely by the kinetic rather than the potential energy. The electrical resistivity of a metal can be considered as a function of two variables, the mean free path l of the electrons and the effective number n of free electrons.

$$\rho \sim m/ln.$$

Assuming that the effective mass m of an electron is constant, we shall obtain the following expression for the coefficient of strain sensitivity:

$$\frac{d\rho}{\rho d\epsilon} = -\frac{dl}{ld\epsilon} - \frac{dn}{nd\epsilon}. \quad (2)$$

If a weak potential is assumed, then the energy E of the electrons can be adequately evaluated by the following equation:

$$E = (\hbar^2/2m)k^2, \quad (3)$$

where k is the wave vector. The number of free electrons

$$n = \frac{2}{\pi} \frac{m}{\hbar^2} \left(\frac{\partial E}{\partial k} \right)_{k=k_{\max}}$$

or, using Eq. (3),

$$n = (2\pi)k$$

and

$$dn/nd\epsilon = dk/kd\epsilon.$$

The wave vector $k \sim 1 - \beta\epsilon$ where β has different values for different metals but always is of the order of unity, therefore

$$dn/nd\epsilon \sim -1. \quad (4)$$

The change of the mean free path l can be evaluated from the relationship $l \sim 1/\langle X^2 \rangle_{Av}$, where $\langle X^2 \rangle_{Av}$ denotes the mean square of the displacement of the vibrating atom. It is easy to show¹⁷ that for $T > \theta$, $\langle X^2 \rangle_{Av} \sim 1/\theta^2$; therefore $l \sim \theta^2$. On the other hand, the change of the characteristic temperature θ with change of volume V is

$$\frac{d \ln \theta}{d \ln V} = -\gamma,$$

where γ is Grüneisen's constant, conveniently calculated from the following relationship:

$$\gamma = \frac{\alpha C_v}{\chi V_0},$$

where α is the volume coefficient of thermal expansion, C_v the molar specific heat at constant volume, χ compressibility and V_0 the atomic volume. If we assume that electrons in metals are not constrained to any specific path, then, as $d \ln V = (1 - 2\mu)d\epsilon$,

$$\frac{dl}{ld\epsilon} = \frac{2d\theta}{\theta d\epsilon} = -2\gamma(1 - 2\mu). \quad (5)$$

¹⁷ N. F. Mott and H. Jones, *The Theory of the Properties of Metals and Alloys* (Oxford University Press, London, 1936).

TABLE II. Strain coefficients of resistivity of metals.

Metal	μ	γ	$\left(\frac{d\rho}{\rho d\epsilon}\right)^a$	$\left(\frac{d\rho}{\rho d\epsilon}\right)^b$	$\left(\frac{d\rho}{\rho d\epsilon}\right)^c$	$\left(\frac{d\rho}{\rho d\epsilon}\right)^d$
Cu	0.35	1.96	1.60	1.60	0.90 to 1.60	2.18
Ag	0.38	2.40	2.16	2.06	...	2.15
Au	0.42	3.03	3.01	2.27	...	1.97
Al	0.34	2.17	2.62	-0.68	1.40	2.39
Pt	0.39	2.54	3.09	3.44	2.60	2.22
Pd	0.39	2.23	1.85	2.70	...	2.00
Pb	0.45	2.73	...	1.12	...	1.55
Sn	0.33	2.14	...	3.65	3.75	2.46
Mo	0.32	1.57	...	1.10	0.80	2.13
W	0.32	1.62	...	1.46	1.16	2.16
Fe	0.28	1.60	0.48	...	0.44	2.42
Co	0.33	1.87	0.37	...	0.84	2.27
Ni	0.30	1.88	-3.40	-9.0	-3.0 to -13.6	2.50
Sb	0.33	0.92	3.90	1.62
Bi	0.33	1.14	-8.7	-3.52	-11.0 to -11.7	1.775

^a Refers to Bridgman work, references 5, 6, and 7.

^b Refers to Kolnick work, reference 8.

^c Refers to present investigation.

^d Refers to Eq. (7).

Equations (2), (4), and (5) yield

$$\frac{d\rho}{\rho d\epsilon} = 1 + 2\gamma(1 - 2\mu). \quad (6)$$

The values of the strain sensitivity coefficient as calculated from Eq. (6) are tabulated in the last column of Table II along with values of μ and γ used in its computation. The agreement is by no means satisfactory but, with a few exceptions, the calculated values are of the right order of magnitude although always too large. It is believed that this is due to the overestimation of effect of free electrons. Also the effect of strains upon the mean free path, as expressed by Eq. (5), is oversimplified. Bridgman⁷ rightly states that the difference in longitudinal and transverse coefficients of strain sensitivity seems to indicate that the mean free path of the electrons must be somewhat constrained. However, the assumption of a fully constrained path does give too high longitudinal effects and does not account satisfactorily for the variety of the transverse effects.

The large increments of conductivity induced by elastic strains in nickel and the abnormally low increments of resistivity observed in cobalt and iron have been explained adequately by von Englert.¹⁸ The elastic

TABLE III. Some physical properties of arsenic, antimony, and bismuth.

Property	As		Sb		Bi	
		⊥		⊥		⊥
ρ (10^{-6} ohm cm)	28.0		35.6	42.6	143.0	109.0
A cgs units			0.20			-10.00
C (10^{-6} cgs units) (magnetic susceptibility)			-0.497	1.38	-1.00	-1.50
property measured along trigonal axis						
⊥ property measured perpendicular to the trigonal axis						

¹⁸ E. von Englert, *Ann. Physik* **5**, 14, 589 (1932).

field applied to these metals produces appreciable magnetization which in turn increases the conductivity of nickel and decreases that of cobalt and iron. The theoretical treatment of this phenomenon was given by Mott.¹⁹

It remains to discuss the abnormal strain sensitivity factors exhibited by the metals of the fifth group of the periodic table, antimony and bismuth. These metals as well as arsenic, the third in the same group, crystallize in the rhombohedral system and exhibit peculiar physical properties, placing them on the borderline of the metallic state. Some of their physical properties such as electrical resistivity, Hall constant A , and magnetic susceptibility are listed in Table III. A schematic representation of the unit cell of the bismuth crystal is represented in Fig. 4. All atoms in this cell are equidistant a with the exception of an inside atom. This atom C , is not placed in the middle of the trigonal axis but slightly below its center. As a consequence, the distance b of atom C from its nearest neighbors is shorter than a . This effect is even more pronounced in antimony and arsenic, as can be ascertained from Table IV, where the relative position of this atom on the trigonal axis is listed as u ; therefore the bond is stronger in the b than in the a direction and the cell does not deform isotropically under applied stress. The bonds b having greater rigidity will resist deformation more than the bonds a . Therefore, because of transverse contraction accompanying tension in the AB direction, atom C may travel towards the center of the AB axis and the parameter u would approach the value $\frac{1}{2}$.

Jones²⁰ found that the Brillouin zone for bismuth is bounded by $(1 \bar{1} 0)$, $(2 2 1)$ and $(\bar{2} \bar{2} 1)$ planes and contains exactly five electrons per atom, so that it should be completely filled. However, there must be a small number of electrons overlapping because the Hall coefficient for bismuth is large and negative (Table IV),

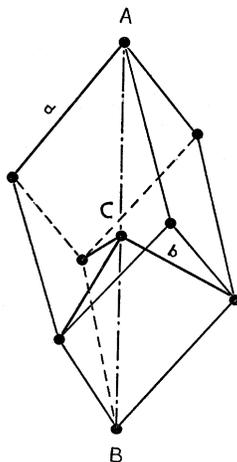


FIG. 4. Unit cell of bismuth.

TABLE IV. Crystallographic data on the structure of arsenic, antimony, and bismuth.

Metal	As	Sb	Bi
a in A	4.1350	4.4976	4.7356
θ	$54^\circ 7'$	$56^\circ 36'$	$57^\circ 16'$
u	0.452	0.466	0.474
S_{221}	1.369	1.018	0.794

indicating a small number of free electrons. The structure factor, proportional to the energy gap over the plane (hkl) , for the rhombohedral system, is

$$S_{hkl} = 2 \cos\{2\pi u(h+k+l)\}$$

where hkl are Miller indices of a plane and u the ratio CB/AB (see Fig. 4). The values of this structure factor for the $(2 2 1)$ plane, together with the elements of the unit cells of arsenic, antimony and bismuth, are given in Table IV. If u were equal to 0.5, $S_{221} = 0$ or there would be no energy discontinuity over this plane. On the other hand, this discontinuity increases sharply with even a slight decrease in u . As mentioned before, the electrical conductivity of bismuth depends chiefly on the overlapping electrons. The number of these electrons, n , is proportional to the probability of an electron jumping over the energy gap, which in turn can be assumed to be a function of the structure factor S . According to Zener²¹ this probability is proportional to $\exp(-cS^2)$ where c can be regarded as constant. Therefore, if due to the unequal strength of bonds in the unit cell, anisotropic deformation produces even a slight increase of u , the structure factor S decreases sharply and in consequence the number of overlapping or conducting electrons increases, manifesting itself in a high negative strain sensitivity coefficient. In the case of antimony, the Hall coefficient is large and positive (Table IV), indicating that the zone is not completely filled and in consequence the electric current is largely conducted by holes. Anisotropic deformation does not control the change in number of electrons. However, elastic deformation produces the contraction of the Brillouin zone and appreciable decrease of conducting holes and manifests itself in a large positive coefficient of strain sensitivity. The same is probably true for white tin, whose zone contains 4.24 electrons per atom,¹⁷ hence more electronic states than valency electrons. There should be therefore some overlapping of electrons in some crystallographic direction and holes in others. As the zone is almost filled, deformation will produce a large decrease in $\partial E/\partial k$ or in the number of conducting electrons. Therefore the strain coefficient of sensitivity of tin should be large and positive, as observed.

ACKNOWLEDGMENT

The author is indebted to Baldwin-Lima-Hamilton Corporation in Philadelphia for their generous support of this research.

¹⁹ N. F. Mott, Proc. Roy. Soc. (London) **A156**, 368 (1936).

²⁰ H. Jones, Proc. Roy. Soc. (London) **A147**, 396 (1939).

²¹ C. Zener, Proc. Roy. Soc. (London) **A145**, 521 (1934).