Proton-Neutron Mass Difference

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SUPPOSE all deviations from isotopic spin symmetry are due solely to electromagnetic effects. Then such things as the mass difference of charged and neutral π mesons, and the neutron-proton mass difference would have to be just electrodynamic.

We have investigated this point and have found that it is a reasonable possibility. For particles of zero spin, like π mesons, assumed to be elementary particles, the self-energy is quadratically divergent. If the photon propagation function[1] $1/k^2$ is cut off by a convergence factor $C(k^2)=[\Lambda^2/(\Lambda^2-k^2)]^2$, the resulting energy is about $3e^2\Lambda^2/8\pi m$, where Λ is the cut-off energy and m is the π-meson mass, and we assumed $\Lambda \gg m$. This gives the observed mass difference of about 11 electron masses for a cut-off Λ of about 1.0 proton masses.[2]

It is usually assumed that the negative value of the proton-neutron mass difference speaks against an ultimate electromagnetic explanation. The following calculation shows that this is an unwarranted assumption. The particles are not simple and there is uncertainty as to the correct law of coupling to the electromagnetic field. But for low energy, the proton can be represented by the Dirac equation with an additional Pauli term to represent the anomalous moment. Since we do not know to how high an energy this may be a reasonable approximation, we have tried providing the moment coupling term with a cut-off factor of its own. We write for the proton self-energy

$$\Delta M=(e^2/\pi i)\int\left[\gamma_\mu-\frac{\mu}{4M}(\gamma_\mu \boldsymbol{k}-\boldsymbol{k}\gamma_\mu)G(k)\right](\boldsymbol{p}-\boldsymbol{k}-M)^{-1}\times\left[\gamma_\mu+\frac{\mu}{4M}(\gamma_\mu \boldsymbol{k}-\boldsymbol{k}\gamma_\mu)G(k)\right]k^{-2}d^4k\,C(k)$$

in the notation of reference 1. We used $G(k)=-\lambda^2(k^2-\lambda^2)^{-1}$ to cut the moment coupling off at energies about λ, and $C(k)=-\Lambda^2(k^2-\Lambda^2)^{-1}$ to cut off the photon propagation function at energy Λ. The expression for the neutron is the same, except that the γ_μ coupling terms are omitted and the value of μ, the anomalous moment in nuclear magnetons, is -1.91 instead of 1.79 for the proton. M is the nucleon mass.

For the proton the term for $\mu=0$, representing coupling of current with current, is positive, as is also the term in μ^2. But the cross term, linear in μ, is negative and quite large if the moment is not cut off too soon. Thus the proton-neutron mass difference can easily turn out negative. For example, if Λ and λ are both taken at $1.4M$, the experimental value of -2.5 electron masses results for this difference (in this case for the neutron ΔM is roughly 1.5, for the proton -1.0 electron masses). No small difference of large numbers is involved. If the cut-off λ is reduced below about $0.75M$, a negative mass difference cannot be obtained. For $\lambda=1.0M$, $\Lambda=4M$ gives the experimental result.

The high cutoff for the anomalous moment implies that the charge responsible for the moment must be spread over only a small distance (of order $\hbar/Mc$). This is also suggested by the relatively small changes that the nucleon moments undergo when nucleons form nuclei.

The cutoff for the propagation function may be interpreted in two ways. Firstly, electrodynamics may fail at high energies, the failure being represented in a crude way by the cutoff. If this is so we could guess from our results that the failure occurs at energies in the neighborhood of the nucleon mass. Another possibility is that the electrodynamics is correct, but the cutoff represents, roughly, the error committed in assuming that the particles are elementary. For example, in the case of the π meson, we have assumed the π meson in virtual states acts as a simple particle. But for energies as high as M, strongly coupled virtual nucleon pairs may be formed. They, rather than failure of electrodynamics, may provide the convergence at energies of order M. In a like manner, the complex of virtual mesons presumed to be associated with nucleons may have the effect that at sufficiently high energy the electromagnetic coupling of neutron and proton may be nearly the same, so that the integral representing the difference of their masses may converge without modification of electrodynamics. In this way, the presumed convergence of the mass differences might tell us something about the character of coupling with the electromagnetic field at high energy.

We conclude that all of the deviations from isotopic spin symmetry could be due solely to coupling with the electromagnetic field.

[1] R. P. Feynman, Phys. Rev. **76**, 769 (1949). We use the notation in this reference.
[2] This result was given by one of us (RPF) at the International Conference in Theoretical Physics, Paris, 1950 (unpublished).

Polarization of Elastically Scattered Nucleons from Nuclei*

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NUCLEONS of low or moderate energy which are elastically scattered from nuclei should be partially polarized[1] by the strong spin-orbit potential underlying the predictions of the shell model of the nucleus. This spin-orbit potential is a consequence of the collective action of many nucleons on the particular nucleon. Thus for incident nucleons whose wavelength is greater than the nuclear spacing ($E \lesssim 50$ Mev), it would be expected that the spin-orbit potential of the shell model would make itself felt. For progressively higher energies the incident nucleon begins to see only one nucleon at a time and while a spin dependence of the elastic scattering can still be expected, it would be more a reflection of the individual nucleon-nucleon interactions than of the spin-orbit potential of the shell model. It will be supposed that even at these higher energies the spin dependence has the form of the usual spin-orbit potential. In either case this spin dependence of the elastic scattering can be investigated phenomenologically by treating the interior of the nucleus in terms of a spin-dependent complex index of refraction[2]—an obvious generalization of the optical model of the nucleus.[3]

For low or moderate energies there is no suitable approximate method for treating the elastic scattering—a phase-shift analysis is necessary. Also, at high energies any polarization calculations using conventional approximation methods[4] are made uncertain by the direct dependence of the polarization on the phase of the scattered wave. A phase-shift analysis for various energies is therefore being undertaken on the Univac at the University of California Radiation Laboratory at Livermore in collaboration with S. Fernbach.

An estimate for small angles of scattering, though rough at best, may be readily obtained by making several simplifying assumptions. The magnitude of the polarization is given by

$$P=\left(\frac{AB^*+A^*B}{d\sigma/d\Omega}\right)\sin\theta. \qquad (1)$$

Here A and B represent the scattering amplitudes corresponding to the spin-independent and spin-dependent parts of the interaction, respectively. The known experimental value for the differential cross section $d\sigma/d\Omega$ may be used. The amplitude, B, for spin-dependent scattering may be estimated by using the Born approximation. Then only the imaginary part of A contributes to P. For small angles this is approximately proportional to the total cross section.

For 300-Mev neutrons incident on carbon, for example, a square-well spin-orbit interaction ($R=1.4A^{\frac{1}{3}}\times10^{-13}$ cm) of 2-Mev depth gives a polarization of 40 percent at five degrees. Though this is probably an overestimate, it suggests that the existence of a small