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A Connection between Pion Photoproduction and Scattering Phase Shifts*

MARC ROSS
Brookhaven National Laboratory, Upton, New York
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It is found in a rough approximation that meson photoproduction matrix elements can be expressed in terms of meson-nucleon scattering phase shifts and some energy-independent parameters, for photons in the laboratory of 200 to 500 Mev. The parameters are fitted to experiment in the lower portion of this energy region. It is then found that the higher-energy photoproduction data is consistent only with a $p_{\frac{3}{2}}$, isotopic spin $\frac{3}{2}$, phase shift which passes through 90° at a scattering energy of about 180 Mev. It is indicated that the s-wave phase shifts remain small.

INTRODUCTION

A SEMIPHENOMENOLOGICAL connection between photoproduction of mesons and meson-nucleon scattering phase shifts is developed in the energy region E_γ(lab) 200–500 Mev. It is assumed that the photoproduction matrix element to a particular angular momentum isotopic spin state is the sum of a Born approximation term plus a term in which the photoproduced meson is scattered before leaving the nucleon. The Born approximation portion of the matrix element is calculated in the (relativistic) weak coupling limit of the symmetric pseudoscalar coupling theory. The coupling constant $G^2/4\pi=16$ is selected beforehand to be in agreement with the (unrenormalized) Tamm-Dancoff theory of meson scattering.[1] The other portion of the photoproduction matrix element will be expressed in terms of parameters which can be fitted to experiment in the low-energy region where fairly complete data on scattering and photoproduction are available.† At higher energies, where scattering experiments have not as yet led to unique phase shifts, the behavior of the phase shifts can be determined from photoproduction data.

PHOTOPRODUCTION MATRIX ELEMENTS

Let $l_{j,m}{}^{0,+}$ be the photoproduction matrix element to a state of orbital angular momentum l, total angular momentum j, and its projection m, with 0,+ being the type of meson produced from photons on protons, such that the differential cross section is

$$\frac{d\sigma^{0,+}}{d\Omega}=\sum_m\langle|\sum_{l,j} l_{j,m}{}^{0,+}\phi_{lj,m}(\theta,\sigma)|^2\rangle_\sigma \qquad (1)$$

in microbarns/sterad in the center-of-mass system, where we define

$$\phi_{s\frac{1}{2},\frac{1}{2}}=\chi^+,$$

$$\phi_{p\frac{1}{2},\frac{1}{2}}=-\cos\theta\chi^+-\sin\theta e^{i\phi}\chi^-,$$

$$\phi_{p\frac{3}{2},\frac{1}{2}}=\sqrt{2}\cos\theta\chi^+-\frac{1}{\sqrt{2}}\sin\theta e^{i\phi}\chi^-,$$

$$\phi_{p\frac{3}{2},\frac{3}{2}}=\sqrt{\tfrac{3}{2}}\sin\theta e^{i\phi}\chi^+.$$

Higher angular momentum states ($l>1$) are treated in a group as a function of θ, m, σ. Thus,

$$\sum_{l>1,j} l_{j,m}{}^{0,+}\phi_{lj,m}=\sum_s K_{m,s}{}^{0,+}(\theta)\chi^s. \qquad (2)$$

To obtain the matrix elements $l_{j,m}$ consider the wave function of the one-nucleon one-meson component of the final state to have the form

$$\sum_\Gamma \psi_\Gamma \underset{r\to\infty}{=} \sum_\Gamma e^{i\delta}i^l\frac{\sin(kr-\frac{1}{2}l\pi+\delta)}{kr}\phi_\Gamma, \qquad (3)$$

with the ϕ_Γ a set of orthonormal angular momentum isotopic spin functions, where Γ represents the appropriate quantum numbers. δ is the scattering phase shift in the state Γ, k is the center-of-mass momentum. The explicit dependence of the photoproduction on Γ is given in Eq. (3).

It is easily seen that

$$\int\left(\frac{k'}{k}\right)^2 dE'\frac{1}{E-E'}\frac{\sin(k'r-\frac{1}{2}l\pi)}{k'r}\underset{r\to\infty}{=}\pi\frac{\cos(kr-\frac{1}{2}l\pi)}{kr}, \qquad (4)$$

where E is the energy of the system. Then we can write (for all r)

$$\psi_\Gamma=\int\left(\frac{k'}{k}\right)^2 dE' e^{i\delta}\cos\delta\left[\delta_{(E-E')}-\frac{g_E(E')}{E-E'}\right]i^l f_l(k'r)\phi_\Gamma, \qquad (5)$$

where, from Eqs. (3) and (4), $g_E(E)=-\pi^{-1}\tan\delta$. The function $g_E(E')$ for $E'\neq E$ determines the wave function at small distances. The quantity in the bracket is seen to be the Γth partial wave of the standing wave solution

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[1] Dyson *et al.* (to be published); F. J. Dyson, in *Proceedings of the Rochester Conference on High Energy Physics, December, 1952* (Interscience Publications, New York, 1953); Sundaresan, Salpeter, and Ross, Phys. Rev. **90**, 372 (1953).

† *Note in proof.*—General properties of the problem which are independent of this model are discussed by K. Aizu, Proceedings of the International Conference on Theoretical Physics, Tokyo (Sept. 1953), and K. M. Watson, Phys. Rev. (to be published). A formal treatment of the model in question is being prepared by the author [for a brief discussion see M. Ross, Phys. Rev. **92**, 855 (1953)]. For earlier phenomenological work see K. A. Brueckner and K. M. Watson, Phys. Rev. **86**, 923 (1952).

of the Goldberger formalism.[2] When the matrix element is computed with ψ_Γ, the delta function term yields $e^{i\delta}\cos\delta B_E(E)$ while the other term is $e^{i\delta}\cos\delta\int dE'(k'/k)^2 \times B_E(E')g_E(E')/(E-E')$, where $B_E(E')$ is the Born approximation matrix element to the state Γ with off the energy shell components indicated by $E'\neq E$.

The photoproduction matrix element to a particular angular momentum isotopic spin state can then be written

$$(B_E(E)\cos\delta+A\sin\delta)e^{i\delta}, \tag{6}$$

where

$$A=\frac{1}{\tan\delta}\int\left(\frac{k'}{k}\right)^2 dE'\frac{g_E(E')}{E-E'}B_E(E').$$

In order to calculate these matrix elements restrictive assumptions must be made. As stated we shall calculate the Born approximation in lowest-order perturbation theory. In this theory B_E is roughly independent of the energy E.[3] We shall assume this independence to be exact. In addition, it is assumed that $g_E(E')/(E-E')$, the scattered wave in the meson scattering problem, has the same shape, or relative dependence on the variable $E-E'$, independent of E. Only the magnitude is considered to depend on energy. The magnitude of g is assumed to behave as $g_E(E)$,[4] i.e., as $\tan\delta$, so we have that A is roughly independent of E and δ. It should be kept in mind that this is a crude approximation.

The details of the calculation of photoproduction in the weak coupling limit can be found elsewhere.[5] We shall merely state results here. In the problem at hand we require the photoproduction matrix elements $B_E(E)$ and an analysis into their s and p components. The following notation is employed: $\hbar=c=1$. Results are stated in the center-of-mass system, with the Z axis given by the incoming photon. Let μ= meson mass, M= nucleon mass, $\mathbf{k}$= meson momentum, θ= angle between $\mathbf{k}$ and Z, $\omega=(k^2+\mu^2)^{\frac{1}{2}}$, $\epsilon=(k^2+M^2)^{\frac{1}{2}}$, and $E=\omega+\epsilon$. The matrix elements to the plane-wave final state are functions of θ and are characterized by the projections of final spin and of total angular momentum, s and m; and by the isotopic spin state. The polarization of the initial photon is taken to be $\mathbf{x}+i\mathbf{y}$.

In the symmetric pseudoscalar coupling theory[6] the photoproduction proceeds in lowest order by the three diagrams of Fig. 1. The matrix elements associated with these diagrams are

$$\begin{aligned} B^{(1)}&=CI^{(1)}\frac{\sqrt{2}a}{-\omega+k\cos\theta},\\ B^{(2)}&=-CI^{(2)}\frac{a+b}{\epsilon+k\cos\theta},\\ B^{(3)}&=CI^{(3)}\frac{b}{\sqrt{2}E},\end{aligned} \tag{7}$$

where

$$C=-\frac{ieG}{4\pi}\left(\frac{k(\epsilon+M)}{2(E^2-M^2)}\right)^{\frac{1}{2}}\frac{1}{E-M},$$

$$a=\frac{k\sin\theta e^{i\phi}}{\sqrt{2}}\left(\chi^s,\left[-\frac{\mathbf{k}\cdot\boldsymbol{\sigma}}{\epsilon+M}+\frac{E-M}{E+M}\sigma_z\right]\chi^{M-1}\right),$$

$$b=\frac{E-M}{2}\left(\chi^s,\left[\frac{\mathbf{k}\cdot\boldsymbol{\sigma}}{\epsilon+M}+1\right]\frac{\sigma_x+i\sigma_y}{\sqrt{2}}\chi^{M-1}\right),$$

and the I's denote the appropriate isotopic spin factors. If each I is written as the coefficient in the isotopic spin $\frac{1}{2}$ state and the coefficient in the $\frac{3}{2}$ state, respectively, then

$$I^{(1)}=(-\sqrt{\tfrac{2}{3}},\sqrt{\tfrac{1}{3}});\; I^{(2)}=(\sqrt{\tfrac{1}{3}},\sqrt{\tfrac{2}{3}});\; I^{(3)}=(\sqrt{6},\ 0).$$

The matrix elements to final s and p states are[7]

$$\begin{aligned} s_{\frac{1}{2}}&=\frac{C}{2}\left[-I^{(1)}\frac{k}{\epsilon+M}J-I^{(2)}\frac{\sqrt{2}}{\epsilon}\left(\omega+\frac{\epsilon-M}{3}\right)+I^{(3)}\frac{E-M}{E}\right],\\ p_{\frac{1}{2}}&=\frac{C}{2}\left[I^{(1)}\frac{E-M}{E+M}J-I^{(2)}\frac{(E-M)k}{3\sqrt{2}\epsilon^2}\left(1-\frac{\omega}{\epsilon}\right)-I^{(3)}\frac{(E-M)k}{E(\epsilon+M)}\right],\\ p_{\frac{3}{2},\frac{1}{2}}&=\frac{C}{2}\left[\frac{I^{(1)}}{\sqrt{2}}\left\{\left(\frac{E-M}{E+M}-\frac{3\omega}{\epsilon+M}\right)J-\frac{4k}{\epsilon+M}\right\}+I^{(2)}\frac{(E-M)k}{3\epsilon^2}\right],\\ p_{\frac{3}{2},\frac{3}{2}}&=\frac{C}{2}\left[\sqrt{\tfrac{3}{2}}I^{(1)}\left\{\left(\frac{E-M}{E+M}-\frac{\omega}{\epsilon+M}\right)J-\frac{4k}{3(\epsilon+M)}\right\}+\frac{I^{(2)}}{2\sqrt{3}}\left(-\frac{4(E-M)}{E+M}-\frac{\epsilon-M}{\epsilon}\right)\frac{k}{\epsilon}\right],\end{aligned} \tag{8}$$

[2] M. L. Goldberger, Phys. Rev. 84, 929 (1951).

[3] See for example the total cross-section curve in G. Araki, Progr. Theoret. Phys. (Japan) 5, 570 (1951), or the $d\sigma^+/d\Omega(90°)$ curve in J. Steinberger and A. S. Bishop, Phys. Rev. 86, 171 (1952).

[4] Some indication that this may be reasonable comes from the the calculation of g in the Tamm-Dancoff approximation (reference 1). Where it is found in those cases examined, that for E near threshold, $g_E(E')$ is fairly flat from say $E'=E$ to $E+Mc^2$ (where M is the nucleon mass), while it drops sharply as E' decreases below E.

[5] Benoist-Gueutal, Prentki, and Ratier, Compt. rend. 230, 1146 (1950); G. Araki, Progr. Theoret. Phys. (Japan) 5, 507 (1950); K. A. Brueckner, Phys. Rev. 79, 641 (1950); M. F. Kaplon, Phys. Rev. 83, 712 (1951).

[6] We consider the interaction $iG\bar{\psi}\gamma_5\tau_\alpha\phi_\alpha\psi$. The coupling constant G is $\sqrt{2}$ times that usually employed in the charged theory. The latter is used by Kaplon (reference 5), and Steinberger and Bishop (reference 3), among others.

[7] The contribution from $B^{(2)}$ which is relatively small has been approximated for convenience. The first two terms in an expansion in powers of k/E are kept, with a consequent error of ≤ 5 percent in the $B^{(2)}$ contribution in the energy region under consideration.

where

$$J=k\int_{-1}^{1}\frac{(1-x^2)dx}{-\omega+k\cos\theta}=2\left[\left(\frac{\mu}{k}\right)^2\ln\left(\frac{k+\omega}{\mu}\right)-\frac{\omega}{k}\right].$$

One can also calculate the wave function g, and thus the terms A in the photoproduction matrix elements in some approximation, from the meson theory. It is felt, however, that this is a reasonable place for a phenomenological viewpoint. The A's are to be regarded as parameters determined experimentally. Let a_1, a_3, c, d be proportional to the parameters A in the $s_{\frac{1}{2}}$, isotopic spin $T=\frac{1}{2}$; $s_{\frac{1}{2}}$, $T=\frac{3}{2}$; $p_{\frac{3}{2},\frac{1}{2}}$, $T=\frac{3}{2}$; $p_{\frac{3}{2},\frac{3}{2}}$, $T=\frac{3}{2}$ states, respectively. It is assumed for simplicity that the phase shifts in the other states are small enough so that the corresponding parameters in these states exert negligible influence.

If the Born approximation from Eq. (8) at E_γ(lab) $=310$ Mev[8] is used, the photoproduction matrix elements in this theory are

$$\begin{aligned}
s_{\frac{1}{2}}^{+}&=e^{i\delta_1}(-1.27\cos\delta_1+a_1\sin\delta_1)\\
&\quad+e^{i\delta_3}[-0.94\cos\delta_3+(a_3/\sqrt{2})\sin\delta_3],\\
s_{\frac{1}{2}}^{0}&=e^{i\delta_1}(+0.90\cos\delta_1-(a_1/\sqrt{2})\sin\delta_1)\\
&\quad+e^{i\delta_3}(-1.32\cos\delta_3+a_3\sin\delta_3),\\
p_{\frac{1}{2}}^{+}&=-0.41-0.43=-0.84,\\
p_{\frac{1}{2}}^{0}&=+0.29-0.62=-0.33,\\
p_{\frac{3}{2},\frac{1}{2}}^{+}&=-0.34+e^{i\delta}(-0.06\cos\delta+(c/\sqrt{2})\sin\delta),\\
p_{\frac{3}{2},\frac{1}{2}}^{0}&=+0.24+e^{i\delta}(-0.09\cos\delta+c\sin\delta),\\
p_{\frac{3}{2},\frac{3}{2}}^{+}&=-0.70+e^{i\delta}(-0.52\cos\delta+(d/\sqrt{2})\sin\delta),\\
p_{\frac{3}{2},\frac{3}{2}}^{0}&=+0.50+e^{i\delta}(-0.73\cos\delta+d\sin\delta).
\end{aligned}\tag{9}$$

As in the $p_{\frac{1}{2}}$ state for both isotopic spins, in the $p_{\frac{3}{2}}$ state the $T=\frac{1}{2}$ phase shift is assumed to be sufficiently small to be neglected. The symbol δ indicates the $p_{\frac{3}{2}}$, $T=\frac{3}{2}$ phase shift. In the s state δ_1, δ_3 are for $T=\frac{1}{2}$, $\frac{3}{2}$, respectively.

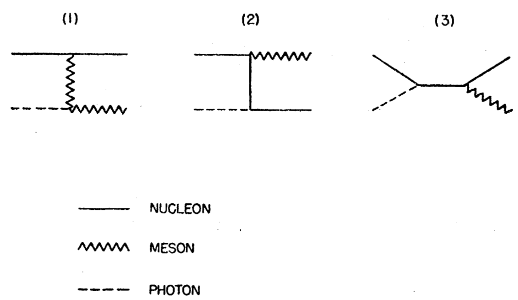

FIG. 1. Lowest order diagrams for photoproduction.

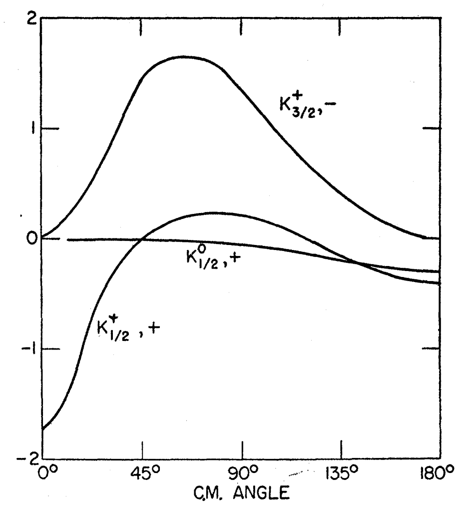

FIG. 2. High angular momentum contribution to Born approximation matrix elements.

Those higher angular momentum matrix elements which are not trivially small are deduced from Eqs. (7), (8) and are expressed in terms of $K_{m,s}^{0,+}(\theta)$, as in Eq. (2). They are plotted in Fig. 2.[9] The phase shifts in these states with $l>1$ are assumed to be sufficiently small so that the Born approximation is valid.[10]

We have some useful *a priori* knowledge of the parameters. Thus, an attractive phase shift will generally increase the Born approximation matrix element to a given angular momentum isotopic spin state. The only deviations from this rule, that might be expected to occur, would be associated with competition of processes. Care must be exercised in the two $p_{\frac{3}{2}}$ states where $E2$ and $M1$ interactions compete. It is found that a_1, a_3, d should be negative, and $c\doteq -d/\sqrt{3}$.[11]

The differential cross sections are calculated from Eq. (9) and Fig. 2, using Eq. (1). An example will be given here because of its particular importance. Thus,

$$\frac{d\sigma^0}{d\Omega}(90^\circ)=\tfrac{3}{2}|p_{\frac{3}{2},\frac{3}{2}}|^2+\left|\frac{1}{\sqrt{2}}p_{\frac{3}{2},\frac{1}{2}}+p_{\frac{1}{2}}\right|^2+|s_{\frac{1}{2}}|^2.\tag{10}$$

In this special case a convenient approximation can be made. The $p_{\frac{3}{2},\frac{3}{2}}$ contribution is found to be about 90 percent of the r.h.s. of Eq. (10). Furthermore, it is easily shown that $p^0_{\frac{3}{2},\frac{3}{2}}\sim\sin(\delta+\alpha)$ within about 1 per-

[8] Convenient formulas for the laboratory energies in which the initial proton is at rest, E_γ(lab), and in which the final nucleon is at rest, E_π(lab), are $E_\pi(\text{lab})=(\mu^2+k^2E^2)^{\frac{1}{2}}-\mu$, $E_\gamma(\text{lab})=E_\pi(\text{lab})+\mu[1+(\mu/2M)]$.

[9] While neutral production can be expressed as $A+B\cos\theta+C\sin^2\theta$, it is seen that this will not be strictly true for positive mesons. At very small angles large deviations from this type of behavior can occur.

[10] Errors introduced in neglecting any small phase shifts are small in photoproduction, though a small d wave phase shift may loom large in the analysis of a scattering distribution.

[11] This relation holds between the matrix elements for a pure $M1$ transition, while the relation $p_{\frac{3}{2},\frac{1}{2}}=\sqrt{2}p_{\frac{3}{2},\frac{3}{2}}$ would hold for an $E2$ transition. With a strong attractive interaction one expects the $M1$ transition to be strongly enhanced over the $E2$. The weak coupling π^+ matrix element in the $p_{\frac{3}{2},\frac{1}{2}}$ state happens to correspond to the $E2$ interaction being stronger than $M1$, so that c has the opposite sign to the Born approximation.

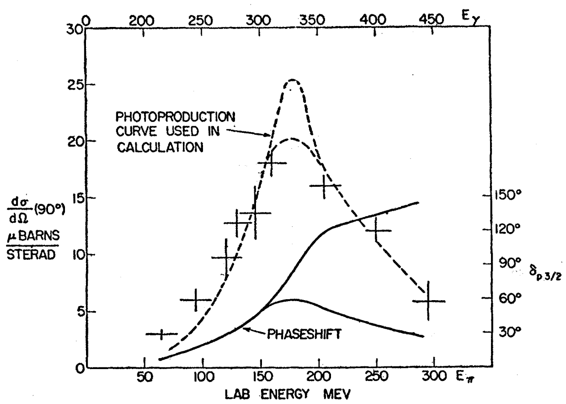

FIG. 3. Neutral photoproduction and the corresponding $p_{\frac{3}{2}}$, $T=\frac{3}{2}$ phase shift from Eq. (11). The experimental points (reference 13) are at 90° in the laboratory, while the theoretical points are in the center of mass.

cent. Then the simple relation,

$$\frac{d\sigma^0}{d\Omega}(90°)\sim\sin^2(\delta+\alpha), \tag{11}$$

where

$$\alpha=\tan^{-1}\left(\frac{0.50-0.73}{d}\right),$$

can be used to determine the $p_{\frac{3}{2}}$, $T=\frac{3}{2}$ phase shift with fair accuracy.

COMPARISON WITH EXPERIMENT

Using measured scattering phase shifts for E_π(lab) ≤ 135 Mev[12] and fitting $d\sigma^0/d\Omega(90°)$ at E_γ(lab)$=285$ Mev (Fig. 3),[13] $d\sigma^+/d\Omega(\theta)$ at 265 Mev (Fig. 4)[14–16] and $d\sigma^+/d\Omega(90°)$ *vs* E (Fig. 5) up to 265 Mev, we obtain the parameters

$$d=-4,\quad \alpha=3\tfrac{1}{2}°,\quad c=2,\quad a_3-a_1=1,$$

(with the *s*-wave parameters especially rough). The π^0 angular distribution could also be helpful in determining the parameters, unfortunately it is now known only roughly. In the neighborhood of 250–300 Mev it is seen that the quantity (approximately) $2a_3+a_1$ determines the asymmetry or $\cos\theta$ term in the π^0 angular distribution. If we assume arbitrarily that there are 10 percent more π^0 forward of 90° than backward,‡ then (roughly)

$$a_3=-1,\quad a_1=-2.$$

The $p_{\frac{3}{2}}$, $T=\frac{3}{2}$ phase shift predicted (above $E=135$ Mev) in accordance with Eq. (11) is shown in Fig. 3. There are two types of behavior. If the maximum in the neutral meson photoproduction at 90° is as high as *about* 25 millibarns/sterad (upper curve), then the $p_{\frac{3}{2}}$ phase shift can pass over 90° (case I). If the photoproduction maximum is lower, then the $p_{\frac{3}{2}}$ phase shift decreases before reaching 90° (case II, lower curve). The experimental values for $d\sigma^0/d\Omega(90°)$ are too rough, as yet, to determine the significant parameter d or to place the maximum with sufficient accuracy to distinguish between the cases.

It is seen that between E_π(lab)$=150$ Mev and 210 Mev, the detailed behavior of phase shift is not given by the measured values of $d\sigma^0/d\Omega(90°)$. In this region, *in order to be definite*, δ was chosen to pass through 90° at 180 Mev and $\cot\delta$ was required to vary smoothly

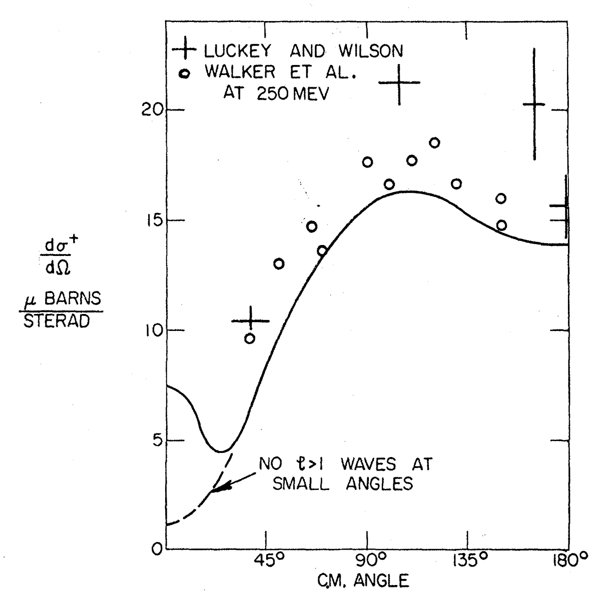

FIG. 4. Photoproduction of π^+ at 265 Mev.

[12] The scattering phase shifts used in the analysis are those of Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953), and Bodansky, Sachs, and Steinberger, Phys. Rev. **90**, 997 (1953).

[13] A complete survey of the π^0 experimental situation was not made. The first two of the following references were arbitrarily chosen for comparison. A. Silverman and M. Stearns, Phys. Rev. **88**, 1225 (1952); Walker, Oakley, and Tollestrup, Phys. Rev. **89**, 1301 (1953); Goldschmidt-Clermont, Osborne, and Scott, Phys. Rev. **89**, 329 (1953); C. G. Andre, UCRL 2425 (1953).

[14] Preliminary results of P. D. Luckey and R. R. Wilson: π^+ photoproduction measurements up to 280 Mev.

[15] Preliminary results of Cal-Tech group (R. L. Walker *et al.*): π^+ photoproduction differential cross sections from 200 to 500 Mev. Some π^0 differential cross sections at high energies have also been measured by this group.

[16] A complete survey of the π^+ experimental situation was not made. References 14 and 15 were arbitrarily chosen for comparison. Experimental work on π^+ photoproduction has also been done by Steinberger and Bishop (reference 3); White, Jakobson, and Schulz, Phys. Rev. **88**, 836 (1952); Jarmie, Repp, and White, Phys. Rev. **91**, 1023 (1953); Goldschmidt-Clermont, Osborne, and Winston, Phys. Rev. **91**, 468 (1953); E. L. Goldwasser and G. Bernadini, Bull. Am. Phys. Soc., Vol. **29**, No. 1, 18 (1954); G. Sargent Janes and W. L. Kraushaar, Phys. Rev. **93**, 900 (1954). With reference to the first three experiments above (from Berkeley), there has been some question of a difference in the calibration used relative to that used at other laboratories. It is believed that there is no such difference here (within 5%) (private communication with R. S. White).

‡ Preliminary indications are that this is incorrect (L. S. Osborne, private communication). The results that follow will not, however, be too sensitive to this behavior.

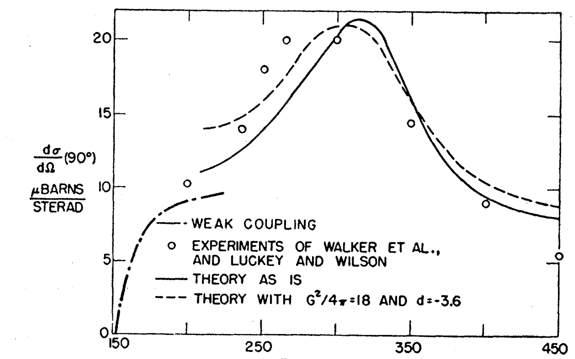

FIG. 5. Photoproduction of π^+ at 90° in the center-of-mass system. (See reference 20.)

with energy. What we shall call the resonance (the passing through 90° by the phase shift) could occur anywhere from say 165–195 Mev and could be considerably narrower or broader than that shown, without straining the fit to the existing π^0 data.

The most sensitive test of whether this resonance occurs (case I) or whether the phase shift remains below 90° (case II) is given by $d\sigma^+/d\Omega$ at higher energies. Since the π^- scattering data[17] at these energies antedates the photoproduction experiments we shall use the former to determine the s-wave phase shifts involved in the latter. The determination of the phase shifts from the scattering data is easy, and in its essentials quite unambiguous, now that the $p_{\frac{3}{2}}$, $T=\frac{3}{2}$ phase shift is given

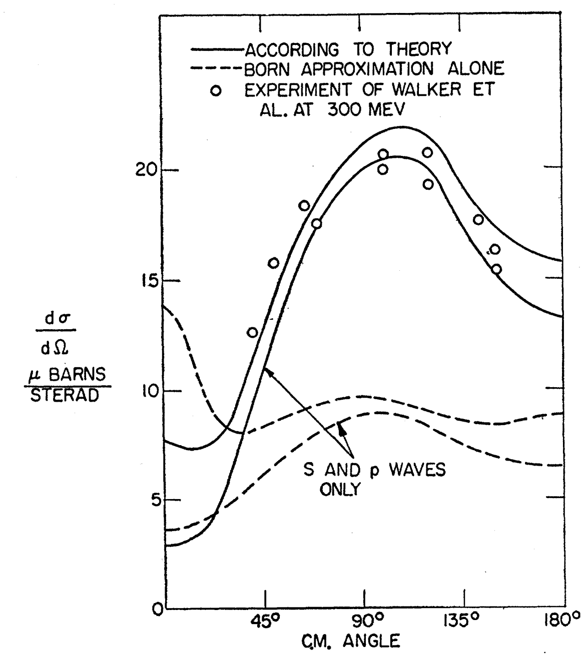

FIG. 6. Photoproduction of π^+ at 310 Mev.

[17] Fermi, Glicksman, Martin, and Nagle, Phys. Rev. **92**, 161 (1953).

Consider only s and p waves; then the scattering cross section is

$$d\sigma/d\Omega = A + B\cos\theta + C\cos^2\theta,$$

where

$$A/\lambda\!\!^{-2} = |a_{p3} - a_{p1}|^2 + |a_s|^2,$$
$$B/\lambda\!\!^{-2} = 2\,\mathrm{Re}\,a_s^*(2a_{p3} + a_{p1}),$$
$$C/3\lambda\!\!^{-2} = |a_{p3}|^2 + 2\,\mathrm{Re}\,a_{p3}^* a_{p1}.$$

The scattering amplitudes, $a_{l,2j}$, for the each of the three processes,

(a) $\pi^+ + p \rightarrow \pi^+ + p$,

(b) $\pi^- + p \rightarrow \pi^- + p$,

(c) $\pi^- + p \rightarrow \pi^0 + p$,

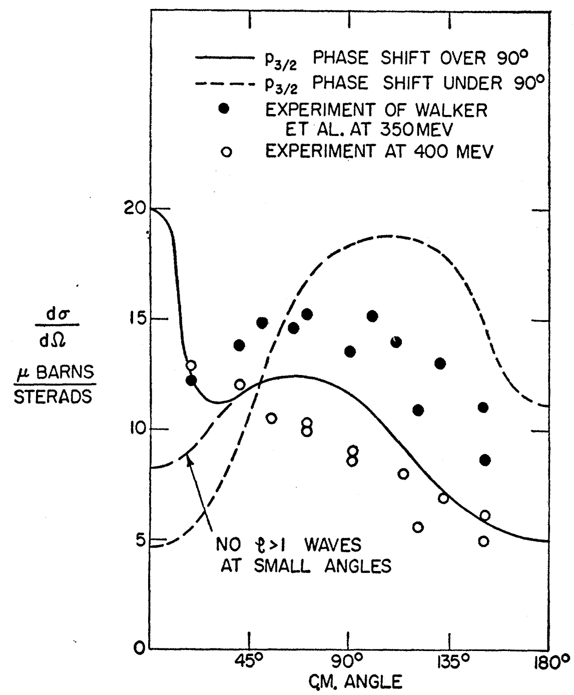

FIG. 7. Photoproduction of π^+ at 375 Mev.

are defined in terms of the scattering amplitudes to the isotopic spin states ($a_{l,j,T} = e^{i\delta}\sin\delta$). These relations are

(a) $a_{lj} = a_{lj,\,T=\frac{3}{2}}$,

(b) $a_{lj} = \frac{1}{3}(a_{lj,\,T=\frac{3}{2}} + 2a_{lj,\,T=\frac{1}{2}})$,

(c) $a_{lj} = \frac{1}{3}\sqrt{2}(a_{lj,\,T=\frac{3}{2}} - a_{lj,\,T=\frac{1}{2}})$.

For case I it is found that the s-wave phase shifts can remain small. We assume then, to simplify the problem further, that the s wave $T=\frac{3}{2}$ phase shift is given by extrapolating from lower energy scattering work:

$$\delta_3 = 0.40 - 0.52(k/\mu). \quad (12)$$

Table I gives the phase shifts chosen at 210 Mev to fit the π^- scattering data under these conditions. The comparison with the data is given in Table II. Also shown in Table I are the phase shifts used to calculate the photoproduction cross sections at $E_\gamma(\text{lab})=310$ Mev and 375 Mev. These curves are shown in Figs. 6, 7. The s-wave phase shifts associated with case II are taken, essentially, from Fermi and Metropolis.[18,19]

The test between case I and case II is made in Fig. 7. The latter is quite unambiguously in disagreement with experiment, while the resonance case is in good agreement. It would be interesting to investigate the π^+ photoproduction experimentally at very small angles to further test this theory. The extension of $d\sigma^+/d\Omega(90°)$ in accordance with this result is given in Fig. 5. The π^0 photoproduction cross section at 375 Mev is given in Table III. The experimental neutral meson data[13] is rough, but is in agreement with a very large $\sin^2\theta$ term. There is also an underermined amount of forward asymmetry.

The π^+ total scattering cross section implied by the theory is shown in Fig. 8. Only the s and $p_{\frac{3}{2}}$ phase shifts are considered here. The experimental cross sections

TABLE I. Phase shifts used for calculations with $E_\pi>135$ Mev.

	Angular momentum isotopic spin state					
	s		$p_{\frac{1}{2}}$		$p_{\frac{3}{2}}$	
	$T=\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
$E_\gamma=310$ Mev[a]	10	-20				60
$E_\pi=210$ Mev[b] Case I	20	-26	10	-10	-10	122
$E_\gamma=375$ Mev[c] Case I	10	-27				128
$E_\gamma=375$ Mev[c] Case II	0	-60				45

[a] Used in Fig. 6.
[b] Used in Table II.
[c] Used in Fig. 7 and Table III.

are, as yet, as rough as the theory!§ It will be recalled that the peak can be narrower or broader and can be shifted 10 Mev or so in either direction.

DISCUSSION

Qualitatively, why does the photoproduction behave as it does? The Born approximation (weak coupling limit) for photoproduction of π^+ is plotted at 310 Mev in Fig. 6. It is essentially isotropic. This is mainly due to

TABLE II. π^- scattering from protons at 210 Mev, $A+B\cos\theta+C\cos^2\theta$ millibarns/sterad.

Process		A	B	C
$\pi^-\to\pi^-$	Theory	1.2	0.5	2.4
	Experiment[a]	1.56±0.34	0.50±0.47	2.14±1.09
$\pi^-\to\pi^0$	Theory	1.4	2.5	3.4
	Experiment[a]	0.84±0.70	1.94±0.73	5.56±2.31
$\pi^+\to\pi^+$	Theory	1.1	1.4	2.7

[a] See reference 17.

the large s-state matrix element. Production to the high angular momentum states ($l>1$) consists of a low isotopic background and a large bump at extreme forward angles arising through interference with the s state matrix element. The asymmetry associated with the interference between the relatively small p-state matrix elements and the s state is over shadowed by the high angular momentum effects, but it is seen to be a small backward effect. The $p_{\frac{3}{2}}$ wave is too small to cancel the effect of the $p_{\frac{1}{2}}$ wave in this respect. In the Born approximation the neutral photoproduction is negligibly small.

When the meson is considered to interact with the nucleon in the final state there results a large increase in the $p_{\frac{3}{2}}$ matrix element. In the neighborhood of 300 Mev the shape for π^+ and π^0 photoproduction approaches that for a pure $M1$ transition to the $p_{\frac{3}{2}}$ state $(2+3\sin^2\theta)$. For the π^0 the s state matrix element remains small. For the π^+ the s state contribution is reduced as a result of repulsion in the $T=\frac{3}{2}$ state, which also helps to wash out the forward bump associated with high angular momenta. The $p_{\frac{3}{2}},\ _{m=\frac{1}{2}}$ matrix element essentially changes sign from the Born approximation as the $M1$ interaction becomes dominant,[10] so that both p states yield a backward asymmetry in interference with the s state. At higher energies the $s-p$ interference which is essentially proportional to the cosine of the difference between the phases of the s and p state *matrix elements*, vanishes and then becomes forward when the $p_{\frac{3}{2}}$ phase shift passes through 90°. The forward bump reappears as the $p_{\frac{3}{2}}$ state reintroduces a large forward component with which the high angular momenta interfere. In case II, as a result of the strong repulsion in the $T=\frac{3}{2}$, s state, the phase of the s-state matrix element is primarily determined by the contribution of the $T=\frac{1}{2}$ component which has a small phase. Thus the difference between the phases of $p_{\frac{3}{2}}$ and s matrix elements remains less than 90° and the asymmetry remains as it was at

TABLE III. Photoproduction of π^0 from photons, $A+B\cos\theta+C\sin^2\theta$ microbarns/sterad.

E_γ(lab) Mev	A	B	C
265	3.7	1.1	3.9
375, Case I	4.1	1.2	11.3
375, Case II	6.9	6.0	8.5

[18] Unpublished report. In more detail, they find the $s_{\frac{1}{2}}$, $T=\frac{1}{2}$ phase shift to be small and negative and the $p_{\frac{1}{2}}$, $T=\frac{3}{2}$ phase shift to be about $+30°$. The latter would, it happens, have little effect if it were considered in the photoproduction curve Fig. 7. Other phase shift solutions have been found and studied by R. L. Martin, Bull. Am. Phys. Soc. **29**, No. 1, 28 (1954), and also by Glicksman, de Hoffmann and Metropolis (private communication).

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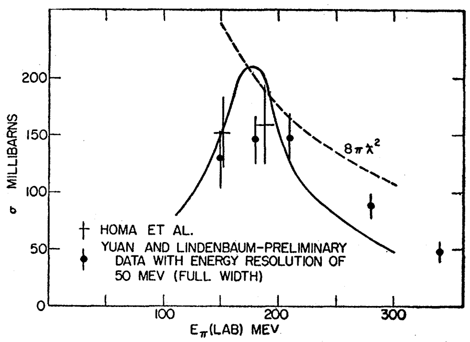

FIG. 8. Total cross section for scattering of positive mesons from protons. Only $p_{\frac{3}{2}}$ and $s_{\frac{1}{2}}$ phase shifts, taken from upper curve Fig. 3 and from Eq. (12), are considered. For experimental work see reference 19.

lower energies, in disagreement with experiment for positive mesons.

There is a rough consistancy between this phenomenological theory and experiment. The limitations of the theory are also quite evident. In the region near 310 Mev, where the assumption of energy independence should introduce only small errors, the agreement between theory and experiment is only within 10 or 20 percent. Of course absolute errors of this order may be present in the data, but this seems an unlikely explanation of some of the present difficulties. Improvement in the theory might be obtained by adjusting the coupling constant (i.e., see Fig. 5). Also the less important angular momentum isotopic spin states could be considered more fully. It is further seen, for example in $d\sigma^+/d\Omega(90°)$ (Fig. 5), how the theory breaks down completely at the high and low ends of the energy region. That the matrix elements should decrease in this region is indicated by examination of the Born approximation term;[3,20] and there is, perhaps, room for extension of the theory by making a detailed examination of the energy dependence of the various terms. Rapid changes are not indicated, however, and it seems clear that unless fairly rapid change of parameters with energy should be predicted, very good agreement with experiment would not be obtained. As our Born approximation term is only calculated in the weak coupling approximation, such difficulty is not surprising.

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[20] In the spirit of the present theory, the photoproduction reduces to the weak coupling limit as threshold is approached (see Fig. 4 with $G^2/4\pi=16$). This is in accord with the idea of N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954). The cross section $d\sigma^+/d\Omega(90°)=6$ at about $E_\gamma(\text{lab})=175$ Mev, recently reported by Bernardini (reference 16), would be fitted in this theory by a coupling constant of about 14.

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A Covariant Treatment of Meson-Nucleon Scattering

MAURICE M. LÉVY
Ecole Normale Supérieure, Paris, France
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A covariant equation for the meson-nucleon system is presented, in which the renormalization of divergent processes is carried out to all orders. A closed expression is given for their contribution to the wave function after renormalization, while the contribution coming from finite processes still involves a series expansion. Exact formulas are derived for the scattering phase shifts.

I. INTRODUCTION

RECENT experiments on pion-nucleon scattering[1] have made apparent the inadequacy of the Born approximation for the calculation of this process and the necessity of a theoretical analysis based on more elaborate methods.

Several attempts have been made[2] to analyze the data by means of the Tamm-Dancoff[3] nonadiabatic method, or an improved form of it.[4] Although this method seems to yield results which are in qualitative agreement with experiment, at least for the p wave, its defects are even more apparent here than in the treatment of nuclear forces.[5] A rapid calculation shows indeed that, even for low-energy scattering, high momenta play a decisive role in intermediate states, and that, consequently, the convergence of the interaction expansion can be expected to be very poor. Moreover, the main contribution to the scattering cross sections

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