discovered by Forbush. It is clear from the observations that this large intensity decrease was not produced by a geomagnetic storm.

It may develop that the solar-terrestrial associations indicated by these neutron intensity measurements will find application in interpreting and predicting the occurrence of solar related phenomena.

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The Origin of Cosmic Rays*

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The isotropy and composition of the primary cosmic radiation suggest that cosmic rays are trapped within the galaxy for an average time of the order of 10^6 years,—a long time compared with the time of escape along straight-line paths, but short compared with the mean life against nuclear collisions with interstellar matter. If one accepts this conclusion, it appears possible to account for the observed properties of cosmic rays under the assumption that cosmic rays acquire their large energies through a gradual acceleration in space, such as suggested by Fermi. In contrast to the original Fermi theory (which denied any possibility of escape from the galaxy), we now find that the energy spectra of protons and heavier nuclei are approximately the same, and that the required injection energies are very modest for all components. We are obliged, however, to assume a much faster rate of acceleration than the original theory required.

In this paper we develop in some detail the consequences of the above assumptions on the basis of a specific model, describing the motion of cosmic rays through the galaxy as a random motion between scattering centers represented by moving magnetized clouds. We briefly discuss the astrophysical implications of our assumptions and the plausibility of the model.

I. GENERAL CONSIDERATIONS

A. Introduction

FERMI[1] has proposed a theory of the origin of cosmic rays according to which the cosmic-ray particles diffuse randomly in interstellar space, gaining energy by collisions against moving magnetic fields, until eventually they loose their accumulated energy catastrophically, by collisions with hydrogen nuclei. This theory explains in a natural way the general isotropy and the observed energy spectrum of the cosmic-ray protons. It fails to account satisfactorily for the considerable flux of alpha particles, and for the heavier nuclei in the primary cosmic radiation. The difficulty is twofold. In the first place, according to Fermi's theory, the injection energy, i.e., the energy required for initiating the acceleration process (an energy at which the rate of energy gain overtakes the rate of loss of energy by ionization), is extremely high for the heavier components. In the second place, the energy spectrum computed for the heavier particles falls off much more steeply at high energy than that of the protons, because their mean free path against collisions with hydrogen nuclei is much shorter. Experimentally, however, the energy spectrum of the various components seem quite similar up to some 10^{13} ev per nucleon at least.[2]

Both difficulties can be overcome if one assumes that cosmic-ray particles diffuse around the galaxy for a time long compared with the average time for escape from the galaxy along straight-line paths, yet short compared with the mean life before collisions with interstellar hydrogen. Under this assumption (which, as originally pointed out by Bradt and Peters, has strong experimental support[3]) the mean life of a cosmic-ray particle in the galaxy is determined mainly by the escape probability, and is thus roughly independent of its mean free path for nuclear collisions. Then the

* Supported in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] E. Fermi, Phys. Rev. **75**, 1169 (1949).

[2] Kaplon, Peters, Reynolds, and Ritson, Phys. Rev. **85**, 295 (1952).

[3] H. Bradt and B. Peters, Phys. Rev. **80**, 993 (1950); see also B. Peters in *Progress of Cosmic Ray Physics*, edited by J. G. Wilson (North-Holland Publishing Company, Amsterdam, 1952).

energy spectra of the various components are approximately the same. The mean life of a cosmic-ray proton in the galaxy is much shorter, according to the present suggestion, than on the original Fermi theory. To account for the observed spectrum, it therefore becomes necessary to assume a rate of energy gain by collisions with moving magnetic fields considerably greater than that assumed by Fermi. The high rate of energy gain decreases the required injection energy to very plausible values, not only for protons, but even more strikingly for the heavy components.

However, it is doubtful whether or not the high rate of energy gain in compatible with astrophysical evidence. We do not intend here to minimize this difficulty. Indeed, we wish it to be clearly understood that the main purpose of this paper is not to uphold a given theory of the origin of cosmic rays, but rather to specify the conditions that must be met by any theory that explains the high energies of cosmic-ray particles by a mechanism of gradual acceleration in the motion through space.

B. Mean Life of Cosmic Rays in the Galaxy

We consider the relevant part of the galaxy as a thin, circular, disk-shaped volume, with negligible azimuthal variations. This shape is the shape observed for the gas clouds, the dust, and the hot stars of the galactic Population I. The radius of the disk is R_0, and its thickness $2h$.

From astronomical data we take the following approximate dimensions for this galactic disk:

$$h=10^3 \text{ light years}, \quad R_0=5\times10^4 \text{ light years},$$

and for the position of the sun:

$$z_s=30 \text{ light years}, \quad R_s=3\times10^4 \text{ light years}.$$

These are generally accepted figures, but no great accuracy can be assigned to them. The most important figure, that for h, is not very well defined.

We assume that interstellar matter consists entirely of hydrogen, spread through space with a mean density of $\frac{1}{2}$ proton per cm^3. We can estimate the mean free paths, l, for nuclear collisions in this interstellar space for the various components of the cosmic-ray beam, using a value of 4×10^{-26} cm^2 for the nucleon-nucleon cross section at cosmic-ray energies and computing the cross sections of the heavier components on the basis of the semitransparent model of nuclei. The mean free path in nuclear matter is taken as 2.7×10^{-13} cm, and the nuclear radius as $1.4\times10^{-13}A^{\frac{1}{3}}$ cm. Table I gives the results.[4]

[4] Of course some of the collisions will result only in a small change of the atomic number in the case of complex nuclei, or in a small change of the kinetic energy in the case of protons. Therefore the *effective* mean free paths may be appreciably larger than the actual collision mean free paths. However, our rudimentary knowledge concerning the density of interstellar matter, the cross sections for high-energy collisions and the events following such collisions, does not justify a more detailed evaluation of l.

TABLE I. Mean free paths, l, for nuclear collisions (density of interstellar hydrogen: $\frac{1}{2}$ proton per cm^3).

Cosmic-ray particle:	Proton	α particle	CNO group	Fe group
Average atomic weight	1	4	14	30
l (measured in 10^6 l.y.)[a]	50	20	7	4

[a] "l.y." = light years.

It is important to remark that, irrespective of the acceleration mechanism, any galactic theory of the origin must ascribe to cosmic-ray particles a mean free path before escape from the galaxy, L, which is long compared with the thickness of the galactic disk, but short compared with the collision mean free path. The upper boundary to L comes from the mass spectrum of the cosmic-ray beam. Considerations based on the relative abundance of protons and heavier nuclei in this beam, and on the numbers of secondary protons which a pure heavy primary beam would make by nuclear collisions with interstellar matter, show that the mean path L of cosmic-ray particles (at least up to 10^{13} ev) cannot be appreciably larger than the collision mean free path, l, of the heaviest components. We thus obtain from Table I:

$$L\lesssim 4\times10^6 \text{ light years.}$$

The lower limit to L comes from considerations of intensity and isotropy. The density of cosmic-ray energy near the earth is of the same order of magnitude as the energy density of starlight. A mean life of cosmic rays in the galaxy approximately equal to that of starlight would then imply a cosmic-ray source of about the same strength as that of starlight. In order to avoid this very unlikely conclusion we must thus assume that $L\gg h$.

The argument based on the observed symmetry was first given by Cocconi,[5] and is here presented in a somewhat different form. One can estimate a lower limit for the anisotropy of cosmic rays at the earth by taking into account only the fact that the solar system is off the median plane of the galaxy by a distance $z_s=30$ l.y. (light years). For this purpose, consider the galaxy as a disk of thickness $2h$ and infinite extension, and assume that cosmic-ray sources are distributed symmetrically with respect to the median plane. Consider two planes parallel to the median plane, at distances z_s and $-z_s$ from it. Let Ψ_1 and Ψ_2 be the numbers of particles per unit area and unit time which cross one of the two planes entering or leaving the intervening volume, respectively. Thus the net number of particles per unit area and unit time leaving this volume through *both* boundaries is $2(\Psi_2-\Psi_1)$. If we neglect nuclear collisions, we then obtain the following equation expressing the conservation of particles:

$$2(\Psi_2-\Psi_1)=\int_{-z_s}^{+z_s} S(z)dz, \tag{1}$$

[5] G. Cocconi, Phys. Rev. 83, 1193 (1951).

where $S(z)$ is the source strength in particles per unit volume and unit time. On the other hand, for a nearly isotropic distribution the following equation holds:

$$\Psi_1+\Psi_2=\tfrac{1}{2}\rho v, \tag{2}$$

where ρ is the density of particles and v their velocity. The ratio between the total number of particles present in the galaxy at any particular time and the number of particles produced per unit time represents the mean life, L/v, of the particles in the galaxy; thus

$$L/v=\int_{-h}^{+h}\rho dz\Big/\int_{-h}^{+h}S(z)dz. \tag{3}$$

If we assume that the density of cosmic-ray particles near the earth is a fair sample of the mean density throughout the galaxy, the above equation becomes

$$L/v\approx 2h\rho\Big/\int_{-h}^{+h}S(z)dz. \tag{4}$$

The asymmetry of cosmic rays with respect to the plane $z=z_s$ may be defined as

$$\delta=2(\Psi_2-\Psi_1)/(\Psi_2+\Psi_1). \tag{5}$$

From Eqs. (1), (2), and (4) one then finds for δ the expression

$$\delta\approx 4\frac{h}{L}\int_{-z_s}^{+z_s}S(z)dz\Big/\int_{-h}^{+h}S(z)dz. \tag{6}$$

For example, in the case of a uniform distribution of sources through the galaxy, Eq. (6) reduces to

$$\delta\approx 4z_s/L. \tag{7}$$

At the mean energy of cosmic rays, about 10 Bev, $\delta\lesssim 10^{-3}$, and therefore

$$L\gtrsim 4000z_s=1.2\times 10^5 \text{ l.y.}$$

Even at energies as high as 10^{13} or 10^{14} ev, $\delta\lesssim 2\times 10^{-2}$, and $L\gtrsim 6000$ light years. These limits would be higher except for the fact that the sun lies close to the galactic equator. It is not likely that the local distribution of the sources is really as symmetrical as the over-all galactic disk model which we use, so that the limits given above are probably conservative.

It may be pertinent to point out that these general conclusions are valid irrespective of the specific mechanism opposing the escape of cosmic-ray particles from the galaxy. They apply equally well to models describing the galaxy as a volume filled with a diffusing medium, or as a volume bounded by a "white" wall. However, if one wanted to determine exact numerical values for the limits of L, rather than orders of magnitude, one would have to take into account the actual distribution of the sources, as well as the detailed mechanism of propagation of cosmic rays in the galaxy.

C. Energetics of the Cosmic Radiation

If we consider a radiation moving in the galaxy, either in straight paths like the starlight or in the tortuous paths of the diffusing cosmic-ray particles, we can write the following relation from energy conservation:

$$(\text{energy density})=(\text{source power density})\times(\text{mean life})\times(\bar{U}/\bar{U}_0),$$

where $\bar{U}_0$ is the mean energy per particle at emission and $\bar{U}$ the mean energy per particle observed in space. For starlight, $\bar{U}/\bar{U}_0=1$. Now it is plain that the mean life of starlight in the disk will be a few times h/c, and we have seen that for the bulk of the cosmic rays the mean life is L/c. It is known from astronomical data that in our location the energy density of light is approximately equal to the energy density of cosmic rays (both being about 1 ev per cm^3). It therefore follows that the energy outputs of the cosmic-ray source and of the light source are in the ratio:

$$\text{cosmic rays/light}\approx h\bar{U}_0/L\bar{U}\approx 10^{-3}\bar{U}_0/\bar{U}.$$

Thus the injection of cosmic rays by stars would be only a small fraction of their total power output, even if there were no considerable acceleration in space. For our model, where the acceleration in space is decisive, we will see that $\bar{U}_0/\bar{U}\approx 10^{-3}$. The energy expenditure for the injection of cosmic rays is then an entirely negligible fraction of the stellar energy output. Even rare stellar events, such as supernovae, might supply the entire cosmic-ray beam injected.

On the other hand, our model does derive the energy of the cosmic-ray beam from interactions with the ion clouds in space, through their magnetic fields. The turbulent energy density of interstellar matter (both kinetic and magnetic) can be estimated roughly, and turns out to be of the same order of magnitude as the density of cosmic-ray energy, perhaps 1 to 10 ev per cm^3. The energy presently stored in the interstellar turbulent motion could then supply the cosmic-ray power requirements only for a few million years, a time very short compared to the age of the galaxy. This implies that cosmic rays represent an important load on the energy input to the turbulent motion, which presumably is the galactic energy of rotation. In our neighborhood the ordered rotational energy density is about a thousand times greater than the turbulent energy.

Such considerations mean that the cosmic rays play an essential role in the dynamical evolution of a spiral galaxy. The energy loss to cosmic rays is an important factor in smoothing out the turbulence. We do not intend to discuss such problems here. We only wish to remark that according to the present picture, cosmic rays derive their energy from some major source of galactic nonnuclear energy, such as the gravitational energy gained by concentration toward the galactic plane and toward the denser galactic center or, likely, some initial kinetic energy of rotation. Cosmic rays are presumably long-lasting features of a spiral galaxy which are not constant, but are related in time to the presence of well-marked, gas-laden, rotating arms.

II. THE DIFFUSION MODEL

A. The Diffusion Equation

We now wish to discuss a more specific model, by considering the galactic volume filled with wandering masses of turbulent, ionized, and magnetic hydrogen plasma moving in an un-ionized substratum. On this model, a cosmic-ray particle travels in nearly straight paths between magnetic clouds of streaming gas; upon entering such a cloud, the particle begins a tortuous trajectory, until it finally drifts out with almost no recollection of its original direction. To describe the random propagation of cosmic rays through the galaxy, we shall use a simple form of the transport theory, the diffusion theory, familiar in the study of slow neutrons. The low accuracy required does not suggest the use of any more elaborate method for the problem. It is worthwhile to remark that our values of such properties of the diffusing medium as the mean free path might well have to be modified if the distribution of types of scattering centers is an unusual one. We can regard the values given below as *effective* values of mean free path, etc., bearing a relation to such physical features as the mean spacing of scattering centers which is not exactly known. But for any reasonable collection of scattering objects, the broad features of diffusion theory still hold.

Fermi assumed that the scattering clouds have random motion and showed that in this case the particle energy will, on the average, increase during a collision. He also showed that the fractional energy gain by a particle in such a collision is a quantity $\alpha=\Delta U/U$ which depends on the velocity of the clouds, but not on U. Here, U is the total energy of the particle (kinetic+rest energy) expressed in units of its rest energy, m_0c^2. In most of our discussion we shall maintain the assumption that α is a constant, without thereby implying that the acceleration mechanism is identical to that originally suggested by Fermi.

We will denote by $\beta(U)$ the actual energy loss per collision (not the fractional loss), measured in units of the rest energy of the particle. This loss may arise from atomic collision in the plasma or in un-ionized gas, or from radiation accompanying accelerations in the magnetic field. We introduce, also, the number of collisions since the ejection of the particle into space, n. We regard the process—which involves n of some millions—as continuous, just as in the Fermi age theory of neutron slowing-down. The Fermi relation between energy and number of collisions is

$$dU/dn=\alpha U-\beta(U). \tag{8}$$

Evidently, particles injected at energies below a minimum energy U_c will not be accelerated:

$$U_c=\beta(U_c)/\alpha. \tag{9}$$

There is strong evidence that beams of cosmic-ray particles arrive at the earth during some solar flares. These particles have energies up to a few Bev per nucleon.[6] They come more or less directly from the neighborhood of the sun. It seems plausible to suppose that the sun is a continuous source of such particles, though perhaps with widely fluctuating intensity and energy spectrum. We may assume that most other stars, probably with still more extreme variations in number and energy of emission, also send fast particles into space (see Sec. IV C). Averaging over the stars, then, we will assign to the galaxy a distributed source density, capable of injecting into space protons, α particles, and other nuclei with energies far above the few kilovolts of the auroral streams, though still low compared to cosmic rays, or perhaps reaching at most a few Bev per nucleon. These emitted particles diffuse throughout the galactic disk, gaining and losing energy until they finally escape from the region of the plasma clouds, or rarely until they change their nuclear identity by a nuclear collision with the protons of space.

Let us write ρ for the number of particles per unit volume, at position $\mathbf{r}$ and time t, which have undergone between n and $n+dn$ collisions since their ejection from a source with energy U_0. Then the familiar scalar flux of particles per cm^2 and per second, useful in all nearly isotropic transport problems, is just $\varphi=\rho(\mathbf{r},t,n,U_0)v(n,U_0)$, where v is the velocity of the particles. The so-called "differential energy spectrum" of primary cosmic rays, j, gives the number of cosmic-ray particles per unit area, per unit solid angle, per second, and per unit energy interval. It is related to the flux φ by the equation

$$j(E)=(1/4\pi)\varphi dn/dE. \tag{10}$$

$E=U-1$ is the kinetic energy of a particle measured in units of its rest energy.

Following the familiar methods[7] of neutron diffusion and age theory, we obtain the differential equation expressing the conservation of particles:

$$\frac{\partial\rho}{\partial t}=\frac{\lambda}{3}\nabla^2\varphi-\frac{1}{\lambda}\frac{\partial\varphi}{\partial n}-\frac{\varphi}{l}+S(\mathbf{r},t,U_0)\delta(n). \tag{11}$$

Here λ is the transport mean free path for collisions with the clouds, and l the mean free path against nuclear collisions. We have written the source density function $S(\mathbf{r},t,U_0)$ for the number of particles injected per second per unit volume with initial energy between

[6] See the very valuable review by L. Biermann, Ann. Rev. Nuclear Sci. 2, 335 (1953).

[7] See, for example, S. Glasstone and M. Edlund, *The Elements of Nuclear Reactor Theory* (D. Van Nostrand Company, Inc., New York, 1952), Chap. VI.

U_0 and U_0+dU_0. The Dirac delta function indicates that the injection is, of course, with $n=0$. For each U_0, then, Eq. (8) gives a connection between U and n. By adding up the injections at all energies, one gets the complete spectrum. We shall generally neglect the initial spread of the injection spectrum.

B. Solution of the Diffusion Equation

It is enough to treat the stationary case, with $\partial\rho/\partial t=0$, and S constant in time. For any finite n, we have

$$\frac{\lambda}{3}\nabla^2\varphi-\frac{1}{\lambda}\frac{\partial\varphi}{\partial n}-\frac{\varphi}{l}=0. \tag{12}$$

Considering Eq. (11) in the neighborhood of $n=0$, we have

$$\varphi(\mathbf{r},t,0,U_0)=\lambda S(\mathbf{r},t,U_0). \tag{13}$$

No other boundary condition in the variable n is needed except the finiteness of φ for all n. The boundary condition to be set in ordinary space is less formal. It depends upon the reflectivity of the boundaries of the diffusing region. We shall set $\varphi=0$ at the extrapolated boundaries,[7] i.e., we shall assume no reflection. This assumption will be discussed later in detail (see Sec. IV B). Here we only remark that a moderate amount of reflection would not substantially alter the present treatment. Only a practically total reflection from the galactic boundaries would invalidate our conclusions.

For our problem it is convenient to introduce cylindrical coordinates with the origin at the center of the galactic disk and the equatorial plane coincident with the median plane. We denote the distance above the equatorial plane by z and the distance from the polar axis by R. We assume axial symmetry and symmetry about the equatorial plane as well.

One can solve Eq. (12) by the familiar method of series expansion into normal modes. For our geometry, this corresponds to an expansion into a Fourier series with respect to z and into a series of Bessel functions with respect to R. It turns out that the solution, or more specifically the functional dependence of φ on n at the location of the solar system, is not affected to any appreciable degree by the actual value of the galactic radius, R_0. This is easily understandable on physical grounds because $h\ll R_0$ and therefore the leakage through the cylindrical surface at $R=R_0$ is negligible compared with the leakage through the plane surfaces at $z=\pm h$. For this reason we can ignore the boundary condition $\varphi=0$ at $R=R_0$, i.e., we can consider the galaxy as having infinite lateral extension.

As a special and particularly simple case, we assume that the cosmic-ray sources are distributed uniformly in the radial direction, and symmetrically with respect to the equatorial plane. The problem then becomes unidimensional and the general solution of Eq. (12) obeying the boundary conditions $\varphi=0$ at $z=\pm h$ can be written as a Fourier series:

$$\varphi(z,n)=\sum_{i=1}^{\infty}A_i\cos[(i-\tfrac{1}{2})\pi z/h]\exp(-n/N_i), \tag{14}$$

where N_i is given by the equation

$$\frac{1}{N_i}=\frac{\pi^2(i-\frac{1}{2})^2\lambda^2}{3h^2}+\frac{\lambda}{l}. \tag{15}$$

In particular,

$$\frac{1}{N_1}=\frac{\pi^2\lambda^2}{12h^2}+\frac{\lambda}{l}. \tag{16}$$

The coefficients A_i are determined by Eq. (13) which, in this case, becomes

$$\lambda S(z)=\sum_{i=1}^{\infty}A_i\cos[(i-\tfrac{1}{2})\pi z/h]. \tag{17}$$

Thus the coefficients A_i are the Fourier coefficients of the source function multiplied by λ.

It is easy to modify the solution to allow for a possible variation of λ with n. It is necessary only to replace λn and $\lambda^2 n$ where they occur in the exponential of Eq. (14) by the integrals $\int_0^n\lambda(n')dn'$ and $\int_0^n\lambda^2(n')dn'$, respectively. The second integral is almost the "age," τ, in Fermi's theory of slowing-down:

$$3\tau=\int_0^n\lambda^2(n')dn'.$$

If we define the integral flux $\Phi(\mathbf{r},n)$ as

$$\Phi(\mathbf{r},n)=\int_n^\infty\varphi(\mathbf{r},n')dn', \tag{18}$$

we obtain from Eq. (14):

$$\Phi(z,0)=\sum_{i=1}^{\infty}N_iA_i\cos[(i-\tfrac{1}{2})\pi z/h]. \tag{19}$$

Notice that $\Phi(z,n)$ is the flux of particles at z which have undergone more than n accelerating collisions with the magnetic clouds of the galaxy, and $\Phi(z,0)$ is the total flux of particles. From Eqs. (3), (19), and (17) we obtain the following expression for the mean path L of cosmic-ray particles in the galaxy:

$$L=\int_{-h}^{+h}\Phi(z,0)dz\Big/\int_{-h}^{+h}S(z)dz$$
$$=\lambda\sum_{i=1}^{\infty}A_iN_i(i-\tfrac{1}{2})^{-1}\Big/\sum_{i=1}^{\infty}A_i(i-\tfrac{1}{2})^{-1}. \tag{20}$$

Unless the higher modes are important (i.e., unless the source strength varies rapidly in the z direction) L is of the order of magnitude of λN_1 and is, therefore, given by

$$\frac{1}{L} \approx \frac{1}{\lambda N_1} = \frac{\pi^2\lambda}{12h^2} + \frac{1}{l}. \tag{21}$$

As discussed in Sec. I B, the mass spectrum of cosmic rays requires that $L \ll l$. Thus L must be of the order of magnitude of $12h^2/\pi^2\lambda$. This means physically, of course, that the mean life of cosmic-ray particles in the galaxy is primarily determined by the leakage through the two planes of the galactic disk. From the upper limit of L (4×10^6 l.y.) one finds a corresponding lower limit of λ: $\lambda > \frac{1}{4}$ l.y. We have also seen that isotropy considerations set a lower limit of about 10^5 l.y. to L. The corresponding upper limit of λ is 10 l.y.

If only the fundamental mode is present, the exact evaluation of δ becomes straightforward. The net flux of outgoing particles through the plane $z=z_s$ may be computed from Eq. (1) together with the expression for the source function:

$$S(z) = S_0 \cos(\pi z/2h),$$

or from the equation:

$$\Psi_2 - \Psi_1 = -\frac{\lambda}{3}\left(\frac{\partial\Phi}{\partial z}\right)_{z=z_s}, \tag{22}$$

together with the expression for Φ:

$$\Phi(z,0) = \lambda S_0 N_1 \cos(\pi z/2h).$$

In either way one obtains (neglecting nuclear absorption):

$$\Psi_2 - \Psi_1 = (2/\pi) h S_0 \sin(\pi z_s/2h). \tag{23}$$

On the other hand, Eq. (2) gives

$$\Psi_1 + \Psi_2 = (6h^2/\pi^2\lambda) S_0 \cos(\pi z_s/2h). \tag{24}$$

Therefore, from Eq. (5):

$$\delta = (2\pi\lambda/3h) \tan(\pi z_s/2h) \approx \pi^2\lambda z_s/3h^2. \tag{25}$$

It happens that Eq. (25) is equivalent to Eq. (7), giving an approximate, but more general expression for δ. It may be pointed out that Eq. (25) applies not only to the asymmetry of the total flux Φ, but to that of the differential flux φ as well. In the latter case it remains valid also when λ is a function of n.

It is interesting to observe that another quite different and perhaps more plausible geometry for the galactic diffusing material will give nearly the same results as the disk described in detail here. The alternative is the confinement of the diffusion within a single spiral arm of the galaxy. Such an arm has a height $2h$, of course, a width perhaps $3h$, or $4h$, and a length comparable to R_0. The curvature can be neglected and the problem treated as diffusion in a long rectangular prism. Different eigenfunctions are appropriate, but the general features of the problem are the same, and the fundamental length is nearly the same as that given by Eq. (21). The additional leakage through the vertical bounding planes is only a correction to the already arge leakage through the top and bottom as before. Whether the fine structure of the galactic magnetic field shows the gaps between spiral arms as markedly as does the gas itself is not known. Since the qualitative results will be quite the same, we shall discuss the somewhat more evident case of disk diffusion in detail, but we do not by any means wish to obscure the possibility that the single-arm geometry, or some intermediate case, may in fact be the real solution.

C. Location of the Sources

In the previous section we have assumed that the sources are distributed uniformly in the plane of the galaxy (although they are not necessarily uniform perpendicularly to this plane). In order to investigate whether or not the details of the source distribution are important we consider next a source function of the following type:

$$S(\mathbf{r}) = S_0 \cos(\pi z/2h) \exp(-R^2/2D^2), \tag{26}$$

i.e., we assume that the source has a gaussian distribution with a rms spread D in the R direction, and is distributed according to the fundamental Fourier mode in the z direction. By integral expansion into Bessel functions, or by direct substitution into the diffusion Eq. (12) one finds the solution corresponding to the source function (26) for a galaxy of infinite radial dimensions:

$$\varphi = \lambda S_0 \cos\left(\frac{\pi z}{2h}\right)\frac{6D^2}{6D^2+4\lambda^2 n} \times \exp\left[-\frac{3R^2}{6D^2+4\lambda^2 n} - \frac{n}{N_1}\right]. \tag{27}$$

We now further assume that $D \ll R$, i.e., that the dimensions of the source are small compared with the distance from the point of observation to the center of the source. The integral of Eq. (27) with respect to n can be expressed, after a suitable transformation, in terms of the Hankel function of zeroth order, $H_0^{(1)}$, so that the total flux of particles at R and z may be written as

$$\Phi(R,z,0) = \text{const} \times \cos(\pi z/2h) i H_0^{(1)}(2ia). \tag{28}$$

In the above equation, which is exact in the limit $D/R \rightarrow 0$,

$$a = \frac{\pi R}{4h}\left[1 + \frac{12h^2}{\pi^2 \lambda l}\right]^{\frac{1}{2}} \approx \frac{\pi R}{4h}. \tag{29}$$

Figure 1 shows a graph of $H_0^{(1)}$ *versus* a. For $a \gg 1$ Eq. (28) becomes approximately

$$\Phi(R,z,0) \approx \text{const} \times \cos(\pi z/2h) \times (2h/\pi R)^{\frac{1}{2}} \exp(-\pi R/2h). \tag{30}$$

One should note that, since we are considering a galaxy of infinite lateral extension, the line $R=0$ merely represents the axis of symmetry of the assumed source, so that the above equations apply to a source located anywhere in the galaxy at a distance R from the solar system. It is also plain that the results obtained do not essentially depend upon the particular form of the source density here used for calculation, provided

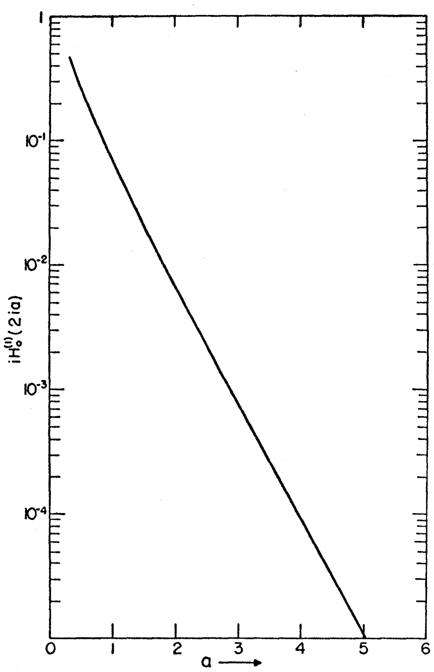


FIG. 1. The Hankel function, $iH_0^{(1)}(2ia)$, of zeroth order plotted *versus* $a=\pi R/4h$. $H_0^{(1)}$ describes the dependence of the total flux of cosmic-ray particles, $\Phi(R,z,0)$, on the radial distance, R, from the center of a concentrated source.

the radial dimensions of the source are small compared with R.

From Eq. (30) one sees that at distances larger than the thickness of the galaxy the contribution of a given source decreases rapidly with distance, the reduction factor being approximately $\exp(-\pi R/2h)$. It is thus difficult to escape the conclusion that the bulk of the cosmic rays—those with energies between 2 Bev and 10 Bev—are locally generated, in our galactic neighborhood, mostly within a few thousand light years from the sun. Any appreciable contribution to the total flux from sources further away than several times the thickness of the galactic disk would imply an extremely uneven distribution of the sources in the galaxy. It would also require an excessively large energy output from these sources. In particular, it seems difficult to assume, as it has been sometimes speculated, that most cosmic rays come from the dense concentration of stars at the center of the galaxy, 30 000 light years from the solar system. In our model the reduction factor for a source located at this distance relative to a local source is, approximately, 10^{-20}!

The above conclusions apply, as we have pointed out, to the bulk of the observed cosmic radiation. The high-energy end of the spectrum (large n) and the low-energy end of the spectrum (small n) require special consideration.

Equation (27) shows that φ does not decrease significantly with increasing R until R becomes larger than $\lambda\sqrt{n}$. If n is sufficiently large as to make $\lambda\sqrt{n}\gg h$, sources located at distances large compared with the thickness of the galactic disk may become as effective as local sources. One can easily see the physical reason for this result by considering that a particle arriving upon the earth after a large number of collisions has had approximately equivalent opportunities to escape from the galaxy whether it has begun its random walk near the earth or a considerable distance away.

To discuss the case of small n, consider again Eq. (27). If $\lambda\sqrt{n}$ is small compared with h, then φ decreases rapidly with increasing R even for distances small compared with the thickness of the galactic disk. Therefore, the observed flux depends critically on the details of the source distribution in the neighborhood of the sun.

One arrives at a similar conclusion considering the results of the preceding section relative to a source distribution depending only on z. For $n\gg N_1$, i.e., $n\gg h^2/\lambda^2$ [see Eq. (16)], the higher modes of the Fourier expansion of φ are negligible; i.e., φ has the same exponential dependence on n, $\exp(-n/N_1)$, irrespective of the source distribution. However, for $n\ll N_1$, the higher modes may be important and, therefore, the functional dependence of φ on n varies with the source distribution.

Note that the conclusions reached in this section are valid under any galactic theory of cosmic rays postulating a three-dimensional random-walk type of propagation through galactic space. In particular, distant sources, such as those situated at the center of the galaxy, could be important only if cosmic rays were confined to the galaxy for times long compared with h/c by a high reflectivity at the galactic boundary, rather than by frequent random changes in their direction of motion within the galaxy.

III. COMPARISON WITH EXPERIMENTS

A. The High-Energy Spectrum. Injection Energies

We have seen in the preceding section that for values of n larger than h^2/λ^2 (but not exceedingly large) the function φ is determined almost entirely by the average strength of cosmic-ray sources in the galactic neighborhood of the sun. In a theory which at best, can only account for the main features of the observed phenomena, we are therefore free to choose any reasonably smooth distribution of sources in the galactic disk. For ease of calculation, we shall assume that the source is distributed according to the fundamental mode in the z direction and is constant in the R direction:

$$S=S_0\cos(\pi z/2h). \tag{31}$$

In this case the solution of the diffusion equation is

$$\varphi=\lambda S_0\cos(\pi z/2h)\exp(-n/N_1), \tag{32}$$

and the integral flux is

$$\Phi(z,n)=\int_n^\infty \varphi(z,n')dn' =\lambda S_0 N_1\cos(\pi z/2h)\exp(-n/N_1). \tag{33}$$

The collision number n and the energy U are connected by Eq. (8). Let us first neglect the ionization loss and examine later whether or not this neglect is justified. We obtain

$$U=U_0e^{\alpha n};\quad n=\alpha^{-1}\ln(U/U_0). \tag{34}$$

From Eqs. (10), (33), and (34) we then compute the integral spectrum of cosmic rays, $J(E)$, i.e., the number of particles per unit area, per unit solid angle and per unit time, with kinetic energy greater than E:

$$J(E)=\int_E^\infty j(E')dE'=\frac{1}{4\pi}\Phi[n(E)]$$
$$=\frac{1}{4\pi}\lambda S_0N_1\cos\left(\frac{\pi z}{2h}\right)\left(\frac{U_0}{1+E}\right)^{1/\alpha N_1}. \tag{35}$$

This is the well-known power law of Fermi's theory.

Figure 2, taken from a paper by Barrett *et al.*,[8] summarizes our present information on the values of $J(E)$ at various energies from 14 Bev to about 2×10^6 Bev. The experimental data refer to all cosmic-ray particles. Little is known on the relative proportion of protons and heavier particles at the highest energies. It is, however, reasonable to assume that protons are greatly predominant here, as they are in the lower energy range. In what follows we shall discuss the data as if they applied entirely to protons. Note that the point at 14 Bev represents the vertical intensity at the geomagnetic equator, where the geomagnetic cutoff for protons is at 14 Bev kinetic energy. The other points result from observations of high-energy nuclear interactions in photoemulsions and from underground experiments on mesons. Data obtained from observations on air showers are also indicated (dashed lines). The measurements are not very accurate, but the range of energies is very great. Over this whole range the experimental data can be fitted to a power law of $(1+E)$ with exponent

$$-1/\alpha N_1=-(1.5\pm0.2). \tag{36}$$

The error refers to the uncertainty in the fit of the experimental data to an assumed power law; there is, of course, no assurance that this law is accurately verified.

Neglecting nuclear collisions, we can evaluate N_1 in terms of λ and h from Eq. (16):

$$N_1=12h^2/\pi^2\lambda^2\approx h^2/\lambda^2. \tag{37}$$

TABLE II. Critical kinetic energies of cosmic-ray particles.

Cosmic-ray particle:	Proton	α particle	CNO group	Fe group
Average atomic number	1	2	7	15
Critical kinetic energy (measured in Mev)	0.9	3.8	65	320

[8] Barrett, Bollinger, Cocconi, Eisenberg, and Greisen, Revs. Modern Phys. 24, 133 (1952).

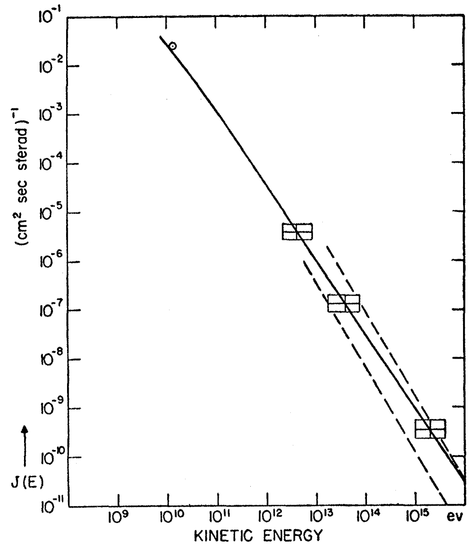


FIG. 2. The integral intensity of primary cosmic rays, $J(E)$, at kinetic energies from 14 Bev to 2×10^6 Bev. The experimental data indicated by squares are taken from a paper by Barrett *et al.* (see reference 8). The point at 14 Bev corresponds to the vertical integral intensity at the geomagnetic equator. The band between the two dashed lines indicates results deduced from the observations on air showers. The solid line represents the theoretical expression $J(E)=\text{const}\times(1+E)^{-1.5}$.

We then obtain from Eq. (36) the following relation:

$$\alpha=0.6\lambda^2/h^2. \tag{38}$$

If we consider the limits on λ discussed in Sec. II B and the fact that, on astronomical grounds, it is difficult to assign too large a value for α or too small a value for λ, we arrive at the following thoroughly tentative set of values

$$\lambda\approx1\text{ light year};\quad \alpha\approx6\times10^{-7}. \tag{39}$$

In Secs. IV A and IV D we discuss the plausibility of these assumptions. Now we must examine the consequences of the neglect of the ionization-loss term in the integration of Eq. (8). The actual ionization losses are rather uncertain. The loss rate in a completely ionized plasma is a good deal larger than that in un-ionized material essentially because of the greater value of the maximum impact parameter for effective collisions. But the division of the actual path traversed into plasma and un-ionized matter is not very well known. Most of the volume of the galactic disk is un-ionized. Taking for simplicity the loss rate in un-ionized hydrogen, we can compute the critical energies for injection of different nuclei from Eqs. (9) and (38). The results are given in Table II. Even if these energies are somewhat increased by the higher plasma losses, they are still very small compared to the mean cosmic-ray energies of 10 Bev or so. Thus the neglect of the ionization loss is justified.

From Eq. (34) and from the value of α given above we may compute the values of n corresponding to the two limits of the spectrum here under consideration (we assume a kinetic injection energy small compared to the rest energy). We obtain for kinetic energies of about 14 Bev, $n\approx 4\times 10^6$, and for kinetic energies of about 2×10^6 Bev, $n\approx 24\times 10^6$. Since $h^2/\lambda^2\approx 10^6$, we see that the conditions specified at the beginning of this section are fulfilled.

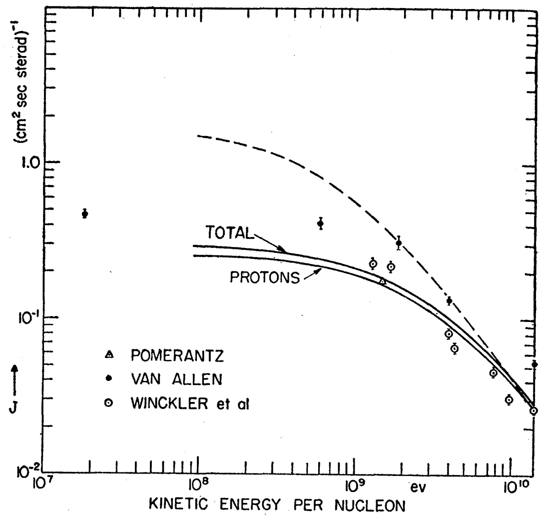
B. The Low-Energy Spectrum

The study of the latitude effect has provided data on the energy spectra of protons and of heavier particles at energies below 10 Bev. Some of these data are summarized in Fig. 3 (a) and (b). The points for the heavy particles are taken from a paper by Kaplan, Peters, Reynolds, and Ritson.[2] The curve for the total number of particles is that suggested by Barrett *et al.*[8] on the basis of Winkler's measurements. The curve for protons has been obtained by subtraction of the numbers of heavy particles from the total numbers of particles. The dashed lines are drawn according to a power law of the type $J=\text{const}\times(1+E)^{-1.5}$.

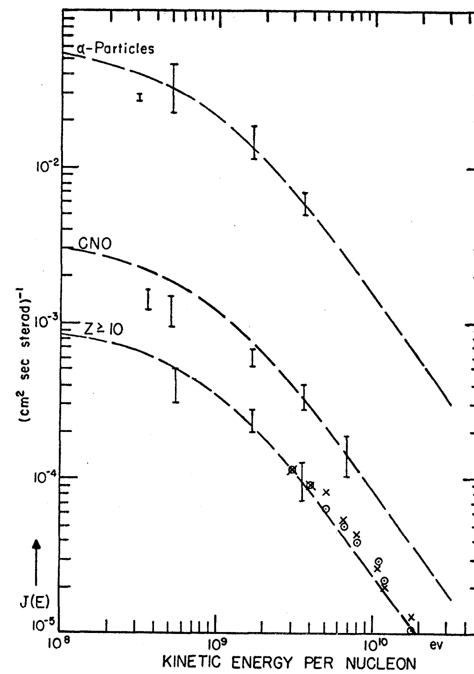
It is necessary to emphasize the fact that most of the experimental results on the latitude effect are not very accurate. They are also difficult to interpret because the theory of geomagnetic phenomena is far from complete. However, it appears that while the data relating to the heavier components do not deviate significantly from the theoretical power law, the more accurate data relating to protons are in definite disagreement with it. Indeed, at energies around 0.1 Bev, the theoretical expression predicts about two or three times as many particles as are actually observed.

More recent results of Neher *et al.*[9] and of Van Allen[10] confirm this conclusion. Neher, working with ionization chambers at an atmospheric depth of 15 g cm^{-2}, finds an increase of only 1 percent in the cosmic-ray intensity between the geomagnetic latitudes of 56° to 66°. The corresponding kinetic cut-off energies for vertical protons are 0.8 Bev and 0.140 Bev, and according to the 1.5 power law, the increase should amount to approximately a factor of two. In a series of rocket experiments, Van Allen found that lowering the cut-off energy from 0.59 Bev to 0.018 Bev increased the intensity by only 14 ± 9 percent as against a theoretical increase of about a factor of two. In both cases, the observed increases are hardly outside the experimental errors and, even if real, can be explained by the opening of the shadow cone as one proceeds toward higher latitudes. (Note that both Neher and Van Allen measured the omnidirectional intensity rather than the vertical intensity, which makes the interpretation of the results subject to greater uncertainty.)

In conclusion, there seems to be little doubt that, in the low-energy region, the proton spectrum becomes much flatter than the theory predicts. It is, however, impossible to say at the present time whether or not there exists an actual "cutoff," i.e., whether or not particles below a certain limiting energy (of the order of 0.8 Bev for protons) are totally absent from the primary spectrum. It is also uncertain whether or not the spectra of the heavier components exhibit a devia-



(a)



(b)

FIG. 3. The integral intensity of primary cosmic rays at energies less than 14 Bev per nucleon. (a) The upper solid curve is that suggested by Barrett *et al.* (see reference 8), and refers to all particles. The lower solid curve, deduced from the former by subtraction of the heavy particles, refers to protons. The circles and the triangles represent experimental determinations by Winkler and Pomeranz, as summarized by Puppi in Chapter VI of J. G. Wilson, *Progress of Cosmic Ray Physics* (North-Holland Publishing Company, Amsterdam, 1952). The dots represent measurements by Van Allen (see reference 10). The dashed line is drawn according to a power law of the type $J=\text{const}\times(1+E)^{-1.5}$ and is the continuation of the curve in Fig. 2. (b) The experimental points refer to measurements by Kaplon *et al.* (see reference 2), on the heavy components of the primary radiation. The dashed lines are drawn according to the power law $J=\text{const}\times(1+E)^{-1.5}$.

[9] Neher, Peterson, and Stern, Phys. Rev. **90**, 655 (1953).

[10] J. A. Van Allen, "The cosmic-ray intensity above the atmosphere near the geomagnetic pole," Department of Physics, State University of Iowa, January, 1953 (unpublished).

tion from the theoretical power law similar to that of the proton spectrum.

It has been suggested that the depletion of the low-energy end of the proton spectrum may not be a property of the primary beam but may be due to a shielding effect by local magnetic fields, such as a magnetic field of the sun.[11] The astronomical data, however, do not seem to support this view.[12] On the other hand, the simple diffusion model predicts, if anything, a deviation from the power law in the direction opposite to that observed. At low energies the ionization loss is not negligible and, in fact, the fractional energy gain per collisions will go to zero at the critical energy for injection, E_c. This effect would tend to *increase* the flux at low energies [see Eq. (10)]. Of course, in the absence of a detailed theory of the collisions between the particles and the magnetic elements, one cannot be sure that the simple equation $dU/dn=\alpha U-\beta$ with $\alpha=\text{const}$ will hold down to the lowest energies.

One should also remark that, at energies slightly above the injection energy, the flux ought to be increased by particles injected above the critical energy and arriving without much acceleration. It is, of course, clear that if the source did not emit particles of energy less than $E_0(E_0>E_c)$, no particles of energy less than E_0 would be found in the incoming radiation. Such an assumption, however, appears rather artificial.

In the absence of any other plausible interpretation of the low-energy end of the spectrum, it is perhaps interesting to note that the theory here under discussion can explain the observed features at the price, however, of introducing another *ad hoc* hypothesis. This hypothesis is that the source density is weaker than average in the neighborhood of the sun. If the sun is between two spiral arms, or within a gap even inside a main arm of the spiral, such an assumption would be reasonable. Formally, we could use as source density a function of the type

$$S=S_0\cos(\pi z/2h)[1-\epsilon\exp(-R^2/2\Delta^2)], \tag{40}$$

where we have chosen as central axis $(R=0)$ a line passing through the sun and perpendicular to the galactic plane. The corresponding solution of the diffusion equation is [compare Eq. (27)]

$$\varphi=\lambda S_0\cos\left(\frac{\pi z}{2h}\right)\exp\left(-\frac{n}{N_1}\right)\times\left[1-\frac{\epsilon F}{F+n}\exp\left(-\frac{3}{4\lambda^2}\frac{R^2}{F+n}\right)\right], \tag{41}$$

where

$$F=3\Delta^2/2\lambda^2. \tag{42}$$

At the position of the solar system $(R=0)$ the integral flux is

$$\Phi(z,0,n)=\lambda S_0N_1\cos\left(\frac{\pi z}{2h}\right)\exp\left(-\frac{n}{N_1}\right)\times\left[1+\frac{\epsilon F}{N_1}\exp\left(\frac{F+n}{N_1}\right)Ei\left(-\frac{F+n}{N_1}\right)\right], \tag{43}$$

where

$$Ei(-x)=-\int_x^\infty e^{-t}dt/t$$

is the exponential integral. From this equation together with Eq. (34) one computes the integral spectrum. This can be easily fitted to the experimental data by proper choice of the parameter $(1-\epsilon)$, which measures the fractional source strength in our locality, and Δ, which gives the size of the local gap in the source distribution. If the sources are stars of some special type (like the T-Tauri stars suggested, wholly speculatively, in Sec. IV C) such a local gap might well be found astronomically. Only better experimental data will decide whether or not it is possible to fit the observations exactly with such a theory. It is at least satisfactory to see that Eq. (43) begins to give a deviation from the simple power law at an energy U_d such that $n(U_d)\approx F$. This implies [see Eqs. (34), (38), and (42)] that $\Delta\approx h[\ln(U_d/U_0)]^{\frac{1}{2}}$. Therefore, the size of the region of weak source would be of the order of h and vary quite slowly with U_d.

An alternative interpretation for the small number of low-energy particles is the assumption of a source of limited dimensions, situated some distance from the sun. But there is no plausible location for such a source apart perhaps from the galactic nucleus itself, which the intensity arguments of Sec. II C make very unsatisfactory. A local weakening of the source density is much more plausible than a relatively concentrated source somewhere a fraction of the galactic radius away from the sun.

C. Y. Fan,[13] in his account of the energy spectrum on Fermi's hypothesis derived the shape of the energy spectrum much as we have done here, though he neglected leakage. He ascribed the low-energy deviations to a finite distance from the source, just as we have done. He succeeded in fitting the whole spectrum by assuming a source in the dense Population II stars of the galactic nucleus and a *spherical* distribution of the diffusing magnetic field with dimensions of the order of R_0. He was required to postulate large injection energies and predicted for the heavy components an energy spectrum much steeper than for protons. We regard his results as sharply contradicted by cosmic-ray evidence alone, without reference to astrophysical plausibility.

C. Absence of Primary Electrons and Photons

It has been quite well established that the number of electrons and photons above 1 Bev arriving on the earth is less than 1 percent of the total particle flux.[14]

Electrons in the primary beam can have two origins: initial injection, together with the protons and the

[11] L. Janossy, Z. Physik **104**, 430 (1937).
[12] For a discussion of this question see also H. Alfvén, Arkiv Fysik **4**, 407 (1952).
[13] C. Y. Fan, Phys. Rev. **82**, 211 (1951).
[14] Critchfield, Ney, and Oleska; Phys. Rev. **85**, 461 (1952).

heavier components, or subsequent collisions of nucleons in space, producing mesons or neutrons. It has been estimated[15] that a few percent of the energy lost in collisions will appear in the form of electrons and photons resulting from the decay of mesons and neutrons. Since in our picture the total number of nuclear collisions suffered in space is quite small, we can expect from this source a negligible contribution to the electronic component. Indeed, the absence of electrons and photons re-enforces the nuclear mass distribution (the presence of the Fe group, doubtfully the absence of Li, Be, and B) in giving evidence that little matter has been traversed by the primary beam.

What of the possibility of electron injection? Our picture predicts that *total* energies are multiplied in the collisions with magnetic turbulence elements. If the electron spectrum at injection is like that of the heavy particles in the *kinetic energy scale*, then the final electron spectrum will be similar to the final proton spectrum, but with the energy scale reduced.

If the injection energy is, say, of the order of 10^7 ev, the reduction factor will be of the order of 100 and thus the electrons will be confined to very low energies. Such a similarity between the electron and the proton kinetic energies at injection would result from a single, more or less linear acceleration by a motional or induced emf. On the other hand, an injection mechanism which involves a cyclic or spiraling type of acceleration tends to produce electrons and protons with approximately the same momentum. In this case the reduction factor in the energy scale of the final spectrum is of the order of 10 and it may be necessary to postulate some mechanism to degrade the electron energy at a fast rate during propagation in space. Collisions with photons and radiation by deflection in magnetic fields have been suggested as possible processes leading to such energy losses.

In conclusion, unless electrons are in fact eliminated by some special mechanism, the theory suggests the existence of a primary electron component of intensity similar to that of the main beam, though of greatly reduced energy. A search for these electrons at high latitudes may be worthwhile.

D. Deviations from the Power Law at Extremely High Energies

For various reasons, one might expect deviations from the power-law energy spectrum at sufficiently high energies.

1. As the energy increases, the maximum distance at which sources are effective increases as well (see Sec. II C). When this distance becomes of the order of magnitude of the galactic radius, the assumption of an infinite uniform distribution of cosmic-ray sources in directions parallel to the galactic plane is obviously no longer justified. Indeed, there are no appreciable sources at $R>R_0$; moreover, near the center of the galaxy the strength of cosmic-ray sources may be far different from that in the neighborhood of the solar system. The maximum distance, R, at which the contribution of a source is important is related to the number of collisions, n, by the equation: $\lambda\sqrt{n}\approx R$. If we take for R half the galactic radius ($R=2.5\times10^4$ l.y.), we obtain for n a value of the order of 6×10^8. This corresponds to an energy of the order of $U_0e^{\alpha n}\approx U_0e^{360}$. We see that the cut-off energy set by the finite extent of the source is so high as to be practically meaningless.

2. Another conceivable cause for a cut-off is a time effect. Very high energy rays must have started a long time ago; on the present theory, the age of a particle is $\lambda n/v$, or about $10^6\times\ln(U/U_0)$ years. If the spiral arms evolve in times of the order of a few rotations, say 10^9 years, we could expect a cutoff for $\ln(U/U_0)\approx10^3$. Indeed, before the evolution of the spiral arms the rays may not have been contained in the galaxy to undergo acceleration. Here, too, however, the cut-off energy lies meaninglessly high.

3. The strict power-law for the energy spectrum follows from the assumption that the transport mean free path λ and the fractional energy gain per collision, α, are independent of energy. At a sufficiently high energy, fixed by the strength and size of the scattering magnetic elements, λ must increase and therefore the spectrum should fall off more rapidly, unless this effect is compensated by a large change in α.

Our experimental knowledge of the high-energy spectrum is not precise enough to exclude the possibility that λ may have already increased appreciably between 10^{10} and 10^{15} ev. In any case, as we shall discuss in Sec. IV A, the energy E_m, at which the increase of λ begins, could not lie much beyond 10^{15} ev. An increase of λ with energy should result not only in a departure from the power spectrum but also in a failure of isotropy.

Let us examine this point in some detail. Neglecting nuclear collisions, the differential energy spectrum for variable λ and α is given by the equation

$$j(E)=\frac{\text{const}}{E\alpha(E)}\cos\left(\frac{\pi z}{2h}\right)\exp\left[-\frac{\pi^2}{12h^2}\int_{E_c}^{E}\frac{\lambda^2(E')}{\alpha(E')}\frac{dE'}{E'}\right]. \quad (44)$$

On the other hand, the spatial asymmetry is given approximately by Eq. (25). One sees that the intensity at E depends on the integral

$$\int_{E_c}^{E}\lambda^2(E')dE'/\alpha(E')E',$$

while the spatial asymmetry is proportional to the value of λ at the given energy E.

Suppose first that α does not change much with energy. One will be able to observe a departure from isotropy before the intensity has dropped to an undetectable value if the increase of λ is sufficiently abrupt. To make a quantitative argument one should consider that the observations refer to integral rather than to differential intensities. For a power-law spectrum, the ratio, f, of integral intensities at the energies E and E_m is given by $f=Ej(E)/E_mj(E_m)$. For a more rapidly decreasing spectrum, such as corresponds to an increasing λ, f is smaller than the above

[15] See Phyllis Greifinger, Ph.D. thesis, Cornell University, Ithaca, New York, 1954 (unpublished).

expression. Therefore, from Eq. (44):

$$f<\exp\left[-\frac{\pi^2}{12h^2}\int_{E_m}^{E}\frac{\lambda^2(E')}{\alpha(E')}\frac{dE'}{E'}\right]=\exp\left[-\xi\frac{\lambda^2(E)}{h^2\alpha(E)}\left(\frac{E-E_m}{E}\right)\right], \quad (45)$$

where ξ is a number of the order of unity for any reasonable dependence of λ on E. Let us assume that λ is constant up to the energy $E_m=10^{15}$ ev. With air-shower detectors it may be possible to measure intensities 10^4 times smaller than that observed at 10^{15} ev. Setting, therefore, $f=10^{-4}$, the above equation yields

$$(E-E_m)/E<9\alpha h^2/\xi\lambda^2. \quad (46)$$

It does not seem likely that one can observe an anisotropy at these high energies unless λ is greater than $h/10$ (for this value Eq. (25) gives $\delta=0.01$; actually the anisotropy may be somewhat greater as already indicated in Sec. I B). One thus obtains

$$(E-E_m)/E<10^3\alpha/\xi.$$

Even if one takes into account the uncertainty in ξ, one sees that λ would have to increase by a factor of 100 in an energy range of less than 1 percent, which is very unlikely. One could of course expect an observable anisotropy if α at 10^{15} ev were much greater than at low energies, because then the required increase in λ would occur over a wider range of energies. Indeed, if we are willing to allow a large value of α at 10^{15} ev, we may assume that λ and α have been increasing smoothly with energy in such a way as to keep the ratio λ^2/α roughly constant. In this way we even may expect at some high energy a measurable anisotropy without appreciable departure from the power-law spectrum. Actually it would seem likely that α increases with energy (see Sec. IV A) but hardly fast enough to hold the intensity at a detectable level by the time a pronounced anisotropy is present. The above conclusions seem to be fairly independent of the details of the diffusion model. They suggest that the beam should remain essentially isotropic to the highest observable energies, if it is true (a) that the cosmic rays gain their energy through a gradual acceleration process, and (b) that the fractional energy gain per collision is not excessively large.

An experimental investigation of the angular distribution at the highest energies is obviously very desirable. Should anisotropy be found, it would require a drastic revision of the model. Moreover, it would furnish information on the geometry of the diffusing volume.

IV. ASTROPHYSICAL IMPLICATIONS OF THE MODEL

We shall now discuss in a perforce incomplete way the nature of the galactic magnetic field, and the astrophysical evidence bearing on the values of the parameters chosen to fit the cosmic-ray data.

A. The Transport Mean Free Path

One can construct without difficulty a model of a field in which, as required by our theory, the transport mean free path is constant up to a high limiting energy. As already mentioned, we picture the galactic space filled with wandering turbulent magnetic elements, each rather well separated from a background of lower field strength. The elements have a kind of hard core of higher gas densities and higher magnetic fields. Cosmic-ray particles wander from core to core in more or less straight-line paths; when they enter each core, they penetrate with a complex spiraling motion about the lines of force until they reflect from the region of strong field, and find their way out again. The mean free path will then depend primarily on the spacing of the cores, but *not* on the energy of the particle. This will remain true until the energy becomes so high that the radius of curvature, R_B, is comparable to the dimensions of the core. At this energy, E_m, the mean free path will increase and the power law will fail.

If one assumes spacings of the order of one or several light years and cores of somewhat smaller dimensions, E_m corresponds to a radius of curvature, R_B, of the order of 1 l.y. One can make a rough estimate of the magnetic fields in clouds by assuming equipartition of energy between magnetic fields and turbulent motion. If one takes one atom per cm^3 in the cores, and velocities of the order of 10 km/sec, one obtains for the magnetic field a value of the order of 3×10^{-6} gauss.[16] The corresponding value of E_m is about 10^{15} ev.

The picture of turbulence suggests that there are cores of different sizes and with different field strengths. As the energy increases a smaller and smaller fraction of the cores will remain able to scatter the particles. Thus, one should expect that beyond a certain energy, the mean free path will increase gradually, in a manner dependent on the distribution of the relevant properties of the cores.

From the crude argument of equipartition, one may expect that the cores of higher field strength will have higher average velocities. In Fermi's model, α is proportional to the square of the velocity. Therefore, one may predict an increase with energy of the fractional energy gain per collision, α, because particles of higher energies scatter preferentially from cores of higher field strengths and therefore higher velocities.

B. Reflectivity of Galactic Boundary

Reasons have been given[6,17] to assign a very large reflectivity to the boundary between the material of the galactic disk and the nearly field-free region outside. A reflectivity as great as 0.9 or 0.95 would have no important effect on our analysis but would require only a re-evaluation of such parameters as λ, h, etc. Only a reflectivity so great that a thousand or more approaches to the boundary should be made before escape, would drastically alter this model. It seems rather unlikely that such a high reflectivity can be maintained since (1) the ionized material is very spottily distributed, (2) it occupies a small fraction of the total volume of the disk or arm, and (3) it may indeed possess many lines of force which do not close, but point outward into extra-galactic space.

Another possible way of trapping cosmic-ray particles should be considered. The galaxy as a whole may possess a general external magnetic field, extending with appreciable intensity to a distance of the order of R_0 in all directions. This field, which may roughly resemble that of a dipole, will bend the trajectories of

[16] Neither the equipartition principle nor the values of density and velocity are reliable; but independent lines of evidence indicate galactic fields of this order of magnitude. See S. Chandrasekhar and E. Fermi, Astrophys. J. 118, 116 (1953).

[17] A. Unsöld, Phys. Rev. 82, 857 (1951).

the escaping particles and may bring them back to the galaxy after they have reached a distance of the order of R_0. In the slowly varying field the particles describe complicated trajectories, so that the time of drift T is long compared with the time of flight along the direct path T_0. Alfvén[18] has shown that T and T_0 are related by the approximate equation

$$T_0/T \approx R_B \partial(\ln B)/\partial r,$$

where B is the absolute value of the magnetic field, R_B is the radius of curvature, and $\partial B/\partial r$ is the derivative of B in the direction perpendicular to the lines of force. In a dipole field, $\partial(\ln B)/\partial r$ is of the order of the inverse distance from the center of the dipole and thus in our case of the order of $1/R_0$. If the particles are to be held in the neighborhood of the galaxy, R_B/R_0 must be small compared with one, say, of the order of $\frac{1}{10}$ at most. Since T_0 is at least R_0/c, the above equation shows that T is greater than 10^6 years. But, unless the particles return to cross the galactic disk some 1000 times in their life history, they will not greatly modify the characteristics of the diffusion process that we have postulated to account for the observed isotropy and intensity. This means that returning particles would have an average age in excess of 10^9 years, which is of the order of the age of the galaxy. Therefore, the properties of the galaxy in times far past are one of the factors that will determine the importance of an external trapping field on cosmic rays now present.

C. Mechanisms of Injection

The strongly reduced energies of injections of the present theory make simpler the problem of the mechanism of injection.

Acceleration of a fair sample of stellar atmosphere to energies of the order 100 Mev per nucleon seems a much more likely process than the production of particles with a mean energy of 10 Bev, and a tail up to 10^{15} ev. Indeed, the ejection of protons up to a few Bev during periods of great solar activity is now fairly certain, and it seems very reasonable to expect a much more constant and widespread mechanism giving rise to protons of considerably smaller kinetic energy even in stars of the type of the sun. The effect of solar flares also suggests that such stars as the T-Tauri variable, which show continually irregular flare-like variations, may be especially effective sources of cosmic rays.

Any electromagnetic mechanism for injection must avoid the heavy short-circuiting produced by an ionized plasma of considerable density.[19] This argument suggests that a sunspot betatron mechanism, such as suggested originally by W. F. G. Swann and elaborated by Butler and Riddiford[20] is not a very likely model if it is located deep in the stellar atmosphere.

We add a purely nuclear argument. Since the primary beam contains heavy nuclei, the paths of acceleration cannot have traversed much matter. The sunspot process involves times of the order of 10^3 or 10^4 seconds and therefore distances of some 10^{14} cm. Since the matter traversed must be less than a mean free path against nuclear collisions for a heavy nucleus, we require a density less than 10^{-13} g cm^{-3}. Such a density is not found below the outer chromosphere of the sun. Using the unjustifiable guide of energy equipartition, with densities of about 10^{-13} g cm^{-3} and velocities of streaming like those seen in sunspots (of the order of 100 km/sec), we get a maximum value for B of about 10 gauss. This seems to suggest that no mechanism which requires large fields for its action is likely to work with the matter densities required.

No dearth of other suggestions exists, the most likely of which use the large spaces and low densities available not in the stellar atmosphere proper, but in the region of the corona or beyond. In particular, the potentials induced by moving magnetized prominence-like jets of matter suggested by Kiepenhauer[21] and Schlüter[22] may be important sources of cosmic-ray particles. The high energies required without a rapid acceleration in space would place severe demands on the structure of the jets. On the other hand, the production of the comparatively low energies required by the present theory can be explained without great effort.

It is important to observe that the process considered here would also accelerate electrons. In the absence of strong magnetic fields it seems right to expect these electrons to enter the subsequent propagation as often as do protons. If there was no further acceleration in space, electrons and protons ought to arrive at the earth with comparable energies and intensities, unless some special process, such as those mentioned in Sec. III C, removes electrons very effectively during their propagation in the galaxy. The same conclusion applies to the case where particles are accelerated in space, but the rate of energy gain is so small as to require injection kinetic energies of the order of the rest energy of protons.

The suggestion here made, that a small fraction of the stellar energy output—of the order of 10^{-6}—goes into protons in the nuclear reaction range, and that this process occurs in regions far away from the regions of thermonuclear reaction, may be fruitful in other problems as well. F. Hoyle[23] has already suggested that such atmospheric processes may produce nuclear reactions enough to explain the presence in stars of such thermally perishable isotopes as deuterium.

[18] H. Alfvén, Z. Physik **105**, 319 (1937).

[19] However, W. F. G. Swann has recently pointed out that the short-circuiting effect of ionized gases might have been overestimated [Duke Conference on Cosmic Rays, December 1953 (unpublished)].

[20] W. F. G. Swann, Phys. Rev. **43**, 217 (1933); S. T. Butler and L. Riddiford, Phil. Mag. **43**, 447 (1952).

[21] K. O. Kiepenhauer, Phys. Rev. **79**, 809 (1950).

[22] A. Schlüter, Z. Naturforsch. **7a**, 136 (1953).

[23] In conversation with E. E. Salpeter.

D. The Speed of the Magnetic Turbulence Elements

Fermi's original theory predicts a value of α equal to about $4V^2/c^2$, where V is the rms random velocity of the magnetic clouds. The high rate of acceleration required by our model would then indicate rms velocities of about 120 km/sec. Such velocities are beyond the maximum of some 80 km/sec, seen by Adams;[24] they are very much beyond his rms value of 30 km/sec and the even lower values, around 10 or 20 km/sec, favored in the later work. Moreover, the assumed small value of the transport mean free path (1 l.y.) goes against the general tendency to see rather larger clouds and a somewhat more uniform distribution of matter and of motion than this picture implies.

Without minimizing this difficulty we wish to submit the following remarks: All the astronomical studies are based on absorption or emission processes in space. The microwave emission studies, in particular, detect only unionized clouds, and do not directly reveal the motion of the plasma. Studies of interstellar spectral absorption lines are necessarily biased in favor of (1) clouds at high densities and large diameters which give stronger absorption, and (2) homogeneous motions, which lead to narrow and well-marked lines. Studies based on the residual polarization of starlight require dust, which probably accompanies rather dense accumulations of gas. Our clouds correspond to a relatively dilute swirling gas in which the denser and dusty clouds seen by the spectroscopists are drifting. The nature of turbulent motion in the plasma is not a problem beyond controversy, but a naive application of the Kolmogoroff spectrum of the homogeneous theory does imply that the more dilute clouds may have higher mean velocities. There may be some astrophysical evidence for this view.[25]

Recently Chandrasekhar and Fermi[26] have presented a picture of the galactic magnetic field as being fairly smoothly oriented along the spiral arms. Their picture is based on the supposed dust-grain orientation derived from polarization measures. Fermi[27] has shown that such a uniform field allows much more rapid acceleration than did his earlier picture of random collisions between particles and magnetic clouds. For the random case, it is only the excess of the head-on over the overtaking collisions which gives a net acceleration. But if the field is relatively smooth, a particle may be trapped along a line of force between two approaching clouds. These form "walls" between which (under certain conditions) repeated reflections will accelerate the particle until it gains enough energy so that it can break out of the trap. The velocities of the clouds do not determine the energy gain, but only the time required. This time can be a very small fraction of the history of the particle.

Fermi's new model accounts for the high rate of energy gain required by the cosmic-ray evidence, without postulating unobserved high velocities of the magnetic clouds. However, this model may be hard to reconcile with the high degree of isotropy of cosmic rays. Perhaps Fermi's acceleration mechanism, which makes use of an over-all regular magnetic field, is superposed on space diffusion through locally turbulent fields. The meaning of our parameter α will change in this more detailed picture: it will cover an average of acceleration rates both over the random collisions with the small, turbulent magnetic clouds which govern the motion in space and over the few but very effective collisions in the regions of uniform field, which govern the change in energy. But the theory as developed here would not be affected in its formulation. For it is from cosmic-ray properties and the general geometry of the gas distribution in the galactic disk alone that our conclusions follow.

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[24] W. S. Adams, Astrophys. J. **109**, 354 (1949).
[25] S. B. Pikelner, Doklady Akad. Nauk S.S.S.R. **88**, 229 (1953).
[26] S. Chandrasekhar and E. Fermi, Astrophys. J. **118**, 113 (1953).
[27] E. Fermi, lecture at the meeting of the American Astronomical Society at Boulder, Colorado, August, 1953 (unpublished).