the validity of the shape-independent approximation the close agreement of the two effective range values is evidence for the reliability of the cross-section measurements.

Figure 9 shows the values of singlet effective ranges, calculated on the shape-independent approximation from recent cross-section measurements, plotted as a function of the neutron energy at which the measurements were made. The errors shown represent only the independent errors in the cross section measurements and do not include the uncertainties in the parameters a_s, a_t, and ρ_t. Figure 9 shows excellent agreement between the various measurements.

The dashed line in Fig. 9 represents the value of the proton-proton effective range[24,25] which is taken as $2.65\pm0.07\cdot10^{-13}$ cm. Though the calculated neutron-proton singlet effective ranges appear to be a little smaller than the proton-proton ranges, the agreement is within the errors of the experiments and constitutes evidence for the charge independence of nuclear forces.

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[25] H. H. Hall and J. L. Powell, Phys. Rev. **90**, 912 (1953).

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Atomic Masses in the Intermediate Region*

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The double-focusing mass spectrometer previously described has been used to measure 28 atomic masses in the region between gallium and niobium. The new information combined with previous determinations permits one to draw a rather complete packing fraction curve for the region from sulfur through xenon. Analysis shows that masses for stable and unstable isotopes of an element can be fitted by parabolas, one for odd A and one for even A. The two parabolas have the same shape (except just above a proton shell) but are displaced in A as well as mass. The displacements are such that, for an isobar, the minimum mass does not occur at the same Z for the four nuclear families (even even, etc.), in contradiction to usual assumptions.

THIS paper presents the results of a portion of an extensive program of mass measurement undertaken with the double-focusing mass spectrometer[1] developed at the University of Minnesota. Previous communications have covered the regions sulfur to zinc[2,3] and palladium to xenon,[4] as well as detailed studies of the triple isobars at mass 40 and 50.[5,6] We now report 28 mass doublets covering isotopes from gallium, $Z=31$, to niobium (columbium), $Z=41$. The combined data is now sufficient to reveal patterns and shell effects which are discussed under the heading of Mass Systematics.

DOUBLET MEASUREMENTS

The method of measurement has not been changed. We measure the atomic mass difference of two ions of the same mass number (a mass doublet). One ion has a known mass—it is usually a hydrocarbon fragment. The ion current is detected electronically, and because of the nature of the feedback arrangement of the control circuits, the measurement of the mass difference reduces to a measurement of a resistance ratio.[2,7] A "run" consists of at least 10 consecutive tracings of the mass spectrum with alternate forward and backward sweep. Runs were taken on different days over a period of months. During these measurements, the width at half height of a mass "peak" corresponded to 1/10 000 of the ion mass. At these masses this resolution is insufficient to separate the C^{13} satellite (one less hydrogen) from the hydrocarbon peak. A correction is possible by measuring the relative intensity of the hydrocarbon ion at the next lower mass number, and where necessary this correction has been applied.

Table I lists the results of the doublet measurements. The errors are probable errors, and are purely statistical. The day-to-day variation usually exceeds that expected from the statistics within the runs. In this case a simple average is computed; otherwise a weighted average is used.

Several stable isotopes are missing. Selenium ions were reliably obtained only from H_2Se gas with the result that Se^{77} and Se^{78} were accompanied by incompletely resolved selenides of other isotopes. Krypton

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† Research supported by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
[1] E. G. Johnson and A. O. Nier, Phys. Rev. **91**, 10 (1953).
[2] Collins, Nier, and Johnson, Phys. Rev. **84**, 717 (1951).
[3] Collins, Nier, and Johnson, Phys. Rev. **86**, 408 (1952).
[4] R. E. Halsted, Phys. Rev. **88**, 666 (1952).
[5] W. H. Johnson, Phys. Rev. **87**, 166 (1952).
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[7] A. O. Nier and T. R. Roberts, Phys. Rev. **81**, 507 (1951).

masses have been reported elsewhere;[8] the present results include those measurements and supersede them. Kr^{80} has been omitted. The hydrocarbon fragment contained a large C^{13} satellite, for which it is difficult to correct, and the doublet value is probably in error. Finally, zirconium masses are far from complete.

ATOMIC MASSES

Table II presents the new atomic masses. Consistent with our previous tables, masses are computed using $H^1=1.008146\pm3$, $C^{12}=12.003842\pm4$ from our own measurements.[2] For completeness, masses for Se^{77}, Se^{78}, and Kr^{80} have been added. The masses may be compared with mass differences from reaction and disintegration energies. Table III shows that the comparison is good. Selenium masses may also be compared with the ratio of selenium mass differences available from microwave measurements of Geschwind *et al.*[9] Table IV shows an excellent agreement.

TABLE I. Mass doublets.

Doublet	ΔM (10^{-4} amu)	No. of runs	Source of ions
$C_5H_9-Ga^{69}$	1447.5 ±0.4	5	metal
$C_5H_{11}-Ga^{71}$	1613.0 ±0.8	5	
$C_5H_{10}-Ge^{70}$	1543.0 ±0.6	6	metal
$C_5H_{12}-Ge^{72}$	1723.5 ±0.5	5	
C_6H-Ge^{73}	845.1 ±0.3	5	
$C_6H_2-Ge^{74}$	946.8 ±0.6	7	
$C_6H_4-Ge^{76}$	1100.5 ±0.4	6	
$C_6H_3-As^{75}$	1017.9 ±0.4	7	metal
$C_6H_2-Se^{74}$	931.4 ±0.7	4	H_2Se gas
$C_6H_4-Se^{76}$	1120.6 ±0.4	4	
$C_6H_9-HSe^{80}$	1461.7 ±0.4	4	
$C_6H_{12}-H_2Se^{82}$	1616.6 ±0.4	4	
$C_6H_7-Br^{79}$	1364.2 ±0.5	5	C_2H_5Br
$C_6H_9-Br^{81}$	1540.5 ±0.5	6	
$C_3H_3-Kr^{78}/2$	634.0 ±0.4	5	gas
$C_3H_5-Kr^{82}/2$	824.19±0.28	9	
$C_6H_{11}-Kr^{83}$	1720.7 ±0.5	5	
$C_3H_6-Kr^{84}/2$	912.20±0.26	10	
$C_3H_7-Kr^{86}/2$	994.07±0.32	6	
$C_6H_{13}-Rb^{85}$	1897.5 ±0.6	5	Rb_2SO_4
$C_5H_{11}O-Rb^{87}$	1717.3 ±1.7	6	
$C_6H_{12}-Sr^{84}$	1807.0 ±1.5	8	metal
$C_6H_{14}-Sr^{86}$	2002.5 ±1.0	10	
$C_5H_{11}O-Sr^{87}$	1720.5 ±0.6	6	
$C_4H_8O_2-Sr^{88}$	1464.6 ±1.1	12	
$C_8H_9-Y^{89}O$	1698.4 ±1.1	5	oxide
$C_7H_6-Zr^{90}$	1426.6 ±2.5	4	oxide
$C_7H_9-Nb^{93}$	1648.1 ±0.8	4	metal

TABLE II. Atomic masses based on $H^1=1.008146\pm3$, $C^{12}=12.003842\pm4$.

Element	Atomic mass	Element	Atomic mass
Ga^{69}	68.94778±6	Kr^{78}	77.94513± 9
Ga^{71}	70.94752±9	Kr^{80}	(79.94194± 7)[b]
Ge^{70}	69.94637±7	Kr^{82}	81.93967± 7
Ge^{72}	71.94462±7	Kr^{83}	82.94059± 7
Ge^{73}	72.94669±4	Kr^{84}	83.93836± 7
Ge^{74}	73.94466±6	Kr^{86}	85.93828± 8
Ge^{76}	75.94559±5	Rb^{85}	84.93920± 8
As^{75}	74.94570±5	Rb^{87}	86.93709±17
Se^{74}	73.94620±8	Sr^{84}	83.94011±15
Se^{76}	75.94357±5	Sr^{86}	85.93684±11
Se^{77}	(76.94459±5)[a]	Sr^{87}	86.93677± 8
Se^{78}	(77.94232±5)[a]	Sr^{88}	87.93408±11
Se^{80}	79.94205±5	Y^{89}	88.93421±11
Se^{82}	81.94285±6	Zr^{90}	89.93311±25
Br^{79}	78.94365±6	Nb^{93}	92.93540± 9
Br^{81}	80.94232±6		

[a] Computed from Se^{76} using (n,γ) reaction data of B. B. Kinsey and G. A. Bartholomew, Can. J. Phys. (to be published).
[b] Computed from Se^{80} using beta scheme reported in Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

MASS SYSTEMATICS

The quantity of precision mass data at intermediate masses is now sufficient to allow an empirical examination for any regularity which might guide the theorist. Because two independent variables (chosen from A, Z, N and T) are required to specify an atom, the description will be in terms of a mass surface. The pattern of stable isotopes shows the necessity of dividing atoms into at least three families: one for odd A and two for even A (even-even and odd-odd). The Bohr-Wheeler formula[10] assumes that these three surfaces are smooth

TABLE III. Comparison with reaction data. Difference between masses expressed as a difference of mass defects $(A-M)$ in milli-mass units.

Atoms	Mass spectrometer difference mMU	Reaction difference mMU	Reactions	Reference
$Ga^{69}-Zn^{70}$	0.01±0.06	0.06±0.2	$(\gamma,n);\beta$	a,b
$Ga^{71}-Zn^{70}$	0.27±0.10	−0.01±0.2	$(\gamma,n);(p,n)$	c,d
$Ge^{74}-As^{75}$	1.04±0.07	0.77±0.2	$\beta^+;(\gamma,n)$	b,e
$As^{75}-Se^{74}$	0.50±0.08	0.51±0.2	$(\gamma,n);\beta$	e,b
$Ge^{74}-Se^{74}$	1.54±0.10	1.28	$\beta^+;\beta$	b
$Se^{76}-As^{75}$	2.13±0.06	2.21±0.06	$(n,\gamma);\beta$	e,b
$Se^{76}-Br^{79}$	0.08±0.06	0.24±0.25	$(n,\gamma);(n,\gamma);(p,n);(\gamma,n)$	e,f,e
$Se^{80}-Br^{81}$	0.27±0.06	0.20±0.14	$\beta^+;(\gamma,n)$	b,e
$Sr^{87}-Rb^{87}$	0.32±0.18	0.30	β	b
$Sr^{87}-Sr^{86}$	0.07±0.12	0.16±0.2	dp	g
$Sr^{88}-Y^{89}$	0.13±0.15	0.38±0.2	$dp;\beta$	g,b
$Zr^{90}-Y^{89}$	1.10±0.25	1.36±0.3	$(p,n);(n,\gamma)$	f,a
$Zr^{90}-Nb^{93}$	2.28±0.25	2.44±0.3	$dp;(n\gamma);(p,n);(\gamma,n)$	g,e,f,e

[a] Hanson, Duffield, Knight, Diven, and Palevsky, Phys. Rev. **76**, 578 (1949).
[b] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).
[c] Sher, Halpern, and Mann, Phys. Rev. **84**, 387 (1951).
[d] C. C. Trail and C. H. Johnson, Phys. Rev. **91**, 474 (1953).
[e] B. B. Kinsey and G. A. Bartholomew, Can. J. Phys. (to be published).
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[g] J. A. Harvey, Phys. Rev. **81**, 353 (1951).

[8] T. L. Collins, National Bureau of Standards Circular 522 (U. S. Government Printing Office, Washington, D. C., 1953), p. 67.
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[10] E. Fermi, *Nuclear Physics Notes* (University of Chicago Press, Chicago, 1950), pp. 6–8.

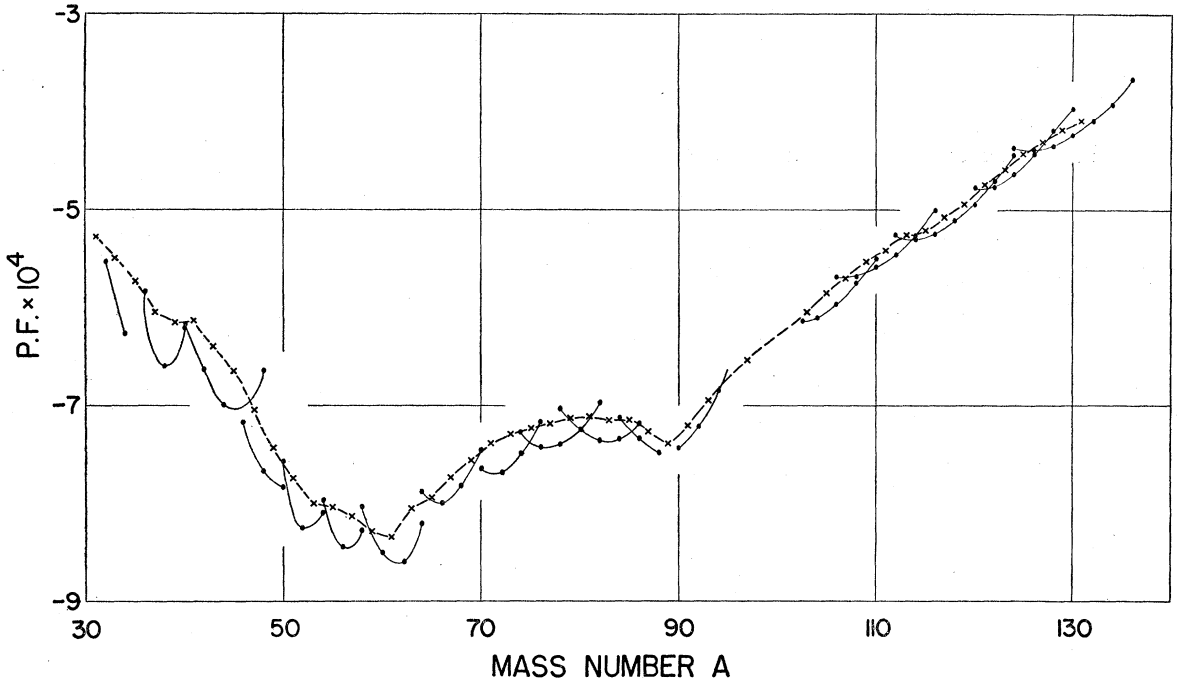

FIG. 1. Packing fraction curve for stable atoms from S^{32} to Xe^{136}. Dashed line is through odd-A atoms; solid lines are through even-A atoms for each element.

and have the same quadratic form for isobars (constant A). The even-A surfaces are assumed to be equally but oppositely displaced from the odd-A surface by an amount of mass which decreases slowly for increasing A. In spite of the fact that numerical constants which have been used in the formula are incorrect, these assumptions have been generally confirmed by beta ray systematics, with two important modifications.[11] First, two surfaces are required for odd A in some mass regions. Second, the surfaces have discontinuities at certain "magic numbers." We can now examine mass surfaces directly. We find much regularity, but the relative position of the three surfaces is considerably more complicated than previously assumed.

For independent variables we shall use A and Z, and for the mass variable we shall use mass defect, $A-M$, in milli-mass units. Imagine that A and Z are horizontal coordinates, and mass defect is downward. Stable atoms are then lower than their unstable neighbors, and the mass surfaces form a narrow valley. Beta-ray data gives information about isobaric (constant A) sections of the valley. We will examine isotopic (constant Z) sections because they contain roughly twice as many points. The minima of the isobaric sections may be joined by a line, representing the greatest stability against beta decay. This line will not coincide with a similar line through the minima of isotopic sections, the difference depending upon the slope of the valley bottom and, hence, upon the mass variable.

A more fundamental variable would be binding energy, which is related to mass defect ΔM by

$$\text{B.E.}=8.986A-0.840Z+\Delta M.$$

Binding energy increases rapidly with A and gives isotopic minima far removed from the region of stability. Our conclusions are not affected by a change to this variable because the transformation is linear. For a plot extending over a wide range, it is common to use $\text{B.E.}/A=8.986-0.840Z/A+\Delta M/A$, or to use the packing fraction, $-\Delta M/A$. The shapes of curves using these variables are almost identical.

Figure 1 gives an over-all picture of our mass measurements. It is a packing fraction curve for stable atoms from S^{32} to Xe^{136}. Errors of the points are about 0.01 unit, too small to show. The relation of the curve to the mass surface is indicated by lines. Odd-A atoms are joined by a single dashed line. There is only one stable atom for each odd A (except 113, 123), so this line is close to the line joining isobaric minima for the odd-A

TABLE IV. Selenium masses from microwave spectroscopy.[a]

	Mass spectroscopy	Microwave
Se^{74}	73.94620±8	73.94659±60
Se^{76}	75.94357±5	(75.94357)[b]
Se^{77}	76.94459±5	76.94456±40
Se^{78}	77.94232±5	77.94219±40
Se^{80}	79.94205±5	(79.94205)[b]
Se^{82}	81.94285±5	81.94275±60

[a] See reference 9.
[b] Two masses assumed in order to compute remaining four masses.

[11] H. E. Suess and J. H. D. Jensen, Arkiv Fysik 3, 577 (1951).

surface. It is apparent that a single line is sufficient for both odd and even Z. Even-A, even-Z atoms are joined by curves for each element. The curves are isotopic sections of the even even surface. The odd-A line shows discontinuities of slope at mass numbers 40 ($20n$,$20p$), 51 ($28n$), 61 ($28p$), 89 ($50n$), and 113 ($50p$). The even even curves accentuate these discontinuities. We suggest that there is a discontinuity near $A=71$ which could be associated with 40 neutrons. Most striking, in Fig. 1, is the second minimum at 50 neutrons.

TABLE V. Mass defect $(A-M)$ for unstable atoms.

Atom	$(A-M)$ mMU	Reaction	Reference
Ni^{57}	40.90±20	(γ,n)	a
Ni^{59}	47.23± 9	(n,γ)	b
Ni^{65}	49.77± 8	β	c
Ga^{66}	46.12± 6	β^+	c
Ga^{67}	50.74± 5	pn	d
Ga^{68}	50.03± 7	β^+	c
Ga^{70}	51.47± 6	pn	d
Ga^{72}	51.08± 6	β	c
Ga^{73}	51.74±20	β	c
Ge^{69}	48.61± 4	β^+	c
Ge^{71}	52.20± 9	pn	d
Ge^{75}	53.08± 6	β	c
Ge^{77}	52.02± 6	β	c
As^{72}	50.70± 6	β^+	c
As^{73}	52.89± 5	pn	d
As^{74}	52.45±10	β^+, β^-, (γ,n)	c,e
As^{76}	53.12± 6	(n,γ); β	f,c
As^{77}	54.66± 6	β	c
Se^{73}	50.01± 6	β	c
Se^{75}	53.34± 4	p,n	d
Se^{79}	56.18± 6	β	c
Se^{81}	56.20± 5	β	c
Br^{75}	50.42± 8	β^+	c
Br^{76}	51.50± 8	β^+	c
Br^{77}	53.95± 8	β^+	c
Br^{78}	53.90±15	(γ,n)	e
Br^{80}	55.92± 7	β^+	c
Br^{83}	58.41± 7	β	c
Br^{84}	56.60± 7	β	c
Br^{86}	53.12±25	β	c
Kr^{79}	54.33± 7	β^+	c
Kr^{85}	60.05± 9	β	c
Kr^{87}	59.01±18	β	c
Sr^{90}	63.90±25	β, β	c
Ag^{111}	59.10±15	β	c
Ag^{113}	57.24±20	β	c
Cd^{107}	59.47±15	β^+	c
Cd^{109}	60.42±15	β^+	c
Cd^{115}	58.10±12	β	c
In^{110}	57.21±14	β^+	c
In^{111}	59.32±25	$(\alpha,2n)$	g
In^{112}	58.19±25	β^+	c
In^{114}	58.23±10	β^+	c
In^{116}	57.56±15	β	c
In^{117}	57.62±15	β	c
In^{119}	55.38±25	β	c
Sn^{113}	59.09±15	β^+	c
Sn^{121}	57.06±15	β	c
Sn^{123}	54.96±20	(γ,n)	h
Te^{127}	53.90±20	β	c
Te^{131}	49.90±42	β	c
I^{124}	53.32±32	β^+	c
I^{125}	55.26±32	β^+	c
I^{126}	53.88±14	β	c
I^{128}	53.37±10	β	c
I^{129}	53.71±15	β	c
I^{130}	51.79±11	β	c
I^{131}	52.23±42	β	c

[a] Ogle, Brown, and Carson, Phys. Rev. **71**, 378 (1947).
[b] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **89**, 375 (1953).
[c] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).
[d] C. C. Trail and C. H. Johnson, Phys. Rev. **91**, 474 (1953).
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[f] B. B. Kinsey and G. A. Bartholomew, Can. J. Phys. (to be published).
[g] Bleuler, Stebbins, and Tendam, Phys. Rev. **90**, 460 (1953).
[h] Hanson, Duffield, Knight, Diven, and Palevsky, Phys. Rev. **76**, 578 (1949).

TABLE VI. Constants for isotopic parabolas found by a least squares fit of the formula $\Delta M=\Delta M_0+c(A-A_0)^2$.

Z	Element	A	ΔM_0(mMU)	A_0(amu)	c	No. of points
28	Ni	even	53.22	62.45	−0.342	4
		odd	51.74	62.62	−0.342	4
30	Zn	even	53.27	67.7	−0.21	4
31	Ga	odd	52.54	70.59	−0.14	4
		even	51.64	70.52	−0.27	4
32	Ge	even	55.47	72.87	−0.11	3
		odd	53.33	73.50	−0.11	3
33	As	odd	54.67	76.66	−0.13	3
		even	53.13	76.27	−0.13	3
34	Se	even	58.03	79.46	−0.141	5
		odd	56.38	79.70	−0.141	5
35	Br	odd	58.39	83.28	−0.115	5
		even	56.75	82.82	−0.115	4
36	Kr	even	61.79	85.25	−0.133	5
		odd	60.19	85.61	−0.133	3
46	Pd	even	63.36	104.69	−0.110	5
47	Ag	odd	60.98	107.44	−0.090	4
48	Cd	even	61.58	109.84	−0.094	6
		odd	60.36	109.96	−0.094	5
49	In	odd	59.58	112.62	−0.103	4
		even	58.32	113.26	−0.103	4
50	Sn	even	60.65	116.20	−0.091	7
		odd	59.77	115.78	−0.091	7
52	Te	even	58.9	118.3	−0.055	5
		odd	56.9	119.7	−0.055	4
53	I	odd	55.36	123.70	−0.059	4
		even	53.88	126.05	−0.134	4
54	Xe	even	55.47	127.72	−0.0873	7

We have examined the isotopic sections of the mass surface in detail for elements with Z from 28 to 36 and from 46 to 54. We supplemented masses from this and previous papers[3,4] with masses for unstable atoms listed in Table V. Unstable atomic masses were computed from stable neighbors with the aid of reaction and disintegration data. For 16 elements there is sufficient data to show that parabolas are adequate to fit isotopic mass defect curves. Each element requires two parabolas, one for odd A and one for even, which were determined by the method of least squares. For most elements it is possible to fit similar parabolas, that is parabolas which have the same second-order term. This is a least squares fit of two parabolas with five arbitrary constants which we call a double fit. For all parabolas we demand at least one more point than arbitrary constants; at least 4 points for a single fit and at least 3 points each for a double fit.

The result of this numerical exercise is given in Table VI, which lists the constants for the parabolas expressed as

$$\Delta M=\Delta M_0+c(A-A_0)^2,$$

where ΔM is the mass defect $(A-M)$ in mMU, and ΔM_0 and A_0 are coordinates of the minimum of the isotopic section. Double fits (same c) are indicated by a bracket. Residuals of the fits are usually less than 0.1

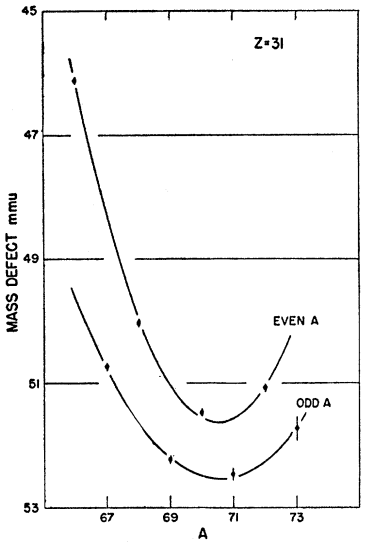

FIG. 2. Parabolas fitted to the mass defects of gallium isotopes.

mMU and are no greater than expected from probable errors. The statistical error of the coefficients in Table VI is difficult to estimate; as a substitute we refer to Figs. 2 to 8 which are representative. (Note the excellent fit of a parabola to the seven, even-A, isotopes of xenon, Fig. 8.) For three elements it was necessary to omit some points. For germanium it was necessary to omit the three isotopes with less than 40 neutrons to obtain a reasonable fit. These three isotopes have greater masses than calculated from the extrapolated parabolas. For indium, an excellent double fit was obtained by omitting In^{115} (Fig. 6). Reaction and disintegration energies do not indicate that the measured mass is incorrect. The fit is never very good for tellurium; however, a double fit is possible if the rare Te^{120} is omitted. Halsted[4] had difficulty measuring tellurium doublets due to the formation of hydrides.

From this examination of isotopic masses we conclude:

1. Parabolas are a good representation of isotopic sections of the mass surfaces within the error of present measurements. Parabolas fit up to and including neutron shells—see Kr^{86} (Fig. 5) and Xe^{136} (Fig. 8).

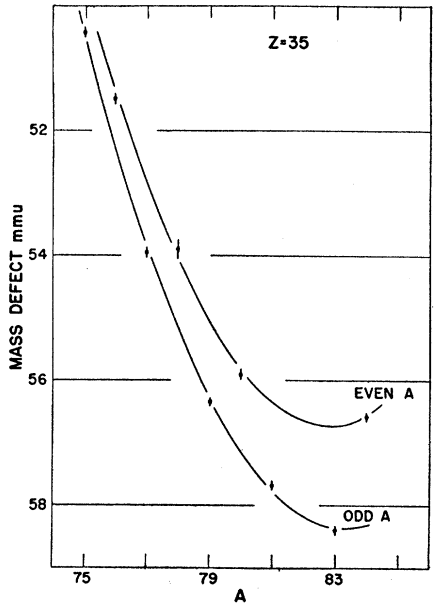

FIG. 4. Similar parabolas fitted to the mass defects of bromine isotopes.

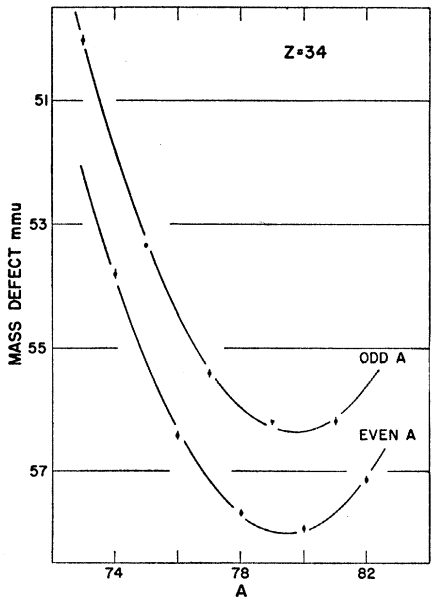

FIG. 3. Similar parabolas fitted to the mass defects of selenium isotopes.

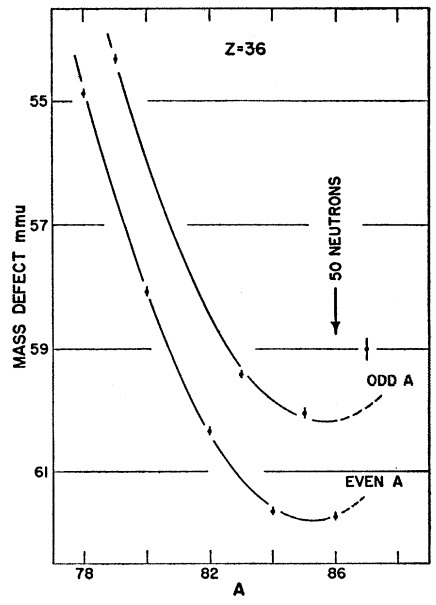

FIG. 5. Similar parabolas fitted to the mass defects of krypton isotopes.

They do not fit across neutron shells—see Kr^{87} (Fig. 5). We find that Sr^{90} ($52n$) has about 4 Mev less binding energy than predicted by extrapolation of a parabola fitted to three strontium isotopes with 50 or less neutrons.

2. In general, the mass surfaces have the same shape for constant Z, indicated by the success of double fits and by the variation of the constant c. There is no reason to believe that this smooth behavior, between shells, should not also apply to isobars. Exceptions are found immediately above proton shells 28 and 50 (only two examined), where the odd-odd surface has about twice the curvature of the corresponding odd-Z, even-N surface (see Ga, Fig. 2). It is not clear from this form of analysis if 3 surfaces are sufficient.

3. A common assumption is that, for a constant A, the minimum mass occurs at the same Z for each of the mass surfaces. Our analysis shows that this cannot be true for masses discussed in this paper. Unfortunately, direct examination of isobaric parabolas is not

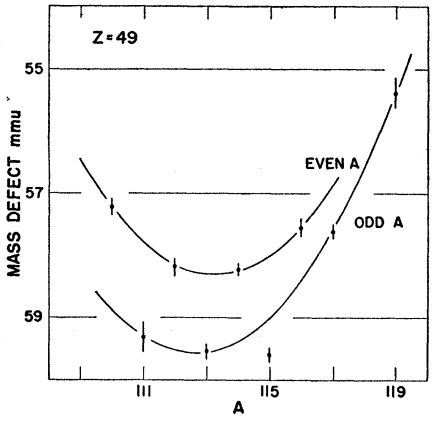

FIG. 6. Similar parabolas fitted to the mass defects of indium isotopes.

useful because there are, at most, two points for each parabola. We reach our conclusion by an examination of minima of isotopic parabolas.

The figures show that there is a real difference in position of the minimum A_0 for the two isotopic parabolas for most elements. For example, the ends of the tin parabolas (Fig. 7) show a difference in spacing of about 1 mMU. Table VII lists differences in A_0 and ΔM_0 from Table VI. The difference is given as coordinates of the odd-A surface minus coordinates of the even-A surface. The signs of ΔM_0 differences come from the position of the odd-A surface between the two even-A surfaces.

It is useful to see what these shifts can mean in terms of isobaric parabolas. Consider the "well behaved" region from arsenic to krypton. The constants here vary with sufficient uniformity so that we can apply the following transformations. Let $\partial A_0/\partial Z=b$ and

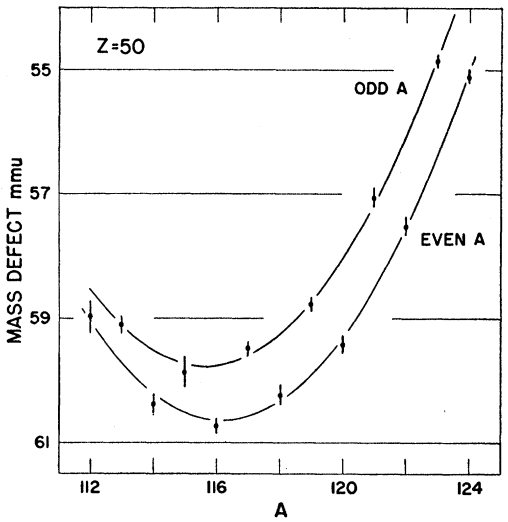

FIG. 7. Similar parabolas fitted to the mass defects of tin isotopes.

$\partial(\Delta M_0)/\partial Z=n$, where A_0, ΔM_0 are the coordinates of the minimum for isotope Z; then the minimum of the constant-A parabola for $A=A_0$ occurs at $Z_0=Z-n/2cb$, with a mass defect $\Delta M_0'=\Delta M_0-n^2/4cb^2$ and a parabolic constant $c'=c/b^2$. As an example we have computed the odd-A parabolas for $A=79$ from data in Table VI. The results are shown in the first two lines of Table VIII. The accuracy is such that we may consider these the same parabola. Line 5 gives a parabola evaluated directly from the three $A=79$ masses (assuming there is one odd-A surface), which shows that the transformation is adequate.

Using the same type of transformation we can calculate, with better accuracy, the difference in the isobaric minima for odd-A and even-A surfaces. Call $a=A_0(\text{odd})-A_0(\text{even})$, and $m=\Delta M_0(\text{odd})-\Delta M_0(\text{even})$,

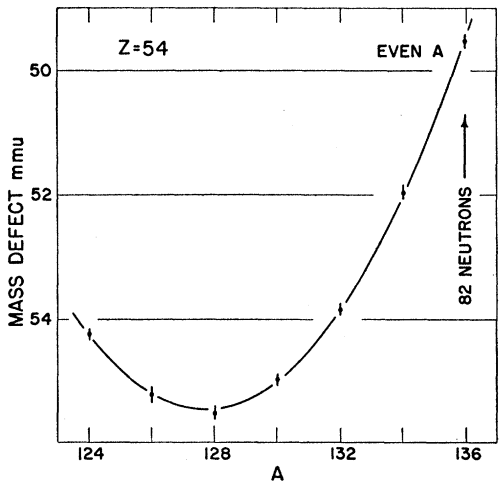

FIG. 8. Parabola fitted to the mass defects of xenon isotopes.

TABLE VII. The isotopic minima (A_0,ΔM_0) of the odd-A mass defect surface minus the minima of the even-A surfaces. A_0(odd) $-A_0$(even)$=a$, ΔM_0(odd)$-\Delta M_0$(even)$=m$.

Z		a(amu)	m(mMU)
28	Ni	0.17	−1.48
31	Ga	0.07	0.90
32	Ge	0.63	−2.14
33	As	0.39	1.54
34	Se	0.24	−1.65
35	Br	0.46	1.64
36	Kr	0.36	−1.60
48	Cd	0.12	−1.22
49	In	−0.64	1.26
50	Sn	−0.52	−0.88

as listed in Table VII; then

$$Z_0(\text{odd})-Z_0(\text{even})=-\frac{a}{b}-\frac{1}{2cb}\frac{\partial m}{\partial Z}+\frac{n}{cb^3}\frac{\partial a}{\partial Z},$$

and

$$\Delta M_0'(\text{odd})-\Delta M_0'(\text{even})=m-(na/b).$$

The spacing m and the shift a for a pair of surfaces does not change fast enough with Z ($\delta Z=2$) to cancel the $-a/b$ term. Lines 3 and 4 of Table VIII list the result of these transformations for our example, $A=79$. Of course, there are no nuclei on an even-A surface at $A=79$, but there is a parabola on a continuous surface. For these isobars the only change necessary in the usual assumptions is to displace the minimum for the odd A parabola 0.1 lower in Z from the even parabolas. This shift is still significant. The transformation has increased the quadratic constant by a factor b^2 ($b\approx 3$)

TABLE VIII. Isobaric parabolas for $A=79$, computed from isotopic constants for the four surfaces, expressed as $\Delta M=\Delta M_0'+c'(Z-Z_0)^2$.

		Z_0	$\Delta M_0'$(mMU)	c'
even Z	odd N	34.58	56.69	−1.17
odd Z	even N	34.5	56.63	−1.3
even even	—even odd	0.10	1.81	
odd odd	—odd even	0.12	−1.76	
odd A	(3 masses)	34.57	56.55	−1.10

so that the effect on isobaric masses is the same magnitude as on isotopic masses.

It is apparent that any mass formula that does not include this displacement of the mass surfaces, and current formulas do not, cannot fit the odd and even masses of an isotope within ½ Mev. The change in sign of the displacement for indium and tin suggest strongly that the effect is related to nuclear shell structure.

4. The results generally show a lack of smoothness in the neighborhood of gallium and germanium (see, for example, Table VII). This could be caused by a subshell at 40 neutrons.

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