species which could be produced by 30-Mev protons on fluorine are Ne^{18}, Ne^{17}, and F^{16}. Since F^{16} is known to be unstable,[1] the activity must be either Ne^{18} or Ne^{17}. If one assumes the activity to be Ne^{17}, the beta-ray energy, coupled with the known mass of F^{17}, gives a minimum possible mass for Ne^{17}. From this value, and the known masses of the other particles involved, one can calculate the threshold energy for the reaction $F^{19}(p,3n)Ne^{17}$. The threshold would be 25.8 Mev. Although the backgrounds and low counting rates prevented the establishment, with high precision, of the threshold for producing the Ne^{18} activity, the activity was solidly in evidence when the incident proton energy was reduced to 24 Mev. Since this energy is below the minimum possible energetic threshold for producing N^{17}, we can eliminate that possibility. Even in the absence of this convincing evidence, one could feel sure that Ne^{17} would have much more energetic positrons (its triad counterpart is N^{17}), and would assign the new activity to Ne^{18} on the grounds that it behaves just as the theory would predict that it should.

CONCLUSION

Our value of 3.2 Mev for the upper limit of the beta-ray spectrum, coupled with the lifetime of 1.6 sec, gives a log ft value for the decay of Ne^{18} of 2.9 ± 0.2. This value clearly places the decay character in the same class as the other known $A=4n+2$ nuclei. The only other known nucleus with such a highly allowed beta decay is He^6, which has a log ft value of 2.95. The close agreement between the calculated mass of Ne^{18} and the observed upper limit of the positrons indicates strongly that the transition goes to the ground state of F^{18}.

ACKNOWLEDGMENTS

We would like to express our appreciation to Dr. Roger Wallace for much valuable assistance, and to Dr. Sumner Kitchen, who also assisted in the experimental program. Our thanks also go to the members of the linear-accelerator operating group who aided the experiment in many ways.

PHYSICAL REVIEW VOLUME 94, NUMBER 2 APRIL 15, 1954

Photon Splitting in a Nuclear Electrostatic Field*

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(Received December 18, 1953)

The cross section for the splitting of a photon into two photons in a nuclear electrostatic field has been calculated from the vacuum polarization Hamiltonian of Euler and others to first order in $e^2/\hbar c$ for low-energy incident photons ($p<mc$). For a favorable experimental case, photons of energy 840 kev incident on lead with antiparallel product photons each emitted at 90° relative to the incident photon, the cross section is 2.3×10^{-33} cm^2/sterad^2.

THE nonlinear terms in the Maxwell equations arising from the polarization of the vacuum[1] result in several interesting effects: coherent photon scattering by a nuclear electrostatic field,[2] scattering of photons by photons,[3] and photon splitting into two product photons in a nuclear electrostatic field, the last briefly discussed by Williams.[4] The cross sections for the first two processes are quite small and hard to verify experimentally, the first because of the difficulty in separating out the coherent scattering by the nuclear electrostatic field from other coherent nuclear and atomic-electron photon scatterings (however, see Wilson[5]), the second because of the lack of gamma-ray sources capable of furnishing enough photons. The cross section for the third process is no less small, but strong gamma-ray sources are available, high-Z nuclei furnish relatively large electrostatic fields, and energy discrimination can be made to eliminate unwanted inelastic scatterings which act as a background for the splitting process.

As calculated in the following, the cross section for the production of two photons oppositely directed and perpendicular to an original photon of energy 1.65 (in units of mc^2—0.84-Mev γ of Mn^{54}) incident on a nucleus of charge $Z=82$ is 2.3×10^{-33} cm^2/sterad^2. (We use energy units of mc^2 throughout.) Of the product photons, nearly all have energy between 0.4 and 1.3

* Assisted by the joint program of the U. S. Office of Naval Research and U. S. Atomic Energy Commission.
† Holder of Shell Fellowship 1952–1954.
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due to a factor $k_1{}^3k_2{}^3$ (a continuous range of energies for each of the two scattered photons is allowed by the conservation laws: $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}$ and $k=k_1+k_2+K^2/2M \approx k_1+k_2$). In the case of double Compton scattering from electrons[6] (a competing process with a cross section per atom very roughly $(2\pi^2)^{-1}\alpha(e^2/mc^2)^2Z=2.4\times10^{-27}$ cm^2/sterad^2), for the same configuration, the conservation equations for the energies of the two product photons have the form $\mathbf{k}=\mathbf{K}$, $\mathbf{k}_1=\mathbf{k}_2$, and $k+1=k_1+k_2+(K^2+1)^{\frac{1}{2}}$. For $k=1.65$, these give $k_1+k_2=0.72$. The energy of a single-Compton scattered photon under these circumstances is 0.62 (random coincidences might compete here because of the relatively huge cross section). The cross section for double Compton scattering from nuclei is relatively negligible (approximately $(2\pi^2)^{-1}Z^2\alpha[(Ze)^2/AMc^2]^2[kmc^2/AMc^2]^2 \approx 10^{-37}$ cm^2/sterad^2). Thus, by biasing counters to accept only photons in coincidence with each photon energy lying between 0.7 and $1.65-0.7=0.95$ and requiring that the two energies add to 1.65, most of the background scattering should be eliminated; moreover, if photons are accepted only in this energy range, the cross section is still about 7×10^{-34} cm^2/sterad^2.

This cross section for the splitting of a photon into two photons has been calculated (in lowest order in α) for small incident momenta ($k<1$) from the equivalent vacuum polarization Hamiltonian of Euler and others:[7]

$$H_I=-(\alpha^2/360\pi^2)[(\hbar/mc)^3/mc^2][(\mathbf{D}^2-\mathbf{B}^2)^2+7(\mathbf{D}\cdot\mathbf{B})^2]$$

of which the contributing terms are (nuclear electrostatic field $\mathbf{D}_n$, photon field $\mathbf{D}_p$, $\mathbf{B}_p$; $\mathbf{D}=\mathbf{D}_p+\mathbf{D}_n$, $\mathbf{B}=\mathbf{B}_p$):

$$-(\alpha^2/180\pi^2)[(\hbar/mc)^3/mc^2] \times[2\mathbf{D}_p{}^2\mathbf{D}_p\cdot\mathbf{D}_n-2\mathbf{B}_p{}^2\mathbf{D}_p\cdot\mathbf{D}_n+7\mathbf{D}_p\cdot\mathbf{B}_p\mathbf{D}_n\cdot\mathbf{B}_p].$$

By using a shielded potential for the nucleus,

$$\mathbf{D}_n=-Ze\,\mathrm{grad}\{[\exp(-r/a)]/r\},$$

the differential cross section for splitting of the incident photon of momentum $\mathbf{k}$ into two photons of momenta $(\mathbf{k}_1,\mathbf{k}_1+d\mathbf{k}_1)$ in $d\Omega_1$, $\mathbf{k}_2$ in $d\Omega_2$, is obtained (wavenumbers in units of $mc/\hbar$, a in units of $\hbar/mc$):

$$\sigma dx d\Omega_1 d\Omega_2=\frac{\alpha^5(\hbar/mc)^2Z^2k^6}{32\cdot81\cdot25\cdot\pi^4} \cdot\frac{x^3(1-x)^3(Ax^2+Bx+C)}{[\mathbf{K}^2/2k^2+1/2a^2k^2]^2}dx d\Omega_1 d\Omega_2 \text{ cm}^2,$$

$$\int_0^1 \sigma dx=Qk^6Z^2 \text{ cm}^2/\text{sterad}^2,$$

where $\mathbf{K}=\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2$, $k_2=k-k_1$, $x=k_1/k$, and A, B, C are functions of the angles involved, say, $\cos(\mathbf{k},\mathbf{k}_1)=\cos\theta_1=u$; $\cos(\mathbf{k},\mathbf{k}_2)=\cos\theta_2=v$; $\cos(\mathbf{k}_1,\mathbf{k}_2)=\cos\theta_{12}=w$. Then in terms of u, v, and $t=1-w$,

$$\mathbf{K}^2/2k^2=tx^2+x(v-u-t)+1-v,$$

and

$$\begin{aligned}A=\;&157t^3+t^2(-193+157v+157u-139v^2-139u^2+157uv)+t(278-278v\\&-278u+157v^2+157u^2-36uv),\end{aligned}$$

$$\begin{aligned}B=\;&-157t^3+t^2(193-278v-36u+278v^2-157uv)\\&+t(-278+363v+193u-278v^2-36u^2+36uv+157uv^2-157u^2v)+(v-u)(278\\&-278v-278u+157v^2+157u^2-36uv),\end{aligned}$$

$$\begin{aligned}C=\;&(1-v)\{t^2(18+139v)+t(85-121v-121u+157uv)+139-139v+157v^2\\&+18u^2-314uv+139u^2v\}.\end{aligned}$$

For the particular case $\theta_1=\pi/2$, $\theta_2=\pi/2$, $\theta_{12}=\pi$, Q is found to be 1.68×10^{-38} cm^2/sterad^2. The variation of Qk^6Z^2 with, for example, deviations of θ_{12} from π for fixed $\theta_1=\pi/2$, $\theta_2=\pi/2$, is relatively slow.

If $k>10^{-2}$, A, B, and C are very small in the region where the term $1/a^2k^2$ becomes significant (i.e., as $\mathbf{K}^2/2k^2\to0$). Then a rough estimate of the total cross section $\sigma(k)$ is $(4\pi)^2$ times the differential cross section for the configuration above, and works out to be about $0.7\times10^{-2}\alpha^5(\hbar/mc)^2Z^2k^6$ cm^2.

For high-energy incident photons ($k\gg1$), a simple calculation by the Weizsäcker-Williams method,[8] using the cross section for high-energy scattering of photons by photons derived by Achieser,[9] shows that

$$\sigma(k)=b\alpha^5(\hbar/mc)^2Z^2\log(\eta k) \text{ cm}^2,$$

where η is of the order of 1; a similar result using a cruder estimate of the cross section for high-energy photon-photon scattering $[\sim\alpha^4(\hbar/mc)^2$ instead of $\sim\alpha^4(\hbar/mc)^2(1/k_1k_2)]$ has been given by Williams.[4] If the expressions for small k and for large k are equated near $k=2$, then $b\approx1/2$.

The author wishes to thank Dr. H. Primakoff for suggesting this problem and for many helpful discussions.

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