respectively. Thus we get:

$$a_j = a_j^0 + (\partial a_j/\partial H)H + (\partial a_j/\partial \mu)_0\mu + (\partial a_j/\partial m_J)_0 m_J + \cdots$$
$$= a_j^0(1+pH+q\mu+rm_J+\cdots). \quad \text{(A-7)}$$

The normalization condition (17) gives

$$1 = S = \Sigma a_i^{0*} a_j^0 |1+pH+q\mu+rm_J|^2 S_{ij} + H^2(\cdots) + H\mu(\cdots) + \mu m_J(\cdots) + \cdots. \quad \text{(A-8)}$$

From this we obtain

$$0 = (\partial S/\partial H)_0 = \Sigma a_i^{0*} a_j^0 S_{ij}(p^*+p);$$

i.e.,

$$p^*+p=0,$$

which shows that p is purely imaginary. Similarly q and r are also shown to be purely imaginary. We can therefore write

$$a_j = a_j^0\{1+i(\alpha H+\beta\mu+\gamma m_J)+\cdots\}$$
$$= a_j^0 \exp\{i(\alpha H+\beta\mu+\gamma m_J)\} + (\text{second order terms}). \quad \text{(A-9)}$$

This shows that, if we multiply the wave function by an appropriate phase factor, the a's can be reduced to the form

$$a_j = a_j^0 + H^2 a_j^{(1)} + H\mu a_j^{(2)} + Hm_J a_J^{(3)} + \mu m_J a_j^{(4)} + \cdots. \quad \text{(A-10)}$$

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Electron-Electron and Positron-Electron Scattering Measurements

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A counter experiment is described which measures the absolute differential electron-electron scattering cross section in the energy interval 0.6 to 1.2 Mev and the absolute differential positron-electron scattering cross section in the energy interval 0.6 to 1.0 Mev. The ratio of these cross sections is also measured with somewhat increased accuracy. The technique of measurement combines good resolution with large energy transfers between the particles to permit a sensitive test of the relativistic features of the Møller and Bhabha formulas. The results verify the Møller formula within the 7 percent experimental error. The Bhabha formula is verified within the 10 percent experimental error. The ratio of the Møller to the Bhabha formula is verified within about 8 percent experimental error.

INTRODUCTION

THE purpose of this report is to collect and summarize the results obtained at this laboratory on electron-electron and positron-electron scattering experiments.[1] The object of these experiments was to check the Møller and Bhabha formulas which are the predictions for e-e and p-e scattering, respectively, based on the Dirac theory. When these experiments were begun the only previous work in this field had been done with cloud chamber techniques.[2] Such experiments were not adequate to check properly either formula, since the relativistic features of the scattering begin to be appreciable only at large energy transfers between the incident and scattered particles, and these are rarely seen in the cloud chamber.

It is customary to discuss scattering cross sections in terms of angular distribution, since this is usually the experimentally measured quantity. However, our apparatus measures directly the energy transferred in the collision and this is probably a more meaningful concept in this particular problem. In any event the conservation laws provide a ready means of going from one to the other. We prefer to talk about the fraction v of the incident kinetic energy transferred in the collision.

The present experiments were designed to study the scattering at large energy transfers using a counter technique. A method has been devised which gives simultaneously a high resolution and good solid angle.

Apparatus was first designed for e-e scattering work, and extensive measurements were made in the energy range 0.6−1.7 Mev with $v=0.5$. Due to the inability to distinguish incident from scattered particles in e-e scattering, the Møller formula is symmetrical about $v=0.5$, the largest distinguishable energy transfer. Thus $v=0.5$ represents the most favorable situation for checking the Møller formula. In the energy range 0.6 −1.7 Mev the electrons are sufficiently relativistic to check the essential features of the Møller formula, which include, besides the Coulomb scattering, additional contributions arising from spin interactions. Since the first reports of this work showing agreement

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[1] Lorne A. Page, Phys. Rev. **81**, 1062 (1951); A. Ashkin and W. M. Woodward, Phys. Rev. **87**, 236 (1952).
[2] Ho Zah-Wei, Compt. rend. **226**, 1083 (1948); Groetzinger, Leder, Ribe, and Berger, Phys. Rev. **79**, 454 (1950); The most recent cloud chamber experiment reported is: G. R. Hoke, Phys. Rev. **87**, 285 (1952).

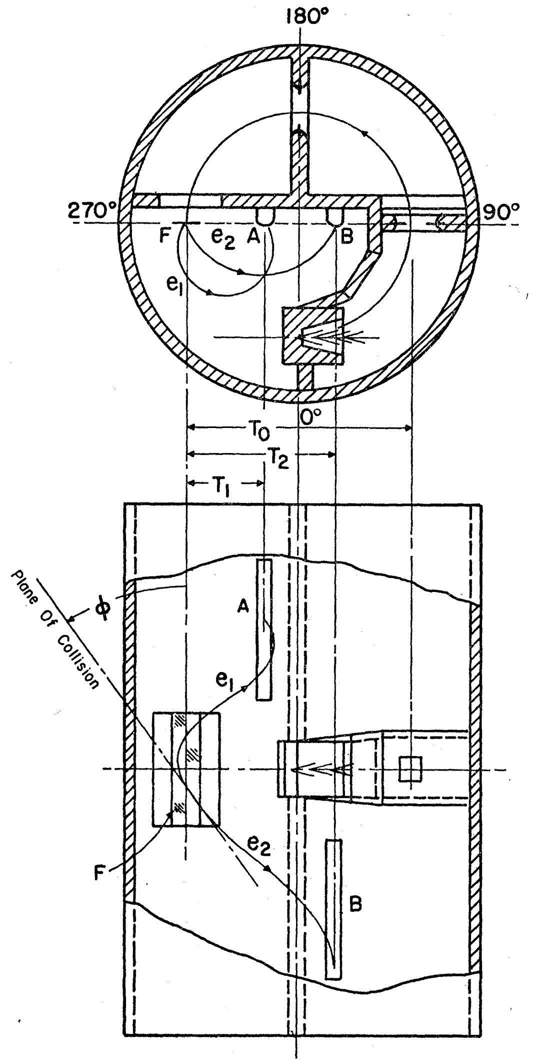

FIG. 1. Schematic view of the 270° scattering chamber used in electron-electron scattering measurements.

with the Møller formula,[1] several other experimenters, using electron accelerators,[3] have also found agreement at higher energies.

Following the work on *e-e* scattering, a new apparatus was built to handle the problem of *p-e* scattering. The positron-electron scattering shows many of the features of *e-e* scattering. The cross section is essentially Rutherford for small v, with deviations expected at large v. The cross section, however, is no longer symmetrical about $v=0.5$, since we have distinguishable particles. Also, in addition to the spin interactions found in the Møller formula, there exists the process called "virtual annihilation," or "exchange interaction." This is a contribution to the scattering cross section arising from the virtual annihilation and recreation of the positron and electron which go off in new directions. The net result is a scattering.

Since the first reports of this work showing agreement with the Bhabha formula,[1] a counter experiment was reported by Howe and MacKenzie[4] measuring the ratio of positron-electron to electron-electron scattering, giving good agreement with theory at 1.3 Mev.

APPARATUS

Geometry

Figure 1 gives a schematic view of the chamber used for the *e-e* scattering work. The chamber was evacuated to a mild vacuum and was surrounded by a solenoid and Helmholtz coils which provided a uniform magnetic field inside the tank. The top view shows the source placed at zero degrees. By a series of slits placed at 90° and 180°, an interval of momentum 12 percent wide was selected from the continuous β spectrum and brought to focus on the scattering foil F placed at 270°. Shown is an *e-e* collision taking place between one of the incident electrons and an atomic electron of the foil. The scattered particles are shown leaving the foil at some angles. They are bent in the magnetic field and are detected as a coincidence by two appropriately placed thin window Geiger counters A and B.

Since the horizontal displacement of the trajectory of the particle is determined only by the component of momentum perpendicular to the scattering foil and this is uniquely related to its kinetic energy, a counter placed parallel to the magnetic field will be sensitive only to particles of a particular energy. As one varies ϕ, the angle between the plane of collision and the vertical, the scattered particles travel along helical paths to different heights along counters A and B. Thus the fraction of all the scatterings observed with a particular v depends only on the length of counter used, and in principle can be made equal to unity with sufficiently long counters.

If, for example, one wants to study the differential scattering cross section for $v=0.4$, one places counter A at a distance from F such that it detects only electrons of energy$=0.4T_0$ where T_0=kinetic energy of the incident electron. By conservation of energy the other electron must have energy $0.6T_0$; therefore, B must be placed at a distance from F such that it detects only electrons of energy $0.6T_0$. Thus, for a fixed position of A there is a unique position for B. This property of the scattering we call the "coherence property." One can check its validity by studying the coincidence rate as a function of the position of B, for example, keeping A fixed. One expects to find a maximum at the position of coherence with a fall off on either side, the shape of which depends upon the particular geometry used. A typical curve is shown in Fig. 2.

Figure 3 gives a schematic view of the chamber used for the *p-e* scattering work. Here 180° rather than 270°

[3] Scott, Hanson, and Lyman, Phys. Rev. 84, 638 (1951); Barber, Becker, and Chu, Phys. Rev. 89, 950 (1953).

[4] H. A. Howe and K. R. MacKenzie, Phys. Rev. 90, 678 (1953).

focusing is used in order to permit more effective shielding of the gamma rays from the positron source.[5] Actually, the 270° geometry is better in one respect, namely that the 90° and 180° slits define the beam which then passes cleanly through the foil at the 270° window, thus reducing slit scattering in the neighborhood of the scattering foil and resulting in low singles rates and correspondingly low backgrounds.

Another modification dictated by the need for adequate gamma-ray shielding was the so-called helical geometry. This refers to the fact that in the positron apparatus the source S is displaced downward from the horizontal plane containing the foil (see Fig. 3). In this situation the incident positron follows a helical path from the source to the foil, where it strikes with an upward component of momentum. As a result the collision products have an added upward component of motion which permits the shifting of the counters upward, thereby providing more room for shielding.

A consequence of this helical geometry is that all scattered particles of a given energy are not brought to focus on a vertical line, but rather on a curve situated roughly symmetrically about a vertical line. This deviation from the simple straight line results in a smearing out of the energy resolution somewhat beyond what would be expected solely from the widths of the counters and source.

The positron chamber can also be used to study *e-e* scattering by merely reversing the magnetic field and placing counter C_2 on the other side of the foil.

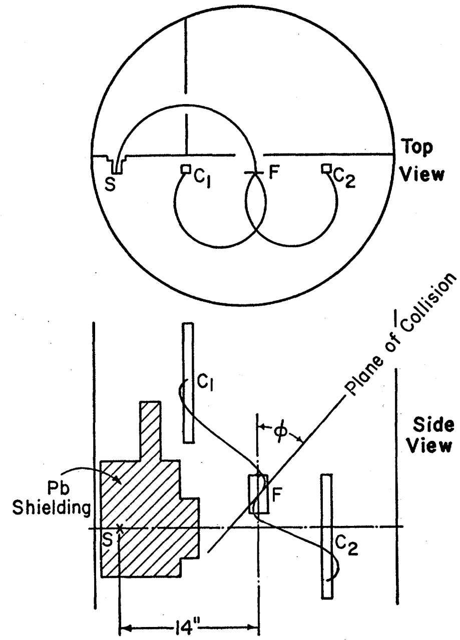

FIG. 3. Schematic view of the 180° scattering chamber used for both positron-electron and electron-electron scattering measurements.

Counters

The Geiger counters used in the 270° chamber were made of ½-in. square tubing with 0.001-in. aluminum windows. The filling was 80 cm Hg of helium with 1 cm Hg of ethyl alcohol. The Geiger counters used in the 180° chamber were made of 0.8-in. square tubing with windows prepared by evaporating aluminum on 0.0005-in. Mylar.[6] The filling was 7.5 cm Hg of argon and 1 cm Hg of ethyl alcohol. These counters, which had superior counting characteristics to the high-pressure ones, suffered from the disadvantage that they had to be filled each time after the chamber was evacuated.

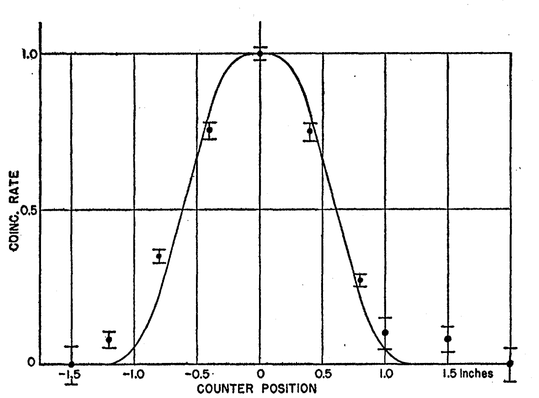

FIG. 2. A typical coherence curve.

Scattering Foils

With the 270° chamber two foils were used, 0.5 mm/cm² of collodion and 4.5 mg/cm² of beryllium. With the 180° chamber Mylar foils[6] of thickness 0.9 mg/cm² and 1.7 mg/cm² were used.

Source Strengths

The *e-e* scattering work using the 270° chamber was done with about 10 mc of $Sr^{90}-Y^{90}$. The *e-e* scattering work using the 180° chamber was done with about 2.5 mc of $Sr^{90}-Y^{90}$. The *p-e* scattering work was done with about 1 mc of Co^{56}.[7]

EXPERIMENTAL PROCEDURE

With the counters located for the desired energy sharing and the magnetic field set for the desired incident energy, one measures the coincidence rate and

[5] The Co^{56} positron sources which were used for reasons of energy and lifetime have approximately three energetic gamma rays for each positron. The $Sr^{90}-Y^{90}$ electron source is free of gamma rays.

[6] Mylar is a plastic film supplied by the DuPont Company. It is a polymer containing hydrogen, carbon, and oxygen.

[7] We are indebted to Professor Martin Deutsch, Massachusetts Institute of Technology; Professor P. V. C. Hough, University of Michigan; and Professor A. J. Allen, University of Pittsburgh for kindly providing us with the cobalt sources used in this work We also would like to thank Professor Norman Bonner of Cornell University for assistance with the chemistry of the source preparation.

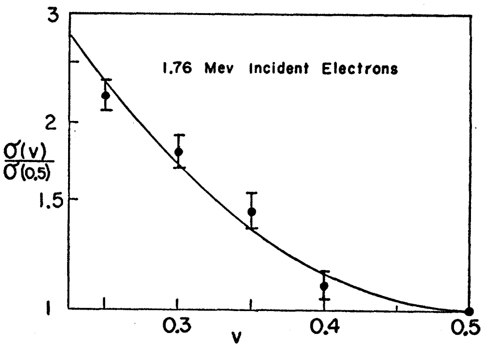

FIG. 4. Comparison with theory of the measured relative differential electron-electron scattering cross section as a function of the fractional energy transfer v taken with the 270° apparatus.

singles rates with the scattering foil in and out of the beam. The difference in coincidence rate, after making proper corrections for the dead time of the counters, is interpreted as being due either to *e-e* or *p-e* scattering. Source strengths were calibrated by placing a small hole at the position of the foil and counting the number of particles that pass cleanly through it with a Geiger counter placed some distance behind the hole. This measurement gives the source strength at one point on the foil. It is necessary to scan horizontally and vertically across the foil to determine the variation of the incident particle intensity with position on the foil.

Knowing the composition and weight of the foil, the above data on coincidence rate and absolute source strength is sufficient to determine the absolute differential cross section for *e-e* and *p-e* scattering. Actually, one must compute a geometrical factor before one can interpret the coincidence rate in terms of a cross section. This factor we call f, the efficiency against vertical loss. It is computed on the basis of the detailed orbits of the scattered particles from different parts of the scattering foil. This factor is a measure of the fraction of all the scatterings of a particular energy sharing which are detected by the counters. It differs from unity because of the restricted length of the counters.[8] As we saw before, this restricts the range of ϕ's which can be detected. This factor takes into account the variation in efficiency with which different parts of the foil are counted. It includes any variations in the number of particles incident at different points on the foil and the variation of the cross section with position on the foil. In the 180° apparatus it includes the smearing out of the energy resolution due to the helical geometry.

The accuracy of the vertical loss factor was checked experimentally in several ways.

In taking data at a particular energy sharing between the particles, for example (0.60−0.40), one can use two possible arrangements of counter positions. One can place the upper counter at a position where it detects particles of energy $0.4T_0$ and the lower at $0.6T_0$. Alternatively, one could place the upper at 0.6 and the lower at 0.4. Either way we should measure the same cross section. This implies that the ratio of the observed counting rates should be equal to the ratio of the computed f's for the two situations. One finds

$$\frac{\text{counts/min}(0.60-0.40)}{\text{counts/min}(0.40-0.60)}=1.55\pm0.09.$$

The computed ratio of the f's is

$$\frac{f(0.60-0.40)}{f(0.40-0.60)}=1.53.$$

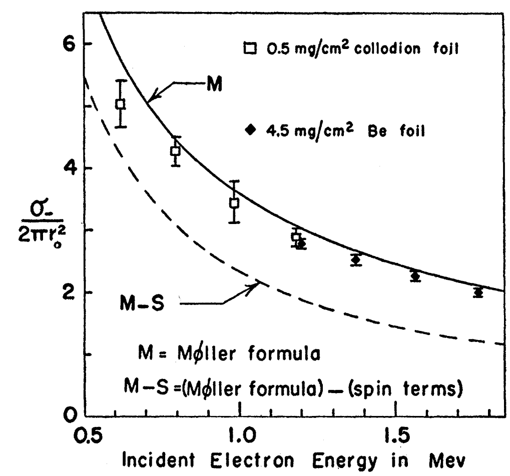

FIG. 5. Comparison with theory of the measured absolute differential electron-electron scattering cross section at $v=0.5$ as a function of the kinetic energy of the incident electron taken with the 270° apparatus.

Another check is to change the actual length of the upper counter in the situation (0.60−0.40). The ratio of the counting rates should again equal the ratio of the computed f's for this situation. One finds

$$\frac{\text{counts/min}(\text{unblocked})}{\text{counts/min}(\text{blocked})}=2.16\pm0.14,$$

whereas

$$\frac{f(\text{unblocked})}{f(\text{blocked})}=2.16.$$

A third check was to measure the *e-e* scattering absolutely with the 180° apparatus. This agreed well with that measured on the 270° apparatus although the f's in the two geometries were entirely independent. This added check particularly gives us confidence in our understanding of the apparatus.

Care was taken to avoid errors due to multiple scattering in the scattering foil. A rather sensitive indication of the presence of multiple scattering is the

[8] The counter lengths had to be restricted for various reasons, such as having to have both counters in the same position, shielding, and the size of the useful magnetic field.

broadening of the coherence curve beyond the computed width. Another check is to study the counting rate as a function of the thickness of the scattering foil wherever possible.

RESULTS

The Møller formula[9] can be written in the following form:

$$\sigma_M(\gamma,v)dv=2\pi r_0^2\frac{\gamma^2}{(\gamma-1)^2}\frac{dv}{(\gamma+1)}\times\left[x^2-3x+\left(\frac{\gamma-1}{\gamma}\right)^2(1+x)\right],$$

where

$$r_0=e^2/\text{mc}^2,\quad \gamma=E_{\text{total}}/\text{mc}^2,\quad E_{\text{total}}=\text{mc}^2+T,\quad x=[v(1-v)]^{-1}$$

and v is the fractional energy transfer. The Bhabha formula[10] can be written in the following form:

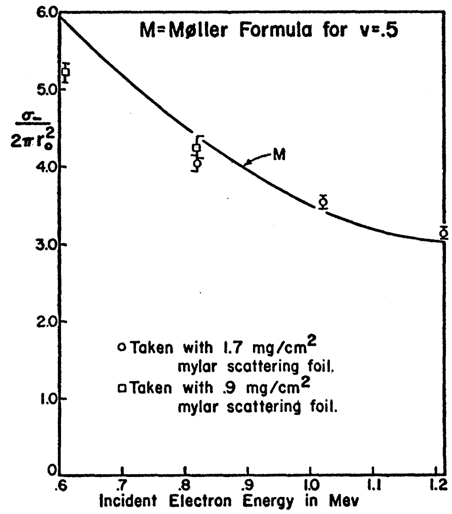

FIG. 6. Comparison with theory of the measured absolute differential electron-electron scattering cross section at $v=0.5$ as a function of the kinetic energy of the incident electron taken with the 180° apparatus.

$$\sigma_B(\gamma,v)dv=2\pi r_0^2\frac{dv}{(\gamma+1)}\left[\frac{1}{v^2}\left(\frac{\gamma}{\gamma-1}\right)^2-\frac{1}{v}\frac{2(\gamma+1)^2-1}{\gamma^2-1}+\frac{3(\gamma+1)^2+1}{(\gamma+1)^2}-v\frac{2\gamma(\gamma-1)}{\gamma^2+1}+v^2\left(\frac{\gamma-1}{\gamma+1}\right)^2\right],$$

where r_0, γ, and v are the same as above.

If one omits the contributions of the virtual annihilation terms, the Bhabha formula becomes

$$\sigma_{B-A}(\gamma,v)dv=2\pi r_0^2\frac{dv}{(\gamma+1)}\left[\frac{1}{v^2}\left(\frac{\gamma}{\gamma-1}\right)^2-\frac{1}{v}\left(\frac{\gamma+1}{\gamma-1}\right)+\frac{1}{2}\right].$$

[9] C. Møller, Ann. Physik 14, 531 (1932).
[10] H. J. Bhabha, Proc. Roy. Soc. (London) A154 195 (1936).

Figure 4 shows the comparison with theory of the measured relative differential *e-e* cross section as a function of the fractional energy transfer v. These data were taken with the 270° apparatus for fixed incident electron energy of 1.76 Mev. They were normalized to $v=0.5$. Similar agreement was found for 1.15-Mev incident energy.

Figure 5 shows the comparison with theory of the measured absolute differential *e-e* cross section at $v=0.5$ as a function of the kinetic energy of the incident electron taken with the 270° apparatus. Curve M is the Møller formula. Curve $M-S$, which is the Møller formula with the spin terms deleted, is drawn to indicate the magnitude of the spin terms. The preference of the data for the Moller formula is apparent.

Figure 6 shows results similar to those of Fig. 5, only this time taken with the 180° apparatus. We notice that the two high-energy points taken with the 1.7-mg/cm² foil are in good agreement with theory. The 0.82-Mev point with this foil thickness lies a little low, presumably because of multiple scattering, since a reduction in the foil thickness to 0.9 mg/cm² improves the agreement with theory. The point at 0.61 Mev is low, presumably because of multiple scattering, even with the 0.9-mg/cm² foil.

In Fig. 7 is displayed the measured energy dependence of the absolute positron-electron cross section for $v=0.5$ in units of $2\pi r_0^2$ taken with the 180° apparatus. Curve B is the prediction of the Bhabha formula. Curve $B-A$ is the prediction of the Bhabha formula less the annihilation term referred to above. As in Fig. 6 for the absolute electron measurements, the agreement with theory is good at the higher energies, but the value at 0.61 Mev is somewhat low, presumably because of multiple scattering. There seems to be little doubt that the

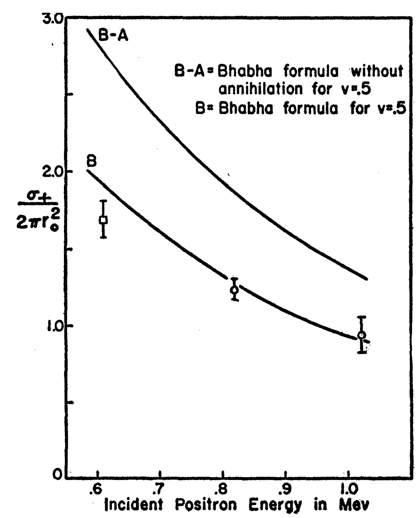

FIG. 7. Comparison with theory of the measured energy dependence of the absolute differential positron-electron scattering cross section for $v=0.5$, taken with the 180° apparatus.

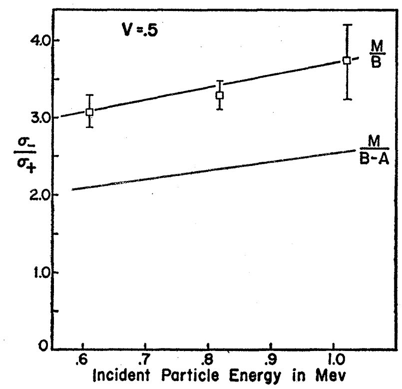

FIG. 8. The ratio of the differential electron-electron to positron-electron cross section, for $v=0.5$, taken with the 180° apparatus. This combines the results of Figs. 6 and 7.

measurements fit the Bhabha curve much better than the $B-A$ curve.

In Fig. 8 we have plotted the ratio of the $e-e$ to the p-e cross section as measured with the 180° apparatus. This combines the results of Figs. 6 and 7. The comparison of the data with the ratio of the Mφller to Bhabha formulas and the Mφller to Bhabha less the annihilation term is also shown. The experimental points are in quite good agreement with the M/B curve. Even the 0.61-Mev point lies on the theoretical curve, which serves to substantiate the supposition that both absolute numbers for the cross section at this energy were low because of multiple scattering.

Figure 9 shows the measured ratio of electron-electron to positron-electron cross section as a function of v for a fixed incident energy of 0.61 Mev. Curves M/B and $M/B-A$ mean the same as before. The experimental evidence again favors M/B over $M/B-A$.

ERRORS

The errors indicated in the above figures are the standard statistical counting errors. These must be combined with other experimental errors to give an over-all error. For the absolute e-e cross section the total error is estimated to be ± 7 percent, while the relative error is about ± 5 percent. The absolute positron-electron cross section has a total error of about ± 10 percent with a relative error of about ± 7 percent. This latter statement is not true, of course, at the points where the positron counting errors approach ± 10 percent.

The measurement of the ratio of the Mφller to the Bhabha formula has an error of about ± 8 percent.

The errors to be associated with the e-e and p-e cross sections as a function of v are the relative errors of ± 5 percent and ± 7 percent, respectively, except at a few points where the statistics are a little worse.

CONCLUSIONS

The Mφller formula is verified in the energy interval 0.6 to 1.2 Mev to an accuracy of about 7 percent standard deviation. The existence of the spin terms is established.

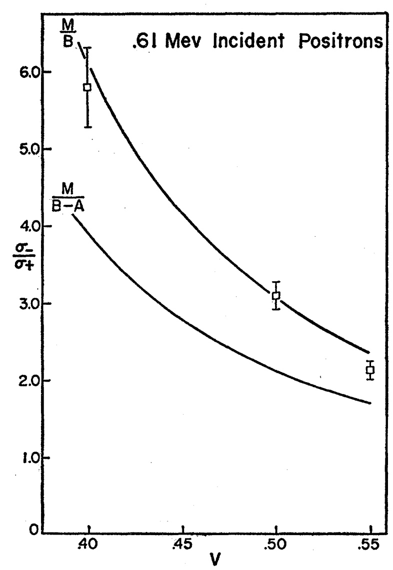

FIG. 9. Comparison with theory of the measured ratio of the differential electron-electron to positron-electron cross section as a function of v for an incident energy of 0.61 Mev.

The Bhabha formula is verified in the energy interval 0.6 to 1.0 Mev to an accuracy of about 10 percent standard deviation. The existence of the virtual annihilation term is established.

The ratio of Mφller to Bhabha formulas is verified to within about 8 percent experimental error.