PHYSICAL REVIEW VOLUME 94, NUMBER 2 APRIL 15, 1954

Some Aspects of Breakdown Streamers*

LEONARD B. LOEB
Department of Physics, University of California, Berkeley, California
(Received December 7, 1953)

Photomultiplier and oscillographic study of positive breakdown streamers yields values of current and distance of tip advance as a function of time. From these the relatively constant number n of excess positive ions left behind by the streamer per cm advance and the tip velocity v_t are obtained. These are related to the linear electron density n_c per cm of streamer channel plasma and the electron drift velocity v_c in the channel by $n_cv_c=nv_t$, with $v_t>v_c$. Since v_c is related to the gradient X_c in the channel by well-known drift velocity data, it is possible to limit the range of values of n_c and v_c. The limiting conditions are that $n_c>n$, $\int_0^x X_c dx<V_x$, where V_x is the applied potential, and finally that X_c must be sufficiently large to maintain the channel conducting despite dissociative recombination and dissociative attachment losses. In longer channels heating cannot be neglected. The dense positive space charge left behind by the electron current in the ionized channel of radius R causes a radial expansion of the excess space charge to a radius R' by incoming electron avalanches leading to a broad transient luminosity for some distance behind the tip. The tip and the conducting channel taking the return stroke, however, retain their small radius R.

THE streamer mechanism of breakdown leading to filamentary sparks initially proposed by H. Raether and independently by J. M. Meek and the writer, has been generally fairly well established through the observations of Lawrence and Dunnington,[1] by Raether[2] for uniform field geometry and by the writer and his students in corona studies, and by Allibone and Meek[3] and Meek and Saxe[4] in point to plane impulse breakdown. The studies of Fisher[5] and his associates and Bandel[6] in the writer's laboratory have shown that while very near threshold the breakdown starts as a low-order Townsend discharge with photoelectric liberation at the cathode, the eventual filamentary spark comes from a streamer breakdown in the space-charge distorted gap. Thus while in plane parallel geometry the threshold for breakdown is set by that for a Townsend discharge, the mechanism at all times is a streamer mechanism. In overvolted plane parallel gaps and in all asymmetrical gap breakdown the breakdown begins with streamer formation, and in some cases the sparking threshold is set by the threshold for a streamer formation.

While both anode and cathode directed streamers occur in midgap breakdown with heavily overvolted plane parallel gaps, by all odds the most effective streamer process, in virtue of its conservative nature, is the cathode-directed positive streamer. It is a study of the more detailed properties of this streamer process that forms the basis of this paper.

Such streamers have been studied in the positive point stepped leader strokes of lightning discharges by McEachron,[7] in the positive point-to-plane impulse breakdown streamer by J. M. Meek and R. Saxe, and recently for the preonset positive corona streamers by Amin[8] in the writer's laboratory. These studies yield certain quantitative data from which the inferences in this article derive.

In brief the mechanism of the streamer process is as follows: Assume that near the anode the geometrically determined field distribution, and the density and mean free path of gas ionizing photons created by the avalanche are adequate. Then the arrival of one adequate electron avalanche, or a suitable convergence of avalanches at the anode, leading to a limiting accumulation of positive ion space charge density and a sufficient number of active photons, permit a streamer to start. Under these conditions the vector addition of the space charge field and the initiating anode gradient are such as to cause the photoelectrons generated by the avalanches to initiate new avalanches along the field axis as these converge on the space charge. Thus the space charge will propagate itself outward towards the cathode as a positive streamer. The important factors in the advance are adequate photon density, the strength of the space charge tip field, and its extent relative to the photoionizing free path. The idealized quantitative threshold for such a process has been derived by the writer and Wijsman.[9]

If the streamer tip field is maintained adequate by current supply from the anode and by appropriate field conditions in the gap, it will advance across the gap to

---

* Some of the investigations underlying this paper were supported by the U. S. Office of Naval Research and by the Research Corporation of America.

[1] E. O. Lawrence and F. G. Dunnington, Phys. Rev. **35**, 396 (1930).

[2] H. Raether, Ergeb. exakt. Naturwiss. **22**, 86 (1949); H. Raether and E. Flegler, Z. tech. Phys. **16**, 436 (1935); Z. Physik **103**, 315 (1936); **104**, 219 (1936). H. Raether, Z. Physik **112**, 463 (1939).

[3] T. E. Allibone and J. M. Meek. Proc. Roy. Soc. (London) **A166**, 97 (1938); **A169**, 246 (1938).

[4] J. M. Meek and R. F. Saxe, Nature **162**, 263 (1948); Allied British Industries and Research Association Report, Section *L*-Dielectrics in General, Reference *L/T* 183 (unpublished).

[5] L. H. Fisher and B. Bederson, Phys. Rev. **81**, 109 (1951); G. A. Kachickas and L. H. Fisher, Phys. Rev. **82**, 318, 519 (1951); **88**, 878 (1952); **91**, 775 (1953).

[6] H. W. Bandel, Phys. Rev. **93**, 649(A) (1954).

[7] K. B. McEachron, J. Franklin Inst. **227**, 149 (1939); also, personal communications between B. F. J. Schonland and L. B. Loeb, February 21 to August 11, 1952.

[8] M. R. Amin, J. Appl. Phys. **25**, (1954).

[9] L. B. Loeb and R. J. Wijsman, J. Appl. Phys. **19**, 797 (1948).

the anode with its relatively dense plasma of electrons and positive ions. At the point of juncture of the streamer with the cathode, the steep potential gradients resulting cause a wave of ionization to move from cathode to anode along the streamer plasma channel. This wave, called the return stroke in lightning and observed by Meek and Saxe in long sparks and in this laboratory on short ones, progresses with velocities up to 10^{10} cm/sec, the velocity depending on the potential gradient at juncture and electron density in the plasma. It increases the existing ion densities of the order of 10^{12} electrons/cm^3 in the streamer channel to the 10^{17} or more corresponding to total ionization characterizing the brilliant filamentary spark.

In what follows, it is our intention to discuss the conditions in the streamer during the advance from anode to cathode. In Fig. 1 is shown the potential fall along the streamer channel in a gap with a streamer that has nearly crossed the gap. The dashed curve A is a pictoralization of the concept which has generally been assumed under varying conditions. Here the plasma is considered very highly conducting, and caculations for example have been made as to the fields ahead of lightning stroke channels assuming it to be equipotential with the anode. Actually the measurements of Meek and Saxe[4] indicate a considerable current flow as streamers cross a 50-cm gap, so that for reasons to be given the actual conditions in the channel must be more like those shown in the solid curve B of Fig. 1. A further peculiar paradox appears in the observations of stepped leader strokes in lightning discharge, and in photomultiplier cell studies of streamers by Meek and Saxe. Here, while the actual observations of the diameter of return stroke channels in lightning show that it is small (20 cm and 0.3 cm, respectively), the diameters of the stepped leaders are some 10 m and photomultipliers show the streamers to have 20-cm diameter. Amin, on preonset streamers, observed shoulders of transient luminosity extending in time well beyond that of the sharply defined tip luminosity. While the advancing avalanche in Raether's cloud track pictures has a certain small diameter, once the anode and cathode directed streamers start to advance in midgap breakdown of overvolted gaps, there is a sudden swelling of the condensed cloud to at least twice the original avalanche radius about such streamer tip, though the later actual spark channels are narrow. The interpretation of these contradictory observations constitute the purpose of this article.

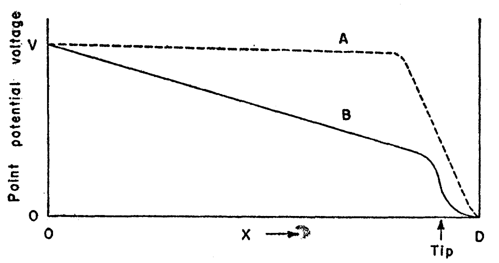

FIG. 1. Potential V of the breakdown streamer channel as a function of distance x from the anode in a gap of length D. Dashed curve A as frequently depicted in the past. Full curve B as indicated by observations of Meek and Saxe and Amin.

Experiment permits measurment of the time between initiation of the streamer at the anode (electrically indicated on the oscillograph) and its arrival at a slit normal to the streamer path and at a measured distance from the anode viewed by photomultiplier and recorded by oscillograph. The plot of distance-time curves at once yields the value of the tip velocity v_t at any point of streamer advance. Simultaneous measurement of current to the anode yields the current i at any distance x from the anode, so that current and tip velocity can be related at that point. Meek and Saxe observed that, except at the start, current and tip velocity increased in proportion for gaps up to 50 cm as the tip advanced. For gaps greater than 50 cm, Allibone and Meek had observed the velocity at first to increase and then achieve a constant value. Amin, with preonset streamers that cross only part of the gap before positive ion space charge distortion near the plane cathode deflects and stops them, observed an initial high velocity followed by a decline as the streamer approached the end of its run. Kip,[10] English,[10] and Amin[8] for preonset streamers, and Meek and Saxe for impulse breakdown streamers for positive point in gaps under 50 cm, observed that the *quantity of charge that flowed per unit length of streamer path, n*, was remarkably constant for a given streamer type. It however varies as between lightning, breakdown, and preonset streamer types. Amin also observed that in general the intensity of luminosity produced by excitation in the corona discharges parallel ionization, and that the light production as the streamer tip advanced was remarkably constant except at the very beginning and during the cessation of advance. It is then possible to state that the quantities of net positive charge n left behind in the channel per *cm* streamer advance and v_t, the tip velocity of the streamer advance, can be measured. It is also likely that in a given streamer type both (n_c+n), the total ionization per cm, and n, the net positive charge per cm left behind in the current, are nearly constant once the streamer is properly launched in its career and advances.

The range of values of observed streamer tip velocities extends roughly from 5×10^6 to in excess of 10^8 cm/sec, with most values above 10^7 cm/sec. Now the current i flowing up the channel requires that there be a potential v_c and a potential gradient X_c active down the channel to maintain it. The drift velocity v_c of the electrons in the channel is related to the field X_c at a given pressure p by the law $v_c=A(X_c/p)^{\frac{1}{2}}$ in the range

---

[10] A. F. Kip, Phys. Rev. **55**, 599 (1939); W. N. English, Phys. Rev. **71**, 648 (1947).

of velocities indicated, where the values of X_c/p and v_c can be taken from observations such as those of Bradbury and Nielsen,[11] or else may be calculated roughly from theory. If v_c is set equal to v_t, then even at 10^7 cm/sec the corresponding values of X_c/p are very high, such that if gas density in the channel is constant, (no heating), the values of X_c observed integrated over the length of the streamer exceed the applied gap potential V considerably. It is thus clear that the drift velocity v_c of the electrons in the channel is less than v_t, the velocity of advance of the tip, which depends on the tip field and mean free path for photoionization. Now since the measured current $i=nev_t$ it must also be related to the number of conducting electrons n_c per unit length of streamer and the drift velocity v_c of those electrons. Hence it is necessary to write that

$$i=nev_t=n_cev_c,$$

such that

$$n=n_cv_c/v_t.$$

With $nv_t=n_cv_c$, it is seen that evaluation of n and v_t yields n_cv_c but does not yield any direct knowledge of either v_c or n_c. However, there are a number of factors relating to the values of n_cv_c and X_c that will serve to help fix n_c and v_c in order of magnitude or better. If the ion production per cm of streamer is in fact constant over most of the length, then the uniformity of the streamer channel may be assumed, so that in what follows the field strength X_c causing the current flow may also be assumed constant along the channel as in Fig. 1.

As the streamer advances, the total number of positive ions and electrons created per cm length of tip advance is given by $n+n_c$. Of these, n electrons are drawn from the channel as current, yielding the charge $ne=\int_0^t idt/x$ electrons per cm of streamer that reach the anode and a linear excess positive ion charge of n per cm along the channel. The number of electrons left behind is n_c and these carry the current along the channel as the streamer advances with a velocity v_c. In consequence of the conditions outlined, the following limiting considerations apply:

(1) Values of v_c must be such, at the existing channel temperature and constant pressure p, that the reduced value of X_c/p corresponding to the potential fall down the channel X_c obeys the condition:

$$\int_0^x X_c dx=V_x,$$

where V_x is the potential drop across the channel of length x. This sets an upper limit to v_c and a lower limit to n_c.

(2) It is clear that n_c must be at least equal to n and probably should be greater than n_c, for it is unlikely that more than half the electrons will be withdrawn from a plasma initially as dense in ionization as $n+n_c$ divided by the cross-sectional area of the channel, i.e., $\sim 10^{12}$ ions/cm^3. This yields a lower limit to n_c in relation to n.

TABLE I. Estimated time in microseconds for recombinative reduction in ion concentration under various assumptions.

$\frac{n_t}{n_0}$	α cm^3/ion sec	n_0	t μsec
0.1	1×10^{-6}	10^{15}	10^{-2}
		10^{13}	1
	1×10^{-8}	10^{15}	1
		10^{13}	100
0.01	1×10^{-6}	10^{15}	10^{-1}
		10^{13}	10
	1×10^{-8}	10^{15}	10
		10^{13}	1000

(3) X_c must be of such value that with the n_c electrons there is enough ionization by collision present to make up losses from the channel caused by (a) diffusion (generally small in $\sim 10\mu$sec), (b) recombination of electrons and molecular ions, (dissociative recombination), and (c) electron attachment to molecules, (dissociative attachment). This condition sets a minimum value to X_c and to v_c, and a maximum value to n_c. It must also be noted that, with the possible exception of the stepped lightning leader stroke, the streamer tip and a short region immediately behind it only are luminous enough to be observed and that the streamer channels under consideration much behind the tip are nonluminous. Hence much active ionization and excitation cannot occur, and this also influences the maximum value of X_c and of temperature in the channels.

The decay processes of (3) will be discussed first, since some definite decisions may be derived. Of these, only dissociative recombination and dissociative attachment are very effective in air, which will be the exemplary gas treated. Considering the recombination in air, data are as yet incomplete. Biondi and Brown,[12] in microwave plasma in O_2, observed coefficients α of the order of 10^{-7} cm^3/ion sec which might be ascribed to this loss mechanism. The loss follows a time decay according to $n_t=n_0/(n_0\alpha t+1)$, where α is the coefficient of recombination, n is the initial ion density, and n_t that after t sec. On this basis Table I has been prepared, showing the time in microseconds for decay of n_0 to n_t in ratios 0.1 and 0.01 for the possible range in values of α and ion densities n_0 found in streamer channels. It is seen that an appreciable decay of n_t/n_0 cannot be expected in streamer channels under a matter of microseconds. Since the advance of all but very long streamers

---

[11] R. A. Nielsen and N. E. Bradbury, Phys. Rev. **49**, 338 (1936); R. A. Nielsen, Phys. Rev. **50**, 950 (1936).

[12] M. A. Biondi and S. C. Brown, Phys. Rev. **76**, 1697 (1949).

TABLE II. Delimitation of electron concentration in streamer channels for various streamer types.

Streamer type	Net charge per cm	v_t cm/sec	n_c/n assumed	v_c cm/sec	X_c/p from v_c volts/cm per mm	T°K assumed	X_c volts/cm	i amp
Corona preonset positive	8×10^9	4×10^7	8	5×10^6	10	300	7600	0.06
Breakdown point to plane positive	5.5×10^{12}	2×10^6	1	2×10^6	3	300	2280	2
		1×10^7	1	1×10^7	25	1200	4600	10
		1×10^7	2	5×10^6	10	470	4000	10
Stepped lightning negative	5×10^{13}	1.6×10^8	20	8×10^6	20	795	5070	1200

such as in the longest sparks and lightning discharge lies under this time scale, recombination will not figure seriously as a loss factor other than in such channels. Attachment of electrons to form O_2^- ions in the relatively hot streamer channels is unlikely because of the low energy of such ion formation. On the other hand, Harrison and Geballe[13] have observed cross sections for dissociative attachment of electrons of 3 volts energy in air with O_2 molecules to form O^- ions to be about 3×10^{-20} cm². The average energy of electrons in air at an X/p of the order of 20 to 30 lies around 3 ev. Attachment is detected down to an $X/p=20$, where it leads to a value of η/p of 5×10^{-3} attachments per cm advance in the field at 1 mm pressure. At $X/p=50$ the value of $\eta/p=1\times10^{-2}$. At 760 mm in air $\eta=3.8$ at $X/p=20$, so that the mean free path L for attachment is 2.6 mm. As electrons attach in such encounters according to $n_x=n_0e^{-x/L}$, the value of x to reduce n_x/n_0 to 0.1 will be 5.8 mm. With a drift velocity of 1×10^7 cm/sec, this reduces the electron concentration for electron energies of 3 volts to 0.1 in 5.8×10^{-8} sec, with electrons advancing 5.8 mm in that time. This process, then, is a serious source of electron removal as long as the average electron energy remains above 3 ev. Once the current ceases to flow and X_c/p falls below 20 or less, all electrons of the swarm above 3 ev stand a good chance of attaching as the electron energy falls below 3 ev in some 10^{-7} sec. Electrons of lower energy will not attach. The effectiveness of such attachment is clearly shown in the Trichel pulse corona at 760 mm where enough electrons attach to form O^- ions and choke off the discharge in 10^{-8} sec within 0.5 mm of the point. It should be noted that once ionization ceases with much dissociative attachment, perhaps half the negative carriers or more will be O^- ions. The ion-ion recombination with positive ions having a large cross section can, at high ion densities, produce a large decrease in carriers in intervals of microseconds.

TABLE II. Radial expansion limits of various streamer channel types.

Streamer	ions/ cm	Channel R radius cm	X_r kv/ cm	$\frac{X_r}{p}$	Channel should expand to R' cm	Radius of observed glow in cm
Pre-onset (+)	5×10^9	1×10^{-2}	150	200	1.0×10^{-1}	Some expansion observed by Amin
Breakdown (+)	4.5×10^{12}	0.5	2600	3400	85	10
Stepped leader (−) or (+)	5×10^{13}	10	1500	2000	1000	100–900

To prevent such loss of electrons, there must be a value of X_c/p in the channel sufficient to create new electrons for all those captured and lost. Harrison and Geballe show that the number of electrons produced by electron impact per cm is less than η until X/p exceeds 32.5 in air. However, the electron current observed amplifies below this, so that some ionization by collision is detectable down to an $X/p=20$. Thus, if the density of excitation and ionization in the channel is so low that ionization of excited states, i.e., ionization by successive impacts, cannot occur, the value of X_c/p should be 20 or above for streamer channels in air. In dense and highly excited channels such as possible in the photographed stepped leader channels of lightning, X_c/p may not have to be so high.

It is now of interest to consider these limitations as applied to observed streamer processes. Before doing so it must be noted that, where currents and current densities in the streamer channel become high and streamer advance lasts of an order of a microsecond, the energy input into the channel of dense current flow can lead to an appreciable heating of the channel. Since pressure is constant, say at 760 mm, the density of the gas is reduced. This decreases the value of X_c needed to yield the effective value of X_c/p causing a drift velocity v_c. Calculations made on the basis of Meek and Saxe's data indicate that heating can take place at 40-cm advance out of the 50-cm gap length to the extent of some 1200°C. Much higher temperatures are precluded since there is no visible luminosity of the channels. The rise in temperature must come as a result of ion movement in the field through collisions

---

[13] M. A. Harrison and R. Geballe, Phys. Rev. **91**, 1 (1953).

with molecules, only some few percent coming from excitations and ionization. The rise to 1200° would reduce the value of X_c needed at 760 mm by a factor of four. Such heating does occur in lightning streamer channels and long sparks where time intervals exceed a microsecond. In the short time intervals of Amin's preonset streamer, advance heating is unlikely.

Table II gives the results of an attempted analysis on three known types of streamers and clearly indicates the limitations considered. Unfortunately data are meagre. It is likely that the velocities of Meek and Saxe's streamers are too low as given. These were estimated from uncorrected time-distance curves of Meek and Saxe by the writer. The correction would certainly have increased the initial velocities by as much as a factor of two, and perhaps more. The data from lightning represent average values and are thus not necessarily self-consistent as to relation of charge, velocity, etc., thus yielding too high a value for the current by a factor of two. It is to be noted that Amin's streamers *which extinguish after some 1.6 cm travel* have an X_c/p of the order of 10, which is inadequate to produce much ionization. The value of X_c of 7600 volts is consistent with the potential drop during the 1-cm advance before current declines and fulfills the $\int X_c dx < V$ requirement. The underestimate of Meek and Saxe's tip velocities at the start gives low values of X_c/p and of X_c. For the same streamer, after 40 cm travel across the gap with 10 amperes of current and a velocity which could easily be 1.3×10^7 cm/sec instead of 1×10^7 cm/sec, n_c/n lies above unity and is more nearly two with X_c/p above 20 and with appropriate heating yielding $X_c = 4000$, a value in harmony with the applied potential. The data in the case of the stepped lightning leaders are not seriously inconsistent and appear reasonable. Actual estimates of X_c for lightning stroke channels range from 1000 to 2000 volts/cm by Schonland,[14] to 3000 by McEachron.[7] With the high electron densities in such channels, these estimates are not unreasonable. In any event, the examples show the type of analysis possible and indicate a need for more comprehensive and accurate measurements of v_t and n for different streamers.

So far the linear number density n_c and n of carriers, or better the number of charges per cm length of streamer, have been dealt with by leaving out of consideration the volume number density. The difficulty in defining the volume density arises from the uncertain values of the diameter of the streamer channels. From theory it would be expected that the diameter of the advancing streamer tip should remain relatively small, because of the required charge density for propagation as well as because of the limited photoionizing free paths and the limited electron diffusion radius of the initiating avalanche head. Further evidence that the actual current-carrying section of the streamer is confined to narrow channels, comes from the diameter of the spark channel illuminated by the return stroke. This indicates that the region in which enough electrons exist for the rapid propagation and multiplication of the return stroke is confined to a small-radius channel along which X_c maintains the channel conducting. If now the observed radius R of the spark channel is taken as that of the streamer tip, evaluation of the quantity n at once permits calculation of the charge density and radial surface field at a distance R from the axis. Such a calculation is shown in Table III for the three types of streamers. It is seen that, with the values of n given, the radial fields at R yield values of X_r/p which are very impressive.

These fields will of course not materialize, for, as soon as X_r/p reaches values in excess of perhaps 35 in air, ionization by electron avalanches converging radially toward the streamer channel of radius R will spread the positive space charge radially outward, the electrons converging into R and some contributing to the current of n_c electrons moving up the streamer channel. Assuming that spreading ceases when X_r/p falls to 20 in the expanded channel, a radius R' of each channel can be calculated. This radius R' carries most of the *excess positive charge of n per cm* at a density of roughly 5×10^{10} ions per cm³. Now it has been observed

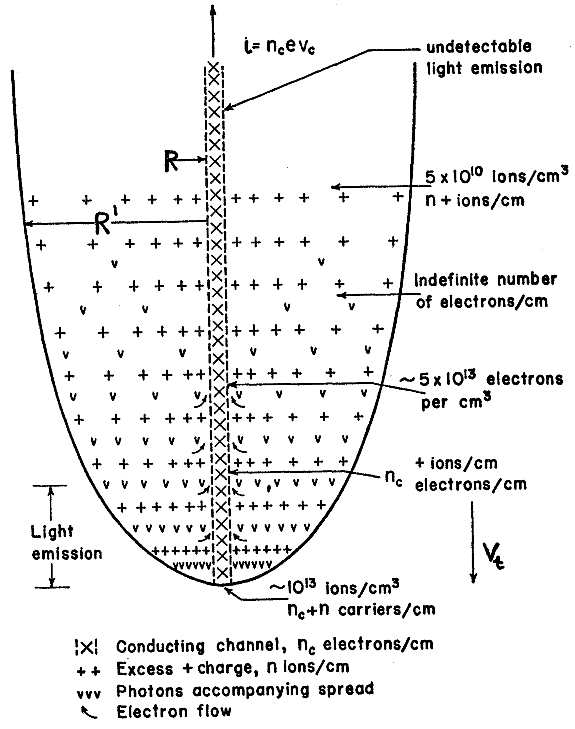

FIG. 2. Schematic drawing of the breakdown streamer tip. The central conducting channel of radius R is indicated as well as the transient luminous flaring of the excess positive space charge by radial flow of electron avalanches. The tip advances downward and the luminosity expands radially with decaying intensity from R at the tip to R' and well behind.

---

[14] B. F. J. Schonland (private communication).

by Meek and Saxe that, during the passage of the streamer tip in long sparks, there is a cusp-shaped luminosity that extends out radially at least 10-cm beyond the streamer axis as sensed by their photomultiplier. After this passes, the streamer channel is largely dark until the return stroke illuminates a channel of some 3-mm radius or less. In the case of lighting discharge, the radius of photographed vigorous steps with heavy currents runs out to some 10 meters. In such strokes, the channel at the cloud end of the stroke may remain visible for some little time. In this case the pilot leader invisible to the camera advances 20 to 200 meters with an active conducting channel of $R=10$ cm illuminated by the return stroke from ground some 10 *milliseconds* later, and has as well the expanded channel of positive space charge of radius R' which, with its roughly 5×10^{10} electrons per cm^3, is illuminated only by the step flash from the cloud end within some 10–100 *microseconds*. After ionization and illumination, the step ionization of radius R' decays, and it is only the original channel of $R=10$ cm which has sufficient conductivity owing to the field X_c to carry the return stroke.

It will be noted from Table III that the observed values of what might be R' are less than those computed. This is not surprising, for, as radial expansion continues, the time rate of ionization and accompanying excitation decline, and what is preceived by photomultiplier or photographic plate corresponds to values considerably less than R' depending on the sensitivity of the detector. Amin could observe only at a value of radius greater than 0.05 cm from the streamer axis. That he did not observe luminosity between 0.05 and 0.1 cm is not surprising. That, however, there was a transient luminosity after the intensely luminous peak passed the slit, is shown by the shoulder of luminosity following the tip and of such shape as only to be accounted for by ionization and excitation occurring long-after even an especially broad tip had passed the slit. The resolving power of Meek and Saxe's system was not such as to have revealed the details of the fine structure of the tip luminosity as observed by Amin, while their radial transient tip expansion could well be observed with their heavy currents. From the evidence presented, even though observed luminosity and calculated values R agree only in order of magnitude, it is believed that the nature of the transient tip shape and luminosity is accounted for. The transient space charge expansion described is illustrated schematically in Fig. 2, which is self-explanatory.

PHYSICAL REVIEW VOLUME 94, NUMBER 2 APRIL 15, 1954

The Scattering of Electromagnetic Waves by Turbulent Atmospheric Fluctuations*

F. VILLARS AND V. F. WEISSKOPF
Department of Physics and Lincoln Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received December 7, 1953)

The statistical theory of turbulence is applied to the problem of density fluctuations in the troposphere and the ionosphere. For suitable wavelengths, for which the so-called similarity region (Kolmogoroff spectrum) of the spectrum of turbulence is relevant, a closed formula can be given for the scattering cross section. It contains as only parameter the turbulent power dissipation S, and its angular dependence is given by $(\sin\frac{1}{2}\theta)^{13/3}$, θ being the scattering angle. The values of S required to explain ionospheric scattering are in excellent agreement with values found from investigations of meteor trails. Tropospheric data cannot be fitted with the assumptions of dry-air turbulence alone. The inference is that humidity fluctuations play an essential part in tropospheric scattering. A preliminary study of these latter fluctuations gives satisfactory results. Further investigations (and experimental data) are needed, however, to work out a quantitative theory.

LIGHT waves are scattered by random fluctuations of the refractive index. In what follows we derive the scattering of elementary waves by random fluctuations which are produced by turbulent perturbations. The general idea underlying this study has been suggested by a number of authors, especially Megaw[1] and Booker.[2] It will be shown that, under certain conditions, the scattering produced by these fluctuations can be expressed in terms of only one parameter, the turbulent energy S dissipated per cm^3 per sec. The conditions of validity of this relation are well fulfilled for the scattering of meter waves in the E layer of the ionosphere; for tropospheric scattering other parameters such as the inhomogeneity of potential temperature and specific humidity play an important part.

A. SIMPLE DERIVATION OF THE SCATTERING FORMULA

1. Scattering Cross Section

In this section we derive the expressions by simple qualitative arguments and leave the exact derivations for Sec. B.

---

* The research in this document was supported in part by the U. S. Army, Navy, and Air Force under contract with the Massachusetts Institute of Technology.

[1] E. C. S. Megaw, Nature **166**, 1100 (1950), and Proc. Inst. Elec. Engrs. (London) **100**, 7 (1953).

[2] H. Booker and W. E. Gordon, Proc. Inst. Radio Engrs. **38**, 401 (1950).