energy-loss values denote the estimated relative "half-widths" of the loss distribution. *a* indicates an estimated "half-width" equal to the unscattered slit image (approximately 1.2 volts), *b* a width twice *a*, *c* three times, etc. Figure 1(a) shows the scattering in germanium and is an example of an *a*-type distribution, while Fig. 1(b) shows the scattering in beryllium which is an example of the *e*-type distribution. Where previous measurements have been made, the extent of agreement varies.

In most cases more than one line has been observed. Where higher values appear to be multiples of some lower one, this would indicate repeated occurrence of the same event. Where the higher values are not multiples of some lower one, it may be an indication that not only lattice interaction but also atomic interaction may be responsible for the observed spectrum. In particular, the highest values for chromium and iron can perhaps be identified with the $M_{II}M_{III}$ x-ray absorption values while the single value found for cadmium is close to its N_{IV} x-ray absorption value.

Pines and Bohm[4] have calculated some values of energy loss by means of their plasma oscillation theory. They give 15.9 ev for aluminum and 18.8 ev for beryllium. Also, Pines[5] has calculated 10.8 ev for magnesium and ∼6 ev for sodium. These values are in reasonable agreement with our measured ones. In a recent paper Wolff[6] has stated that one would expect the widths of the absorption peaks to increase from element to element in the transition series, Sc through Ni, as the *3d* shell is filled. Our estimated band widths indicate that this may not be the case. However, further careful analysis of the bands is necessary before this can be stated more definitely. Similar losses have also been observed by Rudberg,[7] Haworth,[8] Turnbull and Farnsworth,[9] and Reichertz and Farnsworth[10] by measuring the energy distribution of very low-energy electrons reflected from metallic surfaces. It is highly probable that these were due to the same mechanism as the losses observed in transmission.

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Coulomb Excitation by Means of Bombardment with Various Particles

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THE production of Coulomb excitation effects by the impact of different types of projectiles, such as protons, deuterons, and α particles, not only gives a very convincing check on the nature of the process, but also provides a simple method for determining the multipole order of the transition in question.[1]

The cross section for Coulomb excitation produced by the electric multipole field of order λ of the impinging particle may be written[2]

$$\sigma = Z_1^2 \frac{B_e(\lambda)}{e^2 a^{2\lambda}} \cdot \left(\frac{a}{v}\frac{e^2}{\hbar}\right)^2 \cdot f_\lambda\{\xi\}, \tag{1}$$

where

$$a = \frac{Z_1 Z_2 e^2}{m v^2}; \quad \xi = \frac{Z_1 Z_2 e^2 \Delta E}{\hbar m v^3}. \tag{2}$$

Here $Z_1 e$ is the charge, m the reduced mass, and v the velocity of the bombarding particle, whereas $Z_2 e$ is the charge, ΔE the excitation energy, and $B_e(\lambda)$ the reduced transition probability[3] for the target nuclei. The function $f_\lambda\{\xi\}$ has been evaluated for the case $\lambda = 2$ by Alder and Winther.[4] The expression (1), which can be deduced from simple dimensional arguments, is derived by considering the projectile as moving along its classical trajectory, and neglecting the change in the orbit caused by the energy loss ΔE.

Elimination of a and v from (1) and (2) gives

$$\sigma = C_\lambda\{Z_2, \Delta E, B\} Z_1^2 \left(\frac{m}{Z_1}\right)^{2\lambda/3} \xi^{2-4\lambda/3} f_\lambda\{\xi\}, \tag{3}$$

where C_λ is a constant which depends only on λ and the properties of the target nuclei.

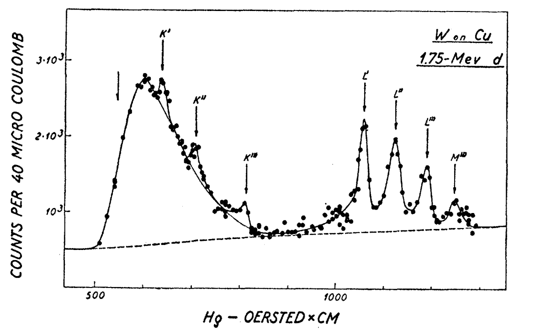

FIG. 1. The β spectrum measured for W bombarded by 1.75-Mev deuterons. The cutoff at small momenta is due to the 0.9-mg/cm² mica window of the counter. The dashed line indicates the general background mainly due to the effect of penetrating radiation from carbon deposits.

One can thus directly determine the multipole order of the excitation process by comparing, for the same value of ξ, the cross sections obtained with particles having different m/Z_1 ratios.

We have tested these conclusions by means of measurements on W bombarded with protons, deuterons, and α particles in the energy range from 1 to 2 Mev.[5] Figure 1 shows the entire β spectrum obtained by bombardment with 1.75-Mev deuterons. For the present purpose we used mainly the L' peak, which corresponds to an excitation energy of 100 kev assigned to the first excited state in W^{182}.[6]

When the background is subtracted, the peak height y of the line can be used as a measure of the corresponding cross section, since the thickness of the target layer was only about $\frac{1}{2}$ mg/cm². Figure 2 shows a plot of $\log_{10}(y/Z_1^2)$ *versus* ξ. The points for deuterons and α particles should be shifted a constant amount

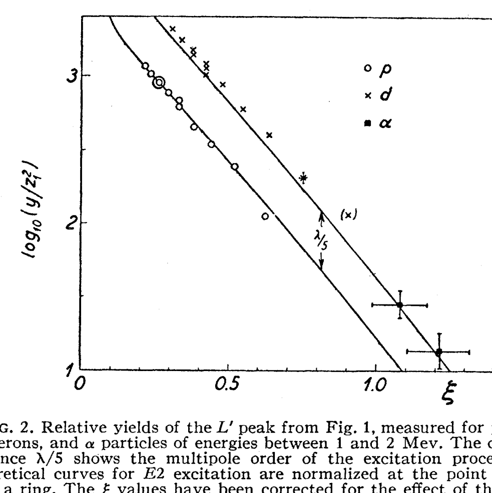

FIG. 2. Relative yields of the L' peak from Fig. 1, measured for protons, deuterons, and α particles of energies between 1 and 2 Mev. The constant distance $\lambda/5$ shows the multipole order of the excitation process. The theoretical curves for $E2$ excitation are normalized at the point marked with a ring. The ξ values have been corrected for the effect of the target thickness, but not for the energy ΔE lost by the excitation.

$\frac{2}{3}\lambda \log_{10}2 = 0.20\lambda$ in the direction of the ordinate axis, as compared to those for protons. The shift determined from the figure has a value of about 0.44, which thus confirms the expected $E2$ character of the excitation. The small difference from the theoretical value 0.40 may well be due to the experimental uncertainties.

Since the function $f_2\{\xi\}$ is known,[4] one can also compare the experimental ξ-dependence with the theory. The theoretical curves shown in Fig. 2 have been normalized at the experimental point corresponding to 1.75-Mev protons. The measurements are seen to confirm the theoretical calculations of the excitation function. The experimental accuracy is not quite sufficient to test to what extent the bombarding energy should be corrected for the energy loss in the excitation process. The comparison provides, nevertheless, still a relatively critical test on the evaluation of ξ, and can therefore be considered as a check on the value used for $Z_2\Delta E$. This can be an aid in cases where the assignment of a particular peak is not clear.

The kind of particle one should use in a given case in order to get the highest absolute yield is determined by the bombarding energy available. This is illustrated by Fig. 3, where the $E2$

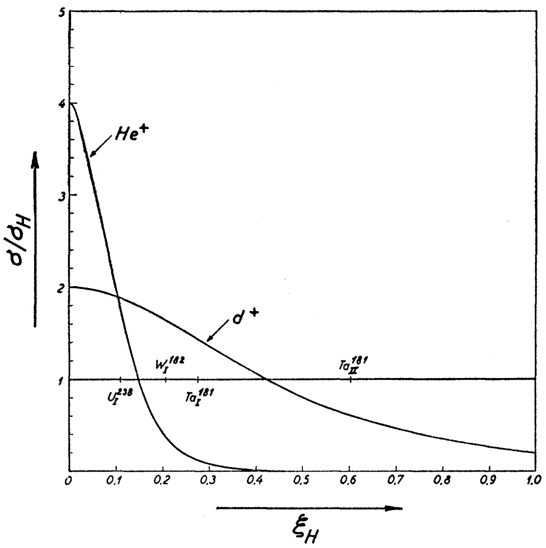

FIG. 3. Ratio of theoretical values for $E2$ excitation cross sections for deuterons and α particles, to those for protons accelerated with the same high voltage, plotted as a function of $\xi_H = Z_2\Delta E/(13U^{3/2})$, where ΔE is the excitation energy in Mev and U the voltage in Mv. $4(\sigma_d/\sigma_H)$ gives the curve for α particles accelerated as He^{++} ions. The marks give the ξ_H values for the first (I) and second (II) excited state corresponding to $U = 2$ Mv.

excitation cross sections calculated by means of Eqs. (1) and (2) are compared for the various particles. The use of protons is evidently advantageous for the higher excitation energies. For lower excitation energies the heavier projectiles are the most suitable, and in particular so, because the background radiation found in this region is much smaller for slow particles. This is demonstrated by the appearance of the K lines shown in Fig. 1. In the spectrum obtained with protons[7] these lines were concealed by the background of stopping electrons, which was an order of magnitude larger than in the case of deuterons.

Similar considerations apply to the γ-ray experiments,[5,8] which are preferable when the conversion coefficients are small. In using Fig. 3, however, it should be remembered that thick-target measurements give relatively larger yields for protons because of their longer range.

The processes responsible for the background radiation from the target atoms are discussed in the following Letter.

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Emission of δ- and X-Rays from Targets Bombarded by Accelerated Ions

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WHEN one bombards elements with protons or heavier ions of such a low energy that they cannot penetrate into the nucleus, then nuclear reactions are confined to Coulomb excitation. In addition one observes other inelastic events, which are due to the ejection of atomic electrons and to bremsstrahlung in the Coulomb field of the nucleus. In the present note we discuss some measurements on these latter radiations, which provide the most important background in Coulomb excitation experiments.

The cross section for the ejection of K electrons from atoms bombarded with heavy charged particles has been calculated by Henneberg.[1] He used plane waves to represent the bombarding particles and nonrelativistic wave functions for the electrons. In this approximation the differential cross section per atom (i.e., per two K electrons) for ejection of a K electron with an energy E_δ, is given by

$$d\sigma_K \simeq 1.4\times 10^{-8} Z_1^2 e^4 (E_1/A_1)^4 E_K^3 (E_K + E_\delta)^{-10} dE_\delta, \qquad E_K \gg E_0 \quad \text{or} \quad E_\delta \gg E_0, \quad (1)$$

where Z_1e is the charge, A_1 the mass number, and E_1 the energy of the bombarding particle. The expression is valid if the binding energy E_K is large compared to the maximum energy E_0 which an electron can obtain in a free collision with the bombarding particle. By integrating (1) over E_δ, one can in this case easily obtain the total cross section for ionization of the K shell. However, regardless of the magnitude of E_K, Eq. (1) should hold for the higher energies in the spectra, i.e., for $E_\delta > E_0$.

In this region the yields of electrons found experimentally by bombardment with protons, deuterons, and α particles[2] correspond to cross sections of the order[3]

$$d\sigma \simeq 10^{-16} Z_1^2 e^4 (E_1/A_1)^4 Z_2^4 E_\delta^{-7} dE_\delta, \quad (2)$$

for $E_1 \simeq 1$ to 2 Mev, $E_\delta \simeq 40$ to 100 kev, and $Z_2 \simeq 50$ to 80. Z_2 is the atomic number of the target material. The electrons were measured in a broad angular region around 90°, and anisotropy has been disregarded in the evaluation of the total 4π cross section (2). Thick target yields can be estimated from (2) by assuming that the electrons are produced in an effective layer of thickness t, where $t \simeq 2\times 10^{-3} E_\delta^2$ mg/cm² for E_δ in kev.

The form of (2) is similar to (1), but the magnitude is much larger. For the heavier elements already the number of electrons in the investigated part of the spectrum is larger than the total number of the ejected K electrons as deduced from x-ray measurements.[4,5] The observed electrons are therefore probably mainly L electrons. For this high-energy region of the spectrum, relativistic effects are important; theoretical estimates indicate that a relativistic treatment may yield cross sections in closer agreement with (2).