Production of K Mesons*

U. HABER-SCHAIM,† *Weizmann Institute of Science, Rehovoth, Israel*

AND

Y. YEIVIN AND G. YEKUTIELI, *Cosmic-Ray Section, Israel Atomic Energy Commission, Tel Aviv, Israel*

(Received September 23, 1953)

Calculations reported in an earlier paper are extended to lower energies using Fermi's explicit statistical method. A few pertinent details of the calculation are given, and the results compared with the observations of the Bristol group.

I. INTRODUCTION

ABOUT a year ago two of us (U.H. and G.Y.) applied Fermi's theory of pion production[1] to the production of K mesons.[2] One of the objects of that paper was to find out whether the characteristic volume in which the K mesons would be produced, according to the Fermi model, is of the order of $(4\pi/3)\lambda\!\!\!^{-}_K{}^3$ or $(4\pi/3)\lambda\!\!\!^{-}_\pi{}^3$, where $\lambda\!\!\!^{-}_K$ and $\lambda\!\!\!^{-}_\pi$ are the Compton wavelengths of the K meson and pion, respectively. The few experimental data available at that time were sufficient to show that both kinds of mesons are produced in the same volume.

In the meantime, more experimental data have been obtained. It seemed, therefore, worthwhile to extend the former calculation which was limited to high energies ($>20Mc^2$) to energies in the vicinity of the threshold.

II. METHOD OF CALCULATION

Following Fermi,[1] we consider the two colliding nucleons and the K mesons as nonrelativistic and the pions as extreme-relativistic particles in the cm system. We restrict the conservation of momentum to the two nucleons; this condition reduces somewhat the injustice done to the K mesons in taking them as nonrelativistic particles. The production of K mesons and pions in a nucleon-nucleon collision has then the statistical weight S_{rs} given by

$$S_{rs}=\frac{2^\alpha \mathfrak{M}^{\frac{3}{2}}}{3^\beta\pi^\gamma\mu^{2\beta}\alpha!}\times\frac{(W-2-rm)^\alpha}{W^\beta(W-2)^{\frac{1}{2}}}, \tag{1}$$

$$\alpha=3s+\tfrac{3}{2}r+\tfrac{1}{2};\quad \beta=r+s;\quad \gamma=s+\tfrac{1}{2}(r-1).$$

W is the total energy in the cm system in units of Mc^2, and m is the mass of the K meson in units of M. Further, $\mathfrak{M}=\frac{1}{2}m^r$ and μ is the mass of the pion.

So far we have taken into account only the density of states in momentum space. We still have to sum over the various possibilities of the distribution of the electric charge among the nucleons and mesons, each of which has a specific statistical weight.[3] Consider, for example, a collision between a proton and a neutron in which four pions are produced. There are seven different charge states which obey the conservation of charge:

$$p+n\rightarrow\begin{cases}2p+\pi^++\pi^0+2\pi^-\\ 2p+\quad 3\pi^0+\pi^-.\\ p+n+2\pi^++2\pi^-\end{cases}$$
$$\text{etc.}$$

The first of these has a statistical weight $\frac{1}{4}$ because there are two pairs of identical particles; the second state gives $\frac{1}{12}$, etc. The spin of the nucleons has been neglected—to include it would cause only minor changes. Now, if instead of four pions one K meson and three pions are produced, the statistical weight factor is increased since we now have fewer identical particles. In other words, the price one has to pay in momentum space in order to produce a heavy meson is reduced by the increase of the statistical weight of the charge states. This point has been overlooked in a recent paper by Kothari,[4] who finds a decrease with energy of the ratio of K mesons to pions. The values of the factors a_{rs} multiplying the S_{rs} are given in Table I.

The average number of K mesons n_K' and the average number of pions n' are given by:

$$\begin{aligned} n'_K&=\Sigma_{rs}ra_{rs}S_{rs}/\Sigma_{rs}a_{rs}S_{rs},\\ n'_\pi&=\Sigma_{rs}sa_{rs}S_{rs}/\Sigma_{rs}a_{rs}S_{rs}.\end{aligned}\tag{2}$$

TABLE I. The statistical weight factors a_{rs} for pn and pp collisions.

	pn			pp		
$s \backslash r$	0	1	2	0	1	2
0	1	2	2.5	1	3	4
1	2	5	6.5	3	8	9.2
2	2.5	6.5	8.76	4	9.2	10.8
3	2.67	5.84	8.0	3.07	7.2	14.2
4	1.46	4.0	5.55	1.8	7.1	10
5	0.8	2.22	...	1.42	4.0	...
6	0.37	1.08	...	0.67	2.05	...

* The results of this paper were briefly reported at the Cosmic Ray Conference at Bagnères-de-Bigorre, July, 1953 (unpublished).

† Now at the Department of Physics, University of Berne, Berne, Switzerland.

[1] E. Fermi, Progr. Theoret. Phys. Japan **5**, 570 (1950); Phys. Rev. **81**, 683 (1951).

[2] U. Haber-Schaim and G. Yekutieli, Phil. Mag. **43**, 997 (1952).

[3] U. Haber-Schaim, "Tables Related to Fermi's Theory of Pion Production," University of Chicago, Department of Physics, 1951 (unpublished).

[4] L. S. Kothari, Nature **171**, 309 (1953); Phys. Rev. **90**, 1087 (1953).

We finally introduce an approximate correction for the conservation of angular momentum $g(W)$:

$$n=g(W)n' \quad (3)$$

for both K mesons and pions, where $g(W)$ decreases from 0.67 at the threshold for pion production to 0.51 at extremely high energies.[1] Within these limits the correction is somewhat arbitrary.

III. RESULTS AND DISCUSSION

The results of these calculations for pn collisions are given in Table II. They are only slightly different for pp collisions. For the mass of the K meson, $m=0.69$ has been used. The values at high energies are taken from our previous paper,[2] and the experimental values were found by the Bristol group.[5]

For the comparison of our results with experiment, one should keep in mind that, although the calculated values are for nucleon-nucleon collisions, the observed values are for nucleon-nucleus collisions. It is difficult to state the effect of the nucleus quantitatively, but, qualitatively, it should reduce the ratio for two reasons (a) the energy available in secondary collisions inside the nucleus is only a fraction of the primary energy, hence the production of K mesons becomes less probable; and (b) there is a chance for a K meson to be reabsorbed in the same nucleus in which it was created, and a pion may even be emitted on this occasion.[6] We would expect this process to occur more frequently at low energies where the K particle is slow than at high energies. In view of these arguments the agreement with experiment does not look unreasonable. One should also note that the whole theory contains only one parameter, the characteristic volume.

TABLE II. Comparison with experiment.

Energy in Mc^2 in lab. system	Calculated			Observed	Kothari (calculated)
	n_K	n_π	n_K/n_π	n_K/n_π	n_K/n_π
4	0.08	0.93	0.09		0.52
6	0.17	1.14	0.14	0.09 ± 0.03	0.040
10	0.30	1.40	0.22		0.036
15	0.47	1.62	0.29	0.20 ± 0.02	0.034
20	0.63	1.78	0.35		
50	1.17	2.34	0.50		
100	1.64	2.85	0.57		
200	2.25	3.43	0.66	0.5 ± 0.2	

[5] D. H. Perkins, Rochester Conference, Dec. 1952 (Interscience Publishers, New York, 1953) and private communication.

[6] B. Peters, Cosmic Ray Conference, Bagnères-de-Bigorre, 1953 (unpublished).

PHYSICAL REVIEW VOLUME 94, NUMBER 1 APRIL 1, 1954

Renormalization

P. T. MATTHEWS, *Department of Mathematical Physics, The University, Birmingham, England*

AND

ABDUS SALAM, *Government College, Lahore, Pakistan*

(Received October 7, 1953)

A theoretical justification for the infinite subtractions, which have to be made in the renormalization of the S matrix, is given along the lines suggested by Gupta and developed by Takeda. It is shown that this is equivalent to working with the renormalized field variables of Dyson, and that the method deals very simply with overlapping divergences and the "wave function" renormalization associated with external lines. It also gives directly Ward's identities and brings out their essential dependence on gauge invariance. The method is applied to free and bound electrons in electrodynamics and all renormalizable meson theories.

In the later sections the new method is related to the original method of Dyson; the Bethe-Salpeter equation is renormalized and closed forms are derived for the renormalization constants.

INTRODUCTION

THE general proof of the renormalization of the charge expansion of the S matrix of interacting fields[1] falls into three distinct parts. Firstly the number of types of infinity (primitive divergents) in the theory is determined. (If this number is finite the theory is renormalizable.) The second step is to define a subtraction procedure which removes these infinities. The third step is to provide a theoretical justification for these subtractions. A general outline of this proof, applied to electrodynamics and various meson theories, has already been given by us.[2] The purpose of the present review is to assume the results of the first two parts of the proof for any renormalizable theory, and to give, in detail, a treatment of the third part, which has previously presented the greatest difficulty. The central idea is to treat all divergences of the theory by means of infinite counter terms, as has always been done for the mass renormalization. This was first suggested by

[1] F. J. Dyson, Phys. Rev. 75, 1736 (1949).

[2] P. T. Matthews and Abdus Salam, Revs. Modern Phys. 23, 311 (1951).