This result is not inconsistent either in magnitude or sign with the octupole moment to be expected for iodine. In fact, it is in good agreement with the result predicted from a detailed theory for a $d_{5/2}$ proton in a single-particle orbit.[5] The authors would like to point out that this result has no relation to that found by Tolansky from a study of the optical spectrum of ionized iodine.[6]

Consideration of the form of the octupole interaction shows why, for a given octupole moment, the interaction energy is appreciably larger in iodine than in the elements of group III or the remaining halogens. The hyperfine structure of some of these elements has been studied by atomic-beam methods with comparable precision and no octupole-like departures have been found.[8] Details of this and other considerations (relativistic effects, configuration interactions, etc.) will be discussed in a forthcoming paper, which will describe the experimental method as well. It will be accompanied by a paper by C. Schwartz on the theory of the hyperfine structure interaction.

We are indebted to Charles Schwartz for his valuable cooperation.

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[1] Here $K=F(F+1)-I(I+1)-J(J+1)$. For $J=3/2$ only these *interactions* exist. This does not preclude, for $I=5/2$, the existence of an electric nuclear 2^4 *moment*.

[2] For a closed shell minus an electron the interaction constants may be expressed as:

$$ha=\mu_0\frac{\mu}{I}\frac{2L(L+1)}{J(J+1)}\langle r^{-3}\rangle,$$

$$hb=-e^2Q\frac{2L}{2L+3}\langle r^{-3}\rangle,$$

$$hc=\mu_0\left\langle\frac{(5z^3-3zr^2)}{2}\,\mathrm{div}\mathbf{M}\right\rangle_{I,I}\frac{2L(L-1)(2L+2)(2L+4)}{(2J+2)(2J+3)(2J+4)}\langle r^{-5}\rangle.$$

The indeterminate form of c for the case of a $p_{3/2}$ electron may be evaluated following reference 3.

[3] H. B. G. Casimir and G. Karreman, Physica **9**, 494 (1942).

[4] The results of these calculations were also obtained independently by C. Schwartz.

[5] Paper on the theory of the hyperfine structure interaction to be published by C. Schwartz.

[6] S. Tolansky, Proc. Roy. Soc. (London) **A170**, 214 (1939). In a detailed paper in which the theory of the hyperfine structure involving the nuclear magnetic octupole moment was first presented, Casimir and Karreman (reference 3) pointed out that the Tolansky octupole moment was some 300 times larger than that expected. Other optical investigations (reference 7) did not support Tolansky's results.

[7] T. Schmidt, Z. Physik **112**, 199 (1939); K. Murakawa, Z. Physik **112**, 234 (1939).

[8] The results of recent high-precision measurements of the hyperfine structure of In^{115}, when suitably corrected for the effects of the neighboring fine-structure level, show the existence of a nuclear magnetic octupole moment in In^{115}, though some four times smaller than the value expected for a $g_{9/2}$ proton. Since, in the group III elements, the effects of configuration interactions for large Z must be considered, this result may not be surprising. We wish to express our gratitude to Professor P. Kusch of Columbia University for making his data available to us before publication [P. Kusch and T. G. Eck, following letter, Phys. Rev. **94**, 1799 (1954)].

Hyperfine Structure of In^{115}. Evidence of a Nuclear Octupole Moment*

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THE hyperfine structure intervals in the $^2P_{3/2}$ state of In^{115} have been measured with a high precision in a search for effects arising from a nuclear magnetic octupole moment. Previous measurements[1] allowed the description of the observed intervals in terms of a magnetic dipole and an electric quadrupole interaction only, within experimental error. The present measurements were made on an apparatus in which the weak field, ranging from 0.43 to 1.48 gauss in the several runs, which determined the magnetic splitting of the lines is extremely uniform. The lines are thus sharp and free of asymmetries and it is possible to determine their frequencies to very high precision. The inhomogeneous deflecting fields of the apparatus are sufficiently low so that the atoms are in the (F,m_F) quantization in these fields. Accordingly almost all transitions $\Delta F=\pm1$, $\Delta m_F=\pm1,0$ are accompanied by significant changes in magnetic moment and it is possible to observe a large number of the Zeeman components of each line. The quadratic terms in the energy levels were small in all cases.

The measured frequencies of the zero field lines are as follows:

$$F=6\leftrightarrow F=5:\quad f_6=1752.6851\pm0.0006\ \text{Mc/sec},$$

$$F=5\leftrightarrow F=4:\quad f_5=1117.1693\pm0.0005\ \text{Mc/sec},$$

$$F=4\leftrightarrow F=3:\quad f_4=\ 668.9638\pm0.0005\ \text{Mc/sec}.$$

It is remarkable that these three frequencies can be very accurately represented by an expression for the energy levels which includes only the dipole and quadrupole interaction. In fact, if f_6 and f_4 are assumed as given, f_5 becomes 1117.1692 Mc/sec. However, the levels of the $^2P_{3/2}$ state are perturbed by the $^2P_{1/2}$ state. The perturbation[2] serves to shift the $F=5$ level upwards by 8.2 kc/sec and the $F=4$ level upwards by 1.0 kc/sec. The attempt to describe the corrected line frequencies by an expression which includes only dipole and quadrupole interaction terms leaves residual discrepancies between observed and calculated line frequencies of the order of 5 kc/sec, far beyond the uncertainties of the experimental data.

If we use the expression for the energy levels given by Jaccarino *et al.* in the preceding letter and which includes magnetic dipole, electric quadrupole, and magnetic octupole terms, we find

$$a=242.16485\pm0.00006\ \text{Mc/sec},$$

$$b=449.5524\pm0.0006\ \text{Mc/sec},$$

$$c=0.000497\pm0.000033\ \text{Mc/sec},$$

where, in each case, the quoted uncertainty is the rms sum of the uncertainties in each of the terms of the linear equation which determines the quantity in terms of the line frequencies. No attempt is made to include uncertainties in the small correction terms which have been applied to the observed frequencies. The quantity c is about fourteen times the uncertainty in that quantity and the reality of an octupole-like interaction term is, therefore, not subject to significant doubt. The determination of the octupole moment itself from the interaction constant cannot be made without further extensive calculation.

These measurements were made in consequence of the observation of a much larger octupole interaction energy in I^{127} by the group at the Massachusetts Institute of Technology whose letter appears immediately before the present letter.[3]

* This work was supported in part by the U. S. Office of Naval Research.

[1] A. K. Mann and P. Kusch, Phys. Rev. **77**, 427 (1950).

[2] We are indebted to Dr. V. Jaccarino and Mr. Charles Schwartz for access to their calculations of the relevant perturbation energies.

[3] V. Jaccarino *et al.*, preceding letter [Phys. Rev. **94**, 1798 (1954)].

Coulomb Effects in Pion-Proton Scattering at Relativistic Energies

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VAN HOVE[1] and Ashkin and Smith[2] have shown how to separate Coulomb and nuclear effects in pion-proton scattering at nonrelativistic energies. One simply considers the Coulomb force negligible inside the region (of radius of the order of the meson Compton wavelength) in which the nuclear forces act, and uses the appropriate Coulomb wave functions outside. It then turns out that the scattering amplitude for not too low energies can be written, to quite good approximation, as the sum of the nuclear amplitude in terms of phase shifts and the Coulomb Born approximation amplitude.

Thus the cross section in the center-of-mass system (including nuclear s and p waves only) is of the form

$$d\sigma/d\Omega=|(1/2ik)(P+Q\cos\theta)+f^{(nf)}(\theta)|^2+|(1/2ik)R\sin\theta+f^{(f)}(\theta)|^2;\quad(1)$$

$\hbar k$ and θ are the momentum and scattering angle in the c.m.

system; for $\pi^+ + p \to \pi^+ + p$,

$$P = \exp(2i\alpha_3) - 1,$$
$$Q = \exp(2i\alpha_{31}) + 2\exp(2i\alpha_{33}) - 3,$$
$$R = \exp(2i\alpha_{33}) - \exp(2i\alpha_{31}),$$

and for $\pi^- + p \to \pi^- + p$,

$$P = \tfrac{1}{3}[2\exp(2i\alpha_1) + \exp(2i\alpha_3)] - 1,$$
$$Q = \tfrac{1}{3}[2\exp(2i\alpha_{11}) + \exp(2i\alpha_{31}) + 4\exp(2i\alpha_{13}) + 2\exp(2i\alpha_{33})] - 3,$$
$$R = \tfrac{1}{3}[2\exp(2i\alpha_{13}) + \exp(2i\alpha_{33}) - 2\exp(2i\alpha_{11}) - \exp(2i\alpha_{31})].$$

The Coulomb non-spin-flip and spin-flip amplitudes can be written in the following form:

$$f^{(nf)}(\theta) = \mp \frac{e^2}{2pV_\pi \sin^2(\theta/2)}, \quad f^{(f)}(\theta) = 0; \tag{2}$$

here $p = \hbar k$ is the momentum in the c.m. system, and V_π is the pion velocity in the laboratory system. The upper and lower signs refer to scattering of positive and negative pions, respectively.

There has been some question about the correct relativistic generalization of the above result. For small angles and high energy, a classical relativistic calculation should give the correct Coulomb cross section; such a calculation yields $(d\sigma/d\Omega)_C = [(e^2/2pV_\pi)(2/\theta)^2]^2$. Comparison of this expression with Eq. (2) shows that the amplitude, as written there, is also correct relativistically for small angles. [Note that the amplitude is given in a different form in references 1 and 2 and hence differs from Eq. (2) at relativistic energies.]

An alternate approach, valid also for large angles, is to write down the scattering amplitudes to first order in $e^2/\hbar c$ for the electromagnetic interaction Hamiltonian:

$$\mathcal{H}^{EM} = -(4\pi)^{\frac{1}{2}} e \psi_p^\dagger \gamma_\nu \psi_p A_\nu + \frac{(4\pi)^{\frac{1}{2}} e\hbar}{2Mc}(\mu_p - 1)\psi_p^\dagger \gamma_\mu \gamma_\nu \psi_p \times \left(\frac{\partial}{\partial x_\mu} A_\nu - \frac{\partial}{\partial x_\nu} A_\mu\right) + \frac{i(4\pi)^{\frac{1}{2}} ec}{\hbar}\left(\varphi^* \frac{\partial \varphi}{\partial x_\nu} - \varphi \frac{\partial \varphi^*}{\partial x_\nu}\right) A_\nu + \frac{4\pi e^2}{\hbar^2} \varphi^* \varphi (A_\nu A_\nu - A_4^2);$$

here ψ_p, φ are the proton and meson wave functions, respectively, and A_ν is the 4-vector potential of the electromagnetic field. We have included a "Pauli term" for the anomalous magnetic moment of the proton (μ_p is the magnetic moment in nuclear magnetons). Using standard methods, we obtain

$$f^{(nf)} = \frac{\mp e^2}{2p(v_\pi + v_p)\sin^2(\theta/2)} \times \left[1 + \tfrac{1}{2}\frac{v_\pi v_p}{c^2}(1 + \cos\theta) - \tfrac{1}{4}(2\mu_p - 1)\frac{v_p^2}{c^2}(1 - \cos\theta)\right],$$

$$f^{(f)} = \frac{\pm e^2}{2p(v_\pi + v_p)\sin^2(\theta/2)}\left[\tfrac{1}{2}\mu_p \frac{v_\pi v_p}{c^2} + \tfrac{1}{4}(2\mu_p - 1)\frac{v_p^2}{c^2}\right]\sin\theta; \tag{3}$$

v_π, v_p are the pion and proton velocities in the c.m. system. We have neglected terms of order $(v_p/c)^3$. It can be seen that the Coulomb cross section based on Eq. (3) reduces correctly to the classical relativistic expression for small angles.

The author is indebted to Professor E. Fermi and Dr. J. Orear for valuable discussion of this problem.

[1] L. van Hove, Phys. Rev. **88**, 1358 (1952).
[2] J. Ashkin and L. Smith, Coulomb Interference Effects in the Scattering of Mesons by Protons, Technical Report No. 1, Carnegie Institute of Technology, February 2, 1953 (unpublished).

Scintillation Response of Anthracene to Soft X-Rays

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BIRKS[1,2] has predicted a difference between the scintillation response of organic phosphors to external electrons and to internal photoelectrons of the same energy. The scintillation response of anthracene to secondary characteristic x-rays of energies between 6 kev and 25 kev has therefore been studied. The scintillations were observed with an E.M.I. type 5060 photomultiplier, and the output pulse amplitude distribution was obtained using a "gray-wedge" pulse-height analyzer, based on the instrument originally developed by Bernstein *et al.*[3] Secondary characteristic x-rays of known energies were obtained by placing a series of scattering foils of different elements in the path of a beam of "white" x-radiation, and were detected at right angles to the primary beam direction by a 2 mm thick crystal of anthracene.

The pulse amplitude distribution photographs, obtained from the analyzer, showed that it was possible to resolve the photoelectron peaks produced by incident x-rays of energy >11 kev, when the photomultiplier was operating at room temperature (20°C). By cooling the photomultiplier to dry ice temperatures, to reduce the dark noise background, adequate resolution was obtained at lower energies from 6 kev to 11 kev.

The mean scintillation pulse height S is plotted as a function of the x-ray energy E in Fig. 1. The experimental data of Taylor

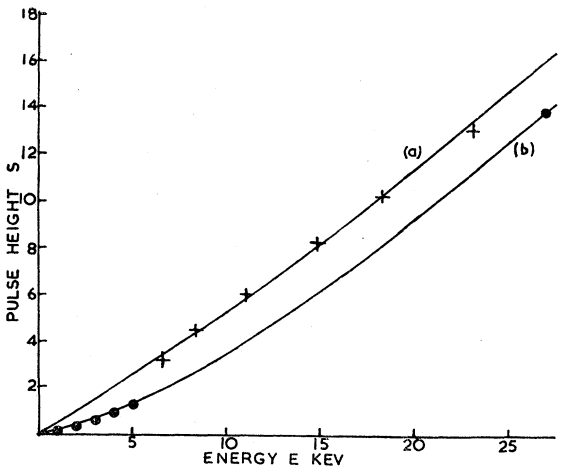

FIG. 1. Scintillation response of anthracene. (a) X-rays. Experimental +; theoretical ——. (b) Electrons. Experimental ●; theoretical ——.

et al.[4] on the scintillation response of anthracene to electrons of energy E, incident externally on the crystal, have been normalized to the same scale of S, and these are also plotted in Fig. 1 for comparison.

The theoretical response curve for photoelectrons, produced internally by x-rays, has been calculated from the formula[2,5]

$$\left(\frac{dS}{dr}\right)_x = \frac{A\,(dE/dr)}{1 + kB\,(dE/dr)},$$

by using experimentally determined values of the constants A and kB, and the range-energy data for electrons, given by Curie.[6] The theoretical response curve for external electrons has been calculated from the formula[1,2] for particles of range r,

$$(dS/dr)_e = \phi (dS/dr)_x,$$

where $\phi = 1 - \frac{1}{2}[\exp(-r/a_0) - (r/a_0)\,\mathrm{Ei}(r/a_0)]$, $\mathrm{Ei}(r/a_0)$ being the exponential integral. A value of $a_0 = 3$ mm air equivalent has been taken, corresponding to a mean free path for the primary photons[2] of about 3 microns.

The two theoretical response curves, which are plotted in Fig. 1, are in excellent agreement with the experimental data. These results provide further evidence in favor of the photon cascade theory of the scintillation process, proposed by Birks.[2,7] Further measurements are in progress on the scintillation response of anthracene and other organic phosphors to x-rays and electrons.