equivalent simple arguments[2] yield a qualitative estimate of the amount and character of the polarization phenomena. The fact that the first Born approximation yields a polarization is a consequence of the use of a complex central potential. The polarization is a result of the selective absorption of nucleons scattered with spins up ($j=l+\frac{1}{2}$) and with spins down ($j=l-\frac{1}{2}$).

It is the purpose of this note to point out that the Born approximation result for the polarization is independent of the shape of the nuclear potential if the spin-orbit potential is taken to be proportional to the gradient of the central nuclear potential.[4] The central potential is taken to be $V_c(r)=-(u_0+iw_0)\rho(r)$ in Mev, where $\rho(r)$ represents the radial dependence of the potential. The spin-orbit potential is taken to be $V_s=-U_s(r)\boldsymbol{\sigma}\cdot\mathbf{L}/\hbar$, where $U_s(r)=-(\mu a^2/r)(d\rho/dr)$. Roughly speaking, μ can be considered as the depth of an equivalent square well of radius a.

In Born approximation, the scattered amplitude $f(\theta)$ is given by

$$(2m/\hbar^2)f(\theta)=A(\theta)+i\boldsymbol{\sigma}\cdot\mathbf{n}\sin\theta B(\theta),$$

where

$$A(\theta)=\int j_0(gr)V_c(r)r^2dr,$$

$$B(\theta)=-\frac{k^2}{g}\int j_1(gr)U_s(r)r^3dr.$$

$\mathbf{n}$ is the unit vector normal to the plane of scattering, k is the incident wave number, g is the momentum transfer, and j_0 and j_1 are spherical Bessel functions.[5] Using the fact that $x^2j_0(x)=(d/dx)[x^2j_1(x)]$ and performing a partial integration on $A(\theta)$, one can show that

$$A(\theta)=-\frac{1}{g}\int j_1(gr)\left(\frac{1}{r}\frac{d}{dr}V_c\right)r^3dr.$$

Accordingly, $A(\theta)$ and $B(\theta)$ have the same functional form and the scattered amplitude can be written as

$$\frac{2m}{\hbar^2}f(\theta)=\left(1-i\boldsymbol{\sigma}\cdot\mathbf{n}\sin\theta\frac{k^2a^2\mu}{u_0+iw_0}\right)A(\theta).$$

The polarization is given then by

$$P=\frac{-2k^2a^2\mu w_0/(u_0^2+w_0^2)}{1+(k^4a^4\mu^2\sin^2\theta)/(u_0^2+w_0^2)}\sin\theta.$$

The polarization is thus independent of the shape of the nuclear potential, except that u_0, w_0, and μ must be adjusted to yield the total and absorption cross sections for a particular choice of the radial dependence, $\rho(r)$. The result is also independent of the target nucleus unless such a dependence is introduced into μa^2 (say, for example, $a\sim A^{\frac{1}{3}}$).

If we take, for example, a square well, the constants obtained in fitting the total nuclear cross sections for 300-Mev neutrons are $u_0\simeq 0$ and $w_0=18$ Mev.[3] If we use $\mu a^2=5\times 10^{-26}$ Mev cm^2 which corresponds to $\mu=\frac{1}{2}$ Mev when a is equal to the radius of carbon, the polarization is given by

$$P=\frac{-8.5}{1+18.1\sin^2\theta}\sin\theta.$$

The polarization obtained in the Born approximation can be expected to hold only for forward scattering angles. For larger angles the angular dependence of $B(\theta)$ relative to $A(\theta)$ changes sufficiently to introduce large corrections to the Born approximation result, particularly in the region of the diffraction minima. Thus, it is principally in these latter regions that model-dependent features of the polarization can be expected to appear. In addition, if non-square well shapes are taken, it has been found that u_0 and w_0 must be increased over their square well values in order to fit the observed cross sections. In some cases the increase is sufficient to invalidate the Born approximation. The results of more accurate calculations for neutrons and protons and for a number of well shapes is in the course of preparation.

I wish to thank Dr. Burton Fried for a discussion during which the observation on the polarization in Born approximation was made.

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[4] W. Heisenberg, *Theorie Des Atomkernes* (Göttingen, 1951), p. 22.
[5] L. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 77.

Hyperfine Structure of I^{127}. Nuclear Magnetic Octupole Moment*

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THE hyperfine structure of the atomic $^2P_{3/2}$ ground state of the stable isotope of iodine has been studied by the atomic-beam magnetic-resonance method. As will be shown, the measured intervals cannot be fitted by dipole and quadrupole-like interactions alone.

The hyperfine structure interaction Hamiltonian, which in the absence of external fields is diagonal in an F, m_F representation, gives for the relative positions of the energy levels[1]:

$$\left(\frac{W}{h}\right)_F=\frac{aK}{2}+b\frac{\frac{3}{4}K(K+1)-I(I+1)J(J+1)}{2I(2I-1)J(2J-1)}+c\frac{5}{4}\left\{\frac{K^3+4K^2+\frac{4}{5}K[-3I(I+1)J(J+1)+I(I+1)+J(J+1)+3]-4I(I+1)J(J+1)}{I(I-1)(2I-1)J(J-1)(2J-1)}\right\},\quad (1)$$

where a and b are the nuclear magnetic dipole and electric quadrupole interaction constants, respectively,[2,3] and c is the nuclear magnetic octupole interaction constant. It can be expressed as

$$hc=\langle r^3P_3\,\mathrm{div}\mathbf{M}\rangle_{I,I}\langle r^{-4}P_3\,\mathrm{div}\mathbf{M}\rangle_{J,J}.\quad (2)$$

(Here $\mathbf{M}$ is defined by the relation $\mathrm{curl}\mathbf{M}=\mathbf{i}/c$. The current density includes spin as well as orbital currents.) We have defined

$$\langle\tfrac{1}{2}(5z^3-3zr^2)\,\mathrm{div}\mathbf{M}\rangle_{I,I}$$

as the octupole moment.

By using second-order perturbation theory, matrix elements of the dipole and quadrupole moment operators which, for a given F, are nondiagonal with respect to J, may be calculated.[4] The effects of these perturbations by the neighboring $^2P_{1/2}$ fine-structure level,

$$[E(P_{1/2})-E(P_{3/2})]/h=2.28\times 10^8\text{ Mc/sec},$$

must be included. The measured zero-field intervals are:

	Uncorrected	Corrected
$F=4\leftrightarrow F=3$:	4226.172±0.015 Mc/sec;	4226.161±0.015 Mc/sec,
$F=3\leftrightarrow F=2$:	1965.884±0.010 Mc/sec;	1965.895±0.010 Mc/sec,
$F=2\leftrightarrow F=1$:	737.492±0.008 Mc/sec;	737.492±0.008 Mc/sec.

Experimental values for each of the three intervals were obtained by observing transitions for which $\Delta F=\pm 1$, in the region of zero external magnetic field.

The corrected intervals give the following values for the interaction constants:

$$a=827.265\pm 0.003\text{ Mc/sec},$$
$$b=1146.356\pm 0.010\text{ Mc/sec},$$
$$c=0.00245\pm 0.00037\text{ Mc/sec (all errors rms)}.$$

From this value of c the nuclear magnetic octupole moment is calculated:

$$\langle\tfrac{1}{2}(5z^3-3zr^2)\,\mathrm{div}\mathbf{M}\rangle_{I,I}=+0.3\mu_n\times 10^{-24}\text{ cm}^2.$$

This result is not inconsistent either in magnitude or sign with the octupole moment to be expected for iodine. In fact, it is in good agreement with the result predicted from a detailed theory for a $d_{5/2}$ proton in a single-particle orbit.[5] The authors would like to point out that this result has no relation to that found by Tolansky from a study of the optical spectrum of ionized iodine.[6]

Consideration of the form of the octupole interaction shows why, for a given octupole moment, the interaction energy is appreciably larger in iodine than in the elements of group III or the remaining halogens. The hyperfine structure of some of these elements has been studied by atomic-beam methods with comparable precision and no octupole-like departures have been found.[8] Details of this and other considerations (relativistic effects, configuration interactions, etc.) will be discussed in a forthcoming paper, which will describe the experimental method as well. It will be accompanied by a paper by C. Schwartz on the theory of the hyperfine structure interaction.

We are indebted to Charles Schwartz for his valuable cooperation.

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[1] Here $K=F(F+1)-I(I+1)-J(J+1)$. For $J=3/2$ only these *interactions* exist. This does not preclude, for $I=5/2$, the existence of an electric nuclear 2^4 *moment*.

[2] For a closed shell minus an electron the interaction constants may be expressed as:

$$ha=\mu_0\frac{\mu}{I}\frac{2L(L+1)}{J(J+1)}\langle r^{-3}\rangle,$$

$$hb=-e^2Q\frac{2L}{2L+3}\langle r^{-3}\rangle,$$

$$hc=\mu_0\left\langle\frac{(5z^3-3zr^2)}{2}\,\mathrm{div}\mathbf{M}\right\rangle_{I,I}\frac{2L(L-1)(2L+2)(2L+4)}{(2J+2)(2J+3)(2J+4)}\langle r^{-5}\rangle.$$

The indeterminate form of c for the case of a $p_{3/2}$ electron may be evaluated following reference 3.

[3] H. B. G. Casimir and G. Karreman, Physica **9**, 494 (1942).

[4] The results of these calculations were also obtained independently by C. Schwartz.

[5] Paper on the theory of the hyperfine structure interaction to be published by C. Schwartz.

[6] S. Tolansky, Proc. Roy. Soc. (London) **A170**, 214 (1939). In a detailed paper in which the theory of the hyperfine structure involving the nuclear magnetic octupole moment was first presented, Casimir and Karreman (reference 3) pointed out that the Tolansky octupole moment was some 300 times larger than that expected. Other optical investigations (reference 7) did not support Tolansky's results.

[7] T. Schmidt, Z. Physik **112**, 199 (1939); K. Murakawa, Z. Physik **112**, 234 (1939).

[8] The results of recent high-precision measurements of the hyperfine structure of In^{115}, when suitably corrected for the effects of the neighboring fine-structure level, show the existence of a nuclear magnetic octupole moment in In^{115}, though some four times smaller than the value expected for a $g_{9/2}$ proton. Since, in the group III elements, the effects of configuration interactions for large Z must be considered, this result may not be surprising. We wish to express our gratitude to Professor P. Kusch of Columbia University for making his data available to us before publication [P. Kusch and T. G. Eck, following letter, Phys. Rev. **94**, 1799 (1954)].

Hyperfine Structure of In^{115}. Evidence of a Nuclear Octupole Moment*

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THE hyperfine structure intervals in the $^2P_{3/2}$ state of In^{115} have been measured with a high precision in a search for effects arising from a nuclear magnetic octupole moment. Previous measurements[1] allowed the description of the observed intervals in terms of a magnetic dipole and an electric quadrupole interaction only, within experimental error. The present measurements were made on an apparatus in which the weak field, ranging from 0.43 to 1.48 gauss in the several runs, which determined the magnetic splitting of the lines is extremely uniform. The lines are thus sharp and free of asymmetries and it is possible to determine their frequencies to very high precision. The inhomogeneous deflecting fields of the apparatus are sufficiently low so that the atoms are in the (F,m_F) quantization in these fields. Accordingly almost all transitions $\Delta F=\pm 1$, $\Delta m_F=\pm 1,0$ are accompanied by significant changes in magnetic moment and it is possible to observe a large number of the Zeeman components of each line. The quadratic terms in the energy levels were small in all cases.

The measured frequencies of the zero field lines are as follows:

$$F=6\leftrightarrow F=5:\quad f_6=1752.6851\pm 0.0006 \text{ Mc/sec},$$

$$F=5\leftrightarrow F=4:\quad f_5=1117.1693\pm 0.0005 \text{ Mc/sec},$$

$$F=4\leftrightarrow F=3:\quad f_4=\ \ 668.9638\pm 0.0005 \text{ Mc/sec}.$$

It is remarkable that these three frequencies can be very accurately represented by an expression for the energy levels which includes only the dipole and quadrupole interaction. In fact, if f_6 and f_4 are assumed as given, f_5 becomes 1117.1692 Mc/sec. However, the levels of the $^2P_{3/2}$ state are perturbed by the $^2P_{1/2}$ state. The perturbation[2] serves to shift the $F=5$ level upwards by 8.2 kc/sec and the $F=4$ level upwards by 1.0 kc/sec. The attempt to describe the corrected line frequencies by an expression which includes only dipole and quadrupole interaction terms leaves residual discrepancies between observed and calculated line frequencies of the order of 5 kc/sec, far beyond the uncertainties of the experimental data.

If we use the expression for the energy levels given by Jaccarino *et al.* in the preceding letter and which includes magnetic dipole, electric quadrupole, and magnetic octupole terms, we find

$$a=242.16485\pm 0.00006 \text{ Mc/sec},$$

$$b=449.5524\pm 0.0006 \text{ Mc/sec},$$

$$c=0.000497\pm 0.000033 \text{ Mc/sec},$$

where, in each case, the quoted uncertainty is the rms sum of the uncertainties in each of the terms of the linear equation which determines the quantity in terms of the line frequencies. No attempt is made to include uncertainties in the small correction terms which have been applied to the observed frequencies. The quantity c is about fourteen times the uncertainty in that quantity and the reality of an octupole-like interaction term is, therefore, not subject to significant doubt. The determination of the octupole moment itself from the interaction constant cannot be made without further extensive calculation.

These measurements were made in consequence of the observation of a much larger octupole interaction energy in I^{127} by the group at the Massachusetts Institute of Technology whose letter appears immediately before the present letter.[3]

* This work was supported in part by the U. S. Office of Naval Research.

[1] A. K. Mann and P. Kusch, Phys. Rev. **77**, 427 (1950).
[2] We are indebted to Dr. V. Jaccarino and Mr. Charles Schwartz for access to their calculations of the relevant perturbation energies.
[3] V. Jaccarino *et al.*, preceding letter [Phys. Rev. **94**, 1798 (1954)].

Coulomb Effects in Pion-Proton Scattering at Relativistic Energies

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VAN HOVE[1] and Ashkin and Smith[2] have shown how to separate Coulomb and nuclear effects in pion-proton scattering at nonrelativistic energies. One simply considers the Coulomb force negligible inside the region (of radius of the order of the meson Compton wavelength) in which the nuclear forces act, and uses the appropriate Coulomb wave functions outside. It then turns out that the scattering amplitude for not too low energies can be written, to quite good approximation, as the sum of the nuclear amplitude in terms of phase shifts and the Coulomb Born approximation amplitude.

Thus the cross section in the center-of-mass system (including nuclear s and p waves only) is of the form

$$d\sigma/d\Omega=|(1/2ik)(P+Q\cos\theta)+f^{(nf)}(\theta)|^2 + |(1/2ik)R\sin\theta+f^{(f)}(\theta)|^2; \quad (1)$$

$\hbar k$ and θ are the momentum and scattering angle in the c.m.