of the one-body problem, and so this method also satisfies condition (3°).

So far we have discussed only the π^+ proton scattering. However, the problem of π^+ neutron can be treated in an exactly analogous fashion. The appropriate assumption there is

$$\psi = a_s^*|0\rangle + b_s^*|2\rangle + \eta'|1\rangle$$

for large g, or

$$\psi = a_s^*|0\rangle + \eta'|1\rangle$$

for small g. In this case, the $\eta'|1\rangle$ term should be included for all values of g, since $|1\rangle$ (real proton) and $|0\rangle$ (real neutron) are always stable. Again, conditions (1°) and (3°) are satisfied but not (2°).

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Renormalization of Meson Theory with a Fixed Extended Source*

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It is shown that the procedures for mass and charge renormalization developed by Dyson, Ward, and others for a covariant local field theory can be applied in the static or fixed source approximation where the interaction is nonlocal. Reasons are given to show that, although charge renormalization is not necessary in this case because the theory is not divergent, it is nevertheless a very sensible procedure.

INTRODUCTION

A WELL-DEFINED and often discussed form of meson theory treats the nucleon as an infinitely heavy source of pions and completely ignores recoil effects.[1] This form of the theory, sometimes called the static approximation, cannot be reached from conventional local pseudoscalar theory by a straightforward limiting process of setting $M=\infty$. Even after renormalization, integrals occurring in the local theory will diverge for an infinite nucleon mass. In the static approximation, convergence is achieved by inserting a cut-off factor, but it has often been pointed out that this factor cannot represent simply a damping due to nucleon recoil.[2] Nevertheless, there are at least two reasons to justify an examination of the static approximation: (1) Mathematically it is much simpler than the relativistic case and can yield a qualitative understanding of many important general features of field theory. (2) A number of experiments suggest that the actual damping of high-energy virtual effects is stronger than that produced by nucleon recoil alone in the local theory.[3] Thus it is possible that in some sense the correct and complete theory will be nonlocal. For example, if three fundamental fields rather than two are required, an approximate theory for the pion-nucleon interaction which does not specifically introduce the third field would have to be nonlocal. If this is the case the cut-off factor in the static approximation, which is equivalent to "spreading out" the region of the pion-nucleon interaction in space, may have a real physical significance.

The main interest in the static approximation thus far has been for the case of strong or intermediate strength coupling, where the relativistic theory has defied attempts at solution. No discussion has heretofore been given of the possibility of renormalizing the static approximation because the theory is finite without charge renormalization and the mass of the nucleon does not appear explicitly. Actually, the identification of self-energies *must* always be done in any field theory and has been done in past treatments of the static approximation. The point is merely that words other than "mass renormalization" have been used to describe the process. However, the technique of mesonic *charge* renormalization has up to now been stated only for a covariant local theory[4,5] and it is the purpose of this paper to show that the same methods can be applied to the static approximation.

The result of renormalizing the static approximation is not to render an infinite theory finite. The theory is finite before and after renormalization. However, renormalization eliminates many unobservable high-frequency effects and shows that observable quantities are much less "cut-off dependent" than is often thought. Also the renormalized coupling constant turns out to be substantially smaller than the original one, so that now perturbation methods become possible. Another way of describing the situation is to say that one has divided the infinite series given by perturbation theory

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[1] See, for example, W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1946).

[2] The point here is that nucleon recoil can damp out momenta much larger than M, if M is the nucleon rest mass, but momenta of the order of M will remain very important. Thus a nonrelativistic approximation to a local pseudoscalar theory is impossible.

[3] J. S. Blair and G. F. Chew, Ann. Rev. Nuc. Sci. 2, 163 (1953).

[4] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

[5] J. C. Ward, Proc. Phys. Soc. (London) **A64**, 54 (1951).

into two subseries, one of which converges much more rapidly than the other. The slowly converging series has been summed completely and the net result shown to be equivalent to reducing the value of the coupling constant appearing in the more rapidly converging series.

FORMULATION OF THE STATIC APPROXIMATION

With a pseudoscalar pion field, linearly and symmetrically coupled to the nucleon source, the interaction energy is

$$\mathcal{H} = (4\pi)^{\frac{1}{2}} \frac{f}{\mu} \sum_{\lambda=1}^{3} \tau_\lambda \int d\mathbf{r}\rho(r)\boldsymbol{\sigma}\cdot\nabla\varphi_\lambda(\mathbf{r}), \tag{1}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the Pauli spin and isotopic spin matrices, φ_λ is the pion field, and $\rho(r)$ is the source distribution function. We shall make our discussion in terms of the specific interaction form (1), but it will become apparent that the only essential feature of (1) is the linearity in φ, which means that $\mathcal{H}$ annihilates or creates pions *one at a time*.

Suppose that we begin by thinking in terms of a conventional perturbation approach. That is to say, we imagine that all transition matrix elements are expanded in powers of the interaction energy $\mathcal{H}$. There will then be a one-to-one correspondence between the terms in this series and diagrams to be drawn according to the following rules: (1) A single solid line running upward (time runs upward) denotes the nucleon. Unlike the nucleon lines in a Feynman diagram,[6] the nucleon line here never turns around, since pair formation is excluded. (2) The pions are denoted by dotted lines which can begin (creation) or end (annihilation) at points along the nucleon line. Dotted lines entering the diagram from below correspond to free pions in the initial state, while dotted lines leaving at the top correspond to free pions in the final state. A moment's thought shows that the pion lines also only move upward (forward in time), and for a virtual pion, creation always precedes annihilation. This, again, is a feature not present in Feynman diagrams, where all time orderings of the vertices are understood.

For purposes of orientation, let us consider a particular fourth-order diagram, Fig. 1a, which occurs in the problem of pion-nucleon scattering. According to the well-known formula of conventional perturbation theory, this diagram corresponds to the following term in the expansion of the transition matrix:

$$\frac{\mathcal{H}_{fn}\mathcal{H}_{nm}\mathcal{H}_{ml}\mathcal{H}_{li}}{(E_i-E_n)(E_i-E_m)(E_i-E_l)}, \tag{2}$$

where the state i contains *one* pion of momentum $\mathbf{k}_i$, the state l contains two pions, one of momentum $\mathbf{k}_i$ and one of momentum $\mathbf{k}$, the state m contains *one* pion of momentum $\mathbf{k}$, and so on. It is clear that we are

[6] R. Feynman, Phys. Rev. **76**, 749 (1949).

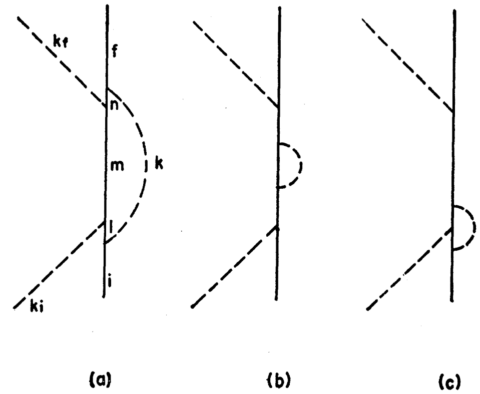

FIG. 1. Some typical diagrams occurring in the pion-nucleon scattering problem.

using the diagram purely as a counting device, to keep track of all possible intermediate states. For each vertex one has a matrix element of $\mathcal{H}$ corresponding to the emission or absorption of a pion. For each segment of the nucleon line between vertices we have an energy denominator.

It is legitimate to think of the energy denominators as propagation functions in a sense very similar to that of Feynman. Let us introduce the function

$$S(E) = 1/(E+i\epsilon), \tag{3}$$

where E is defined as the net energy which the nucleon has "absorbed" from the various pions which it has created or annihilated. E is zero for a "real" nucleon at the beginning or end of a diagram but for intermediate virtual nucleon states may be either positive or negative, to be calculated as if energy were conserved at each vertex. It follows that E is exactly the conventional energy denominator associated with the nucleon line in question.

It has been pointed out[7] that the simple expansion of which (2) is a sample term, is correct, strictly speaking, only in the absence of self-energies. It can easily be shown, however, that the correct expansion is obtained merely by adding the self-energy into each energy denominator.[8] In our problem, therefore, we should define the argument E of the nucleon propagation function as the *total* nucleon energy, including the self-energy. For "real" nucleons, we now have $E=E_S$, where E_S is the self-energy. Later E_S will be removed by a renormalization procedure.

The quantity ϵ, appearing in (3), is an infinitesimal real positive number which is to approach zero after sums over intermediate virtual states are performed. As usual, ϵ guarantees that the proper boundary conditions are satisfied by the transition matrix. If we omit ϵ

[7] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **91**, 398 (1953).

[8] This rule ignores "end effects," that is, terms corresponding to the self-energy acting as a perturbation either at the beginning or the end of a process. It is well known, however, that conventional charge renormalization causes these effects to cancel out.

and take the principal value of integrals when poles arise from the energy denominators, we will be calculating the reactance matrix rather than the transition matrix.[9] Bearing these facts in mind, we shall henceforth dispense with writing down ϵ explicitly.

The matrix element of $\mathcal{H}$ for absorption of a pion of charge type λ ($\lambda=3$ corresponds to a neutral pion, while $\lambda=1$, 2 are linear combinations of positive and negative[10]) and momentum $\mathbf{k}$ is

$$(4\pi)^{\frac{1}{2}}\frac{f}{\mu}\tau_\lambda\frac{\boldsymbol{\sigma}\cdot i\mathbf{k}}{(2\omega_k)^{\frac{1}{2}}}v(k)=fV_\lambda(\mathbf{k}), \tag{4}$$

where $v(k)$ is the Fourier transform of the source function $\rho(r)$.[11] Conventionally, $\int\rho(r)d\mathbf{r}$ is normalized to unity, so that $v(0)=1$. The corresponding creation operator is the complex conjugate of (4). Thus we may write out (2) as follows, if the initial pion is in the state i, the intermediate pion in the state j, and the final pion in the state f:

$$f^4\sum_j V_jS(E_s+\omega_i-\omega_j-\omega_f)V_f{}^*S(E_s+\omega_i-\omega_j)\times V_iS(E_s-\omega_j)V_j{}^*. \tag{2'}$$

Comparing (2′) to Feynman's formalism,[6] note that we have no propagation function for the pions. Actually one could perfectly well introduce a pion propagation function, but, since all lines move forward in time, it is possible immediately to do the integrals over pion energy and get only a residue at the *positive* value, $\omega_k=+(k^2+\mu^2)^{\frac{1}{2}}$. The factor, $1/2\omega_k$, which occurs in $|V_k|^2$ is precisely this residue. The concept of a *nucleon* propagation function is useful here because many of the renormalization effects will be identified as radiative modifications of the function $S(E)$. There are no modifications of the *pion* propagation function in the static approximation, however, since there are no nucleon pairs.

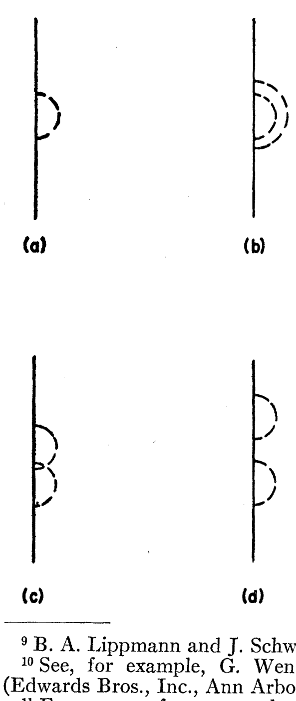

FIG. 2. Some diagrams contributing to $S'(E)$, the modified nucleon propagation function.

People familiar with Dyson's approach[4] will now see how the corresponding idea works here. We have two primary quantities, the propagation function $S(E)$ and the vertex operator V_j. By using diagrams we determine how to combine these quantities to form all the possible terms in the expansion of the transition matrix. One is next led to the notion of reducible and irreducible parts of diagrams. That is to say, certain combinations of lines and vertices can be thought of as merely producing a modification in the functions S and V. For example, among the fourth-order diagrams for pion-nucleon scattering, Fig. 1b corresponds to a modification of the intermediate nucleon propagation in a second-order diagram. Correspondingly Fig. 1c may be interpreted as a modification of one of the *vertices* in second-order scattering. Note on the other hand that Fig. 1a is irreducible. No parts may be interpreted merely as modifications of S or V.

Thus our problem, just as did Dyson's, breaks into two parts: (1) The determination of the modified nucleon propagation function, which we shall call $S'(E)$, and the modified vertex operator, which we shall call V_j'. (2) The evaluation of the irreducible diagrams in terms of S' and V_j'. The problem of renormalization is entirely concentrated in the first part. The remainder of this paper, then, will give an explicit recipe for calculating S' and V_j', taking proper account of mass renormalization and incorporating a mesonic charge renormalization which, as stated before, is not necessary but is extremely useful.

THE MODIFIED NUCLEON PROPAGATION FUNCTION

Following Ward,[5] we base our treatment of the modified nucleon propagation function $S'(E)$ on an auxiliary function, to be called $\Gamma_0(E)$ and to be defined by

$$\Gamma_0(E)=\partial/\partial E[1/S'(E)]. \tag{5}$$

In order to understand the significance of $\Gamma_0(E)$, note that we may write

$$S'(E)=\frac{1}{E-\sum(E)}, \tag{6}$$

where $\sum(E)$ is the sum of all modifications of a bare nucleon line due to overlapping meson lines. For example, the term in $\sum(E)$ of lowest order in f (second order) corresponds to Fig. 2a and is given by

$$f^2\sum_j V_jS(E-\omega_j)V_j{}^*. \tag{7}$$

Similarly the fourth-order terms in $\sum(E)$ correspond to Figs. 2b and 2c which each contain four vertex

[9] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 398 (1953).
[10] See, for example, G. Wentzel, *Quantum Theory of Fields* (Edwards Bros., Inc., Ann Arbor, 1946), p. 61.
[11] For reasons of economy, the two variables $\mathbf{k}$ and λ will usually be summarized by the single index j. For example, instead of $V_\lambda(\mathbf{k})$ we shall write V_j and instead of $\sum_\lambda\int d\mathbf{k}/(2\pi)^3$, we write $\sum_j$.

operators and three propagation functions. Note, however, that there is *no* term in $\sum(E)$ corresponding to Fig. 2d, which contains *nonoverlapping* pions. Such diagrams are produced by expanding (6) in powers of $\sum(E)$. That is,

$$S'(E)=\frac{1}{E}+\frac{1}{E}\sum(E)\frac{1}{E}+\frac{1}{E}\sum(E)\frac{1}{E}\sum(E)\frac{1}{E}+\cdots. \quad (8)$$

From Eqs. (5) and (6), it follows that

$$\Gamma_0(E)=1+\Lambda_0(E),$$

where

$$\Lambda_0(E)=-\frac{\partial}{\partial E}\sum(E), \quad (9)$$

which leads to an interpretation of $\Gamma_0(E)$ if one remembers that in each term which makes up $\sum(E)$ the dependence on E arises solely from the nucleon propagation functions. If n propagation functions occur in a particular term, then taking the derivative with respect to E splits that term into n pieces. A particular piece

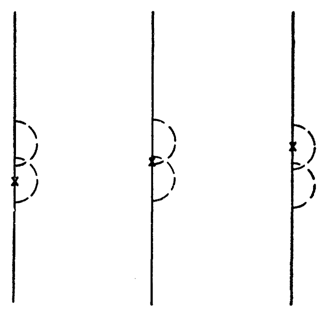

FIG. 3. Sample second-order contributions to $\Lambda_0(E)$.

can be represented by a diagram with an x on the nucleon line which has been differentiated. Thus, for example, the term in $\sum(E)$ corresponding to Fig. 2c is split by differentiation into the three pieces, shown in Fig. 3, which contribute to $\Lambda_0(E)$. Finally note that the operation $-\partial/\partial E$ on a particular propagation function $1/(E-\Omega)$ gives simply $1/(E-\Omega)^2$, so that $\Gamma_0(E)$ is just the so-called vertex modification of the effective coupling of the nucleon to a hypothetical neutral scalar field of low frequency.

In other words, suppose we wanted to calculate the matrix element of interaction of a nucleon with a neutral scalar field containing only very low frequencies so that it cannot change the proton energy appreciably. We would then evaluate a series of terms corresponding to the diagrams in Fig. 4, in which each pion overlaps with at least one other and the point of interaction with the "external" field, indicated by the x, is interlocked with the virtual pions. (One must also calculate diagrams involving nonoverlapping pions, but these would be treated most naturally by modifying the propagation function for the incoming and outgoing nucleon.) The sum of the above diagrams is called the vertex modification of the nucleonic coupling to the external field and is evidently identical with $\Gamma_0(E)$. For example, to second order in f, the vertex modification of the coupling to the external field from Fig. 4 is

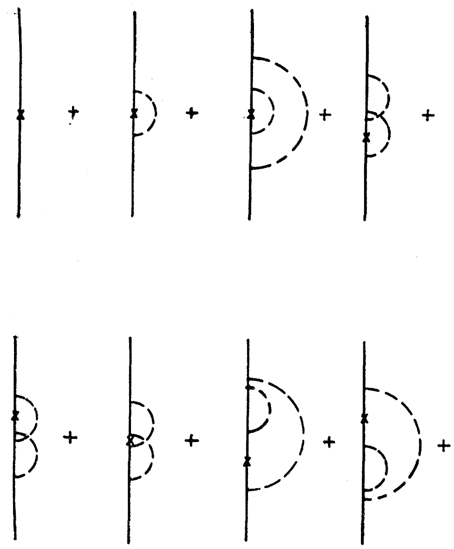

FIG. 4. Vertex modifying diagrams up to fourth order.

$$1+f^2\textstyle\sum_j V_jS(E-\omega_j)S(E-\omega_j)V_j^*, \quad (7')$$

which also follows immediately from (9) and (7).

NUCLEON ENERGY (MASS) RENORMALIZATION

It is convenient both physically and mathematically to shift the scale of the nucleon energy so that it is always referred to the self-energy. That is, if E_S is the self-energy of a single nucleon due to the associated pion field, then we define $E'=E-E_S$. Now from exactly the same considerations as apply to the relativistic theory, E_S may be identified as the position of the pole of $S'(E)$. In other words E_S is the solution of the equation

$$E_S=\sum(E_S), \quad (10)$$

and if S' is now considered as a function of E', then $S'(E')$ has its pole at $E'=0$.[12]

With the boundary condition that $1/S'(E')$ shall vanish at $E'=0$, we may integrate Eq. (5) to obtain an explicit formula for S' in terms of Γ_0,

$$1/S'(E')=\int_0^{E'} d\lambda'\Gamma_0(\lambda'). \quad (11)$$

THE RENORMALIZED PROPAGATION FUNCTION

Following Dyson, we now wish to introduce a renormalized nucleon propagation function,

$$S_r'(E')=Z_2^{-1}S'(E'), \quad (12)$$

[12] We here violate accepted mathematical notation in favor of maintaining physical simplicity. We mean by $S'(E')$ a quantity *equal* to $S'(E)$ for $E'=E-E_S$.

such that

$$S_r'(E') \xrightarrow[E'\to 0]{} 1/E'. \quad (13)$$

There is no overriding motivation in meson theory for the particular condition (13), as there is in electrodynamics for the corresponding condition, because there exist no low-frequency measurements of the pion-nucleon coupling such as the oil-drop experiment which measures the electron-electromagnetic field coupling. Nevertheless, in order to maximize the simplicity of the theory the condition (13) is indicated.[13]

If we simultaneously define

$$\Gamma_{0r}(E') = Z_2\Gamma_0(E'), \quad (14)$$

then relations of the type (5) and (11) are maintained for the renormalized quantities. That is,

$$\Gamma_{0r}(E') = \frac{\partial}{\partial E'}\left[\frac{1}{S_r'(E')}\right], \quad (15)$$

$$\frac{1}{S_r'(E')} = \int_0^{E'} d\lambda' \Gamma_{0r}(\lambda'). \quad (16)$$

From Eq. (15), we see that the requirement (13) implies that

$$\Gamma_{0r}(E') \xrightarrow[E'\to 0]{} 1, \quad (17)$$

and the latter relation allows us to determine Z_2 in terms of $\sum$, or, as will turn out to be convenient later, in terms of $\sum_r = Z_2\sum$ or $\Lambda_{0r} = Z_2\Lambda_0$. Substituting (14) into (9), we obtain

$$Z_2^{-1}\Gamma_{0r}(E') = 1 - \frac{\partial}{\partial E'}\sum(E') = 1 + Z_2^{-1}\Lambda_{0r}(E').$$

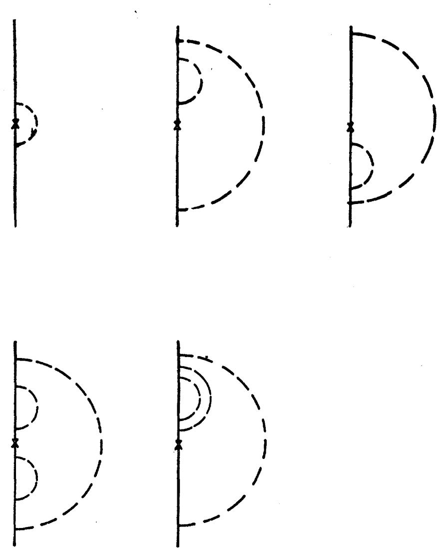

FIG. 5. Sample diagrams illustrating propagation function modification *within* a "fundamental" vertex diagram.

[13] For example, without condition (13) one could not neglect the "end effects" mentioned in footnote 8.

Taking the limit $E'\to 0$ then gives

$$Z_2 = 1 - \Lambda_{0r}(0). \quad (18)$$

It can be shown[14] that Z_2 represents the probability of finding the bare nucleon (zero pion configuration) in a single physical nucleon. Thus we know that $Z_2 < 1$.

An important problem still remains: to find a procedure for evaluating Γ_{0r}, and thus S_r', in which only renormalized quantities occur, Z_2 itself having been completely eliminated. Such a procedure is of course necessary in a divergent theory such as quantum electrodynamics. Here it is useful in eliminating uninteresting high-frequency effects which tend to obscure the essential aspects of the problem. To formulate the desired procedure, however, we must first construct the modified vertex operator.

THE MODIFIED VERTEX OPERATOR AND ITS RENORMALIZATION

The series of diagrams whose sum yields the modified vertex operator is in exact topological correspondence to the series which produces Γ_0 and which is indicated in Fig. 4. Now, however, a relation of the type (5) does *not* hold unless the pion field happens actually to be neutral and scalar. The insertion of an "external" vertex (indicated by the cross in Fig. 4) into one of the $\sum$ diagrams does more than simply square the nucleon propagation function at that point. For example, if the pion field is symmetrical and pseudoscalar, the absorption of one of its quanta may change either the charge or the spin of the nucleon. Mathematically, one would say that the "external" vertex operator does not commute with the "internal" vertex operators. In spite of this fact, it is not hard to convince oneself that the dependence of the modified vertex operator V_j' on $\mathbf{k}$, $\boldsymbol{\sigma}$, and λ will be exactly the same as that of V_j. The difference is that V_j' will also depend on E_1' and E_2', the nucleon energies before and after the vertex. We may summarize these statements by defining $L(E_2',E_1')$ such that

$$V_j'(E_2',E_1') = V_j L(E_2',E_1'). \quad (19)$$

This factorization of V_j' will be justified later by its self-consistency. Note that we consider E_2' and E_1' as independent of $\mathbf{k}$ although in any actual calculation $E_2' - E_1' = \omega_k$.

The failure of the vertex operators to commute will manifest itself in numerical factors not present in the case of Γ_0. For example, to second order in the coupling constant for the symmetrical pseudoscalar theory,

$$L(E_2',E_1') = 1 + (1/9)f^2\textstyle\sum_j V_j S(E_2'-\omega_j) \times S(E_1'-\omega_j)V_j^*. \quad (20)$$

[14] This fact was pointed out to the author by F. Low and its proof can easily be achieved with the techniques developed by M. Gell-Mann and F. Low in a forthcoming paper.

Comparing to (7′), one might say that to this order, only 1/9 as many pions contribute to the vertex modification as contribute to $\sum$, which gives the propagation function modification. In general, we shall express the function $L(E_2',E_1')$ as

$$L(E_2',E_1')=1+\Lambda(E_2',E_1'), \tag{21}$$

where $\Lambda(E_2',E_1')$ is the sum of contributions from all diagrams of the type shown in Fig. 4.

Still following Dyson's scheme, we next define a renormalized vertex operator L_r by

$$L_r(E_2',E_1')=Z_1L(E_2',E_1') \tag{22}$$

and require that

$$L_r(0,0)=1, \tag{23}$$

again following the convention established in electrodynamics. If we simultaneously define $\Lambda_r=Z_1\Lambda$, then we find

$$Z_1=1-\Lambda_r(0,0). \tag{24}$$

We are now in a position to formulate a general procedure for calculating S_r' and L_r, which does not involve Z_1 and Z_2.

RENORMALIZED INTEGRAL EQUATIONS

We shall find it worth while to make the remaining discussion in terms of certain integral equations rather than straightforward expansions in powers of the coupling constant. It seems likely that these equations have a more general validity than the power series expansions but no such claim is being made at this time. Our chief motive here in employing integral equations is to simplify the discussion.

Consider first the quantity $\Gamma_0(E')$ or $\Lambda_0(E')$ from which $S'(E)$ may be derived. The infinite series of diagrams which make up $\Gamma_0(E')$ may be split up into subseries in a systematic manner. For example, a well-defined subseries corresponds to taking the second diagram of Fig. 4, which has the value

$$f^2\textstyle\sum_j V_jS(E'-\omega_j)S(E'-\omega_j)V_j^* \tag{25}$$

and modifying the two nucleon propagation functions. This would generate diagrams of the type shown in Fig. 5. One may alternatively modify the two vertex operators, generating diagrams of the type shown in Fig. 6. Or one may do both and note that simultaneous modifications of S and V does not repeat any diagrams. Each diagram occurs only once. Making both modifications replaces (25) by

$$\begin{aligned} f^2\textstyle\sum_j V_j'S'(E'-\omega_j)S'(E'-\omega_j)V_j'^* \\ =(Z_2f/Z_1)^2\textstyle\sum_j V_jL_r(E',E'-\omega_j)S_r'(E'-\omega_j) \\ \times S_r'(E'-\omega_j)V_j^*L_r(E'-\omega_j,E'). \end{aligned} \tag{26}$$

This series of diagrams may be enlarged even further by inserting at the position of the cross the function $\Gamma_0(E'-\omega_j)$ itself, which gives us an integral equation

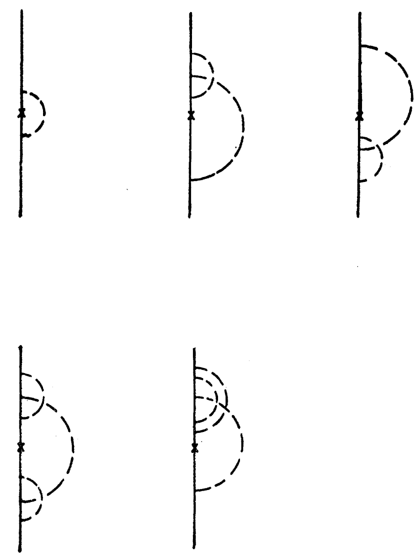

FIG. 6. Sample diagrams illustrating vertex modification *within* a "fundamental" vertex diagram.

for Γ_0. Again no repetitions of diagrams are generated. One might hope that, by modifying S, V, *and* the external vertex, one has included everything but this is unfortunately not so. One cannot, for example, generate in this way the sixth diagram of Fig. 4. This diagram, in fact, can itself be used to generate an entirely new series of terms by modifying within it the four propagation functions, the four internal vertices, and the one external vertex. In fact one can enumerate an infinite series of "fundamental" diagrams which each must be completely modified in order to generate the entire series of terms. The "fundamental" diagrams involving up to three pions are shown in Fig. 7.

Each "fundamental" diagram contains Γ_0 once and a number of V's equal to the number of S's. After modification and renormalization, therefore, each term will contain a factor, $Z_2(Z_2Z_1^{-1}f)^n$, so that it is natural to define a renormalized coupling constant,

$$f_r=Z_2Z_1^{-1}f, \tag{27}$$

as well as $\Lambda_{0r}=Z_2\Lambda_0$, which we have already done. The equation for Γ_{0r} then reads

$$\Gamma_{0r}(E')=Z_2+\Lambda_{0r}(E'), \tag{28}$$

where $\Lambda_{0r}(E')$ is to be evaluated entirely with the renormalized functions S_r' and L_r and the renormalized coupling constant f_r. Using (18) we may eliminate Z_2 from (28), obtaining the final equation

$$\Gamma_{0r}(E')=1+\Lambda_{0r}(E')-\Lambda_{0r}(0). \tag{29}$$

This is formally a linear integral equation for Γ_{0r}, but it contains of course the functions S_r' and L_r which must be codetermined. Equation (16) gives the recipe for obtaining S_r' and already entirely in terms of renormalized quantities. The problem of obtaining a re-

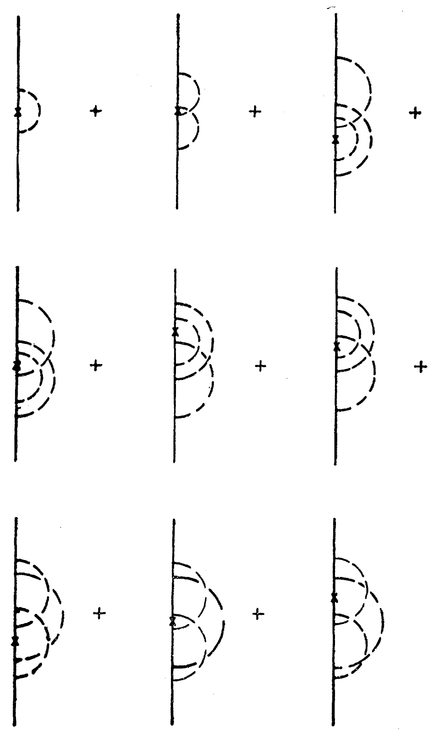

FIG. 7. The fundamental vertex diagrams involving up to three virtual pions.

normalized equation for L_r is completely analogous to that for Γ_{0r}, so we write down the result immediately

$$L_r(E_2',E_1')=1+\Lambda_r(E_2',E_1')-\Lambda_r(0,0). \qquad (30)$$

The quantity Λ_r is to be evaluated by the same recipe as that for Λ_{0r} except that, for the external vertex, L_rV_j is to be used in place of Γ_0. The operator V_j is to be factored out only after the appropriate commutations have been performed to bring it outside the summation over virtual pions.

CONCLUSION

It has been demonstrated explicitly that charge and mass renormalization can be performed even in the static approximation of field theory. The particular recipe given for the renormalization is in terms of coupled integral equations of infinite order for the exact vertex and propagation functions but, as usual, perturbation methods can be applied. In the following paper, one such method will be formulated.

In the final renormalized equations, the nucleon self-energy and the original coupling constant are completely eliminated. It will be shown in the following paper that in a perturbation calculation integrals over virtual pion momenta are now linearly dependent on the cutoff in momentum space, whereas without charge renormalization, quadratically dependent terms would appear. Thus the performance of explicit charge renormalization in advance shows that the theory is less cutoff dependent than might appear at first sight. It will also be demonstrated in the following paper that the renormalized coupling constant in the pseudoscalar theory is sufficiently smaller than the original constant that perturbation approaches have a good chance of success. Our conclusion, then, is that, although charge renormalization is not required in a finite theory, it can be performed and is in many respects a very sensible procedure.

From a practical point of view it is possible that if the static approximation to meson theory is to have a chance of actually corresponding to physical reality it will be necessary to include a quadratic term in the coupling so as to produce S-wave pion-nucleon scattering. The author has every confidence that renormalization procedures can be developed for such a quadratic term, but to date no attempt has been made in this direction.

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