Hard Core Interpretation of the Reaction $p+d\rightarrow\pi^++t$

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The cross section for this reaction is calculated for three Hulthén deuteron wave functions. A hard core in the deuteron at one-half meson Compton wavelength reduces the total cross section and flattens the angular distribution in the backward direction in agreement with experiment.

THE cross section for the reaction $p+d\rightarrow\pi^++t$ has been calculated from that for $p+p\rightarrow\pi^++d$ by Ruderman,[1] who employs the impulse approximation for the production of mesons from the proton in the deuteron, neglects positive pion production from the neutron, and estimates the triton wave function in terms of the deuteron wave function. The purpose of this note is to show that the experiments performed[2] at Berkeley are consistent with a repulsive core in the neutron-proton interaction.

The neutron in deuterium is assumed merely to impart to the struck proton the momentum needed for over-all conservation. In p-n collisions, positive pions can be produced only via intermediate states of isotopic spin $\frac{1}{2}$, while from p-p collisions the isotopic spin state $\frac{3}{2}$ is available as well. The evidence, from experiments on complex nuclei, for the suppression of $p+n\rightarrow\pi^-$, and by charge symmetry $p+n\rightarrow\pi^+$, relative to $p+p\rightarrow\pi^+$ is by now compelling. The large momentum transfers involved in meson production suggest the applicability of the impulse approximation and imply a short-range encounter between the incident and struck protons.

Under these assumptions, the transition rate for meson production in deuterium can be calculated[1] from that in hydrogen, giving in the center-of-mass system of proton and deuteron:

$$\frac{d\sigma}{d\Omega}(pd\pi^+t)=\frac{1}{v_{pd}}[f(\theta)]^2\frac{1}{3}\frac{E_t}{E_\pi+E_t}\left[v_{pp}\frac{E_\pi+E_d}{E_d}\frac{d\sigma}{d\Omega}(pp\pi^+d)\right].$$

Here the E's and v's are the energies and relative velocities occurring in the phase space and flux factors of the cross sections. The quantities in brackets are to be evaluated in the center-of-mass system of the two protons for that energy which produces a meson of momentum $\mathbf{q}$.

The form factor,

$$f(\theta)=\int\psi_D(\mathbf{x})\exp(i\boldsymbol{\Delta}\cdot\mathbf{x}/\hbar)\frac{\psi_T(\mathbf{x},0)}{\psi_D(0)}d\mathbf{x},$$

depends on the deuteron and triton wave functions $\psi_D(\mathbf{x})$, $\psi_T(\mathbf{Z},\mathbf{x})$ where $\mathbf{x}$ is the neutron-proton relative coordinate and $\mathbf{Z}$ the coordinate of the neutron in the triton relative to the center of mass of the proton and remaining neutron. With $\mathbf{k}$ the momentum of the incident proton, $\boldsymbol{\Delta}=\frac{1}{2}\mathbf{k}-\frac{1}{3}\mathbf{q}$ is the relative momentum in the struck deuteron which is required by momentum conservation. The integral $f(\theta)$ is the amplitude for finding in the original deuteron such a relative momentum that the original neutron and the deuteron from $p+p\rightarrow\pi^++d$ will form a bound triton in the final state.

For 341-Mev protons, which produce mesons of 78 Mev in the center of mass, Δ varies from 1.5 to $2.3\mu c$ for meson angles θ from 0° to 180°, so that $f(\theta)$ is sensitive to the deuteron and triton wave functions for $x=0.4$ to 0.6 meson Compton wavelengths. In the absence of information concerning the triton wave function at these distances, one can only show that the cross section observed is consistent with a repulsive core in the interior of the deuteron.

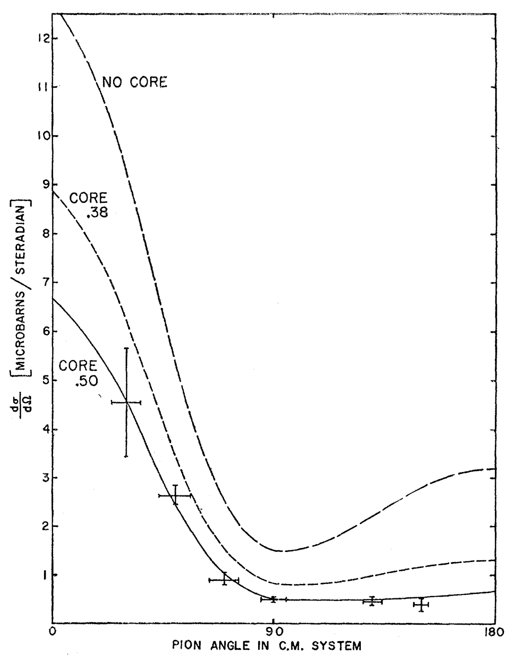

FIG. 1. Differential cross section in the center-of-mass system for $p+d\rightarrow\pi^++t$ with 341-Mev incident protons. The three curves are for deuteron wave functions with hard core at $0.50\hbar/\mu c$, $0.38\hbar/\mu c$, and no core. The experimental points are those of reference 2.

[1] M. Ruderman, Phys. Rev. **87**, 383 (1952). There are several numerical errors in the calculation of the cross section by Ruderman, who considered no core, which prevent comparison of his results with experiment, even in this case.

[2] Frank, Bandtel, Madey, and Moyer, preceding paper [Phys. Rev. **94**, 1716 (1954)].

For the triton wave function, Fröhlich *et al.*[3] find that the simple form

$$\psi_T=N_t\exp\{-\tfrac{1}{2}\alpha(x_{12}+x_{23}+x_{31})\},$$

where x_{ij} is the distance between nucleons i and j, gives 90 percent of the triton binding energy when calculated with nuclear potentials fitted to the two-body data. The optimum value of the parameter α is $\alpha=1.270\mu c/\hbar$, and the correct normalization constant is

$$N_t=(2/7)^{\frac{1}{2}}\alpha^3/\pi.$$

For the deuteron three different Hulthén wave functions,

$$\psi_D=(e^{-\beta r}-e^{-\gamma r})/r,$$

were employed: the foregoing function, and similar functions made to vanish at core radii $r_c=0.38$ and at $0.50\hbar/\mu c$. In each case the Hulthén parameters β and γ were chosen to fit the deuteron binding energy and triplet range. While the differential cross section for meson production is strongly anisotropic, this is probably more evidence for the important role of intermediate states of isotopic spin and angular momentum $\frac{3}{2}$ than for the effects of D state in the deuteron.

The cross section for the production of 78-Mev mesons in the $p+p\rightarrow\pi^{+}+d$ reaction has been extrapolated from the data of Schulz,[4] Durbin, Loar, and Steinberger,[5] and Crawford and Stevenson[6] to be

$$\frac{d\sigma}{d\Omega}(pp\pi^{+}d)=210(\tfrac{1}{3}+\cos^2\theta)\qquad\mu\text{b/steradian.}$$

The results obtained are shown in Fig. 1 together with the experimental points of Frank *et al.*[2] The effect of a core in suppressing the deuteron wave function at small distances is to reduce the total cross section and to flatten the angular distribution in the backwards direction, in agreement with experiment.

A different extrapolation for the proton-proton excitation function would change the absolute cross section without affecting the angular distribution. However, the excitation function for 78-Mev mesons would have to be 50 percent less than as extrapolated in order to bring the calculated cross section anywhere near the experimental points. A less isotropic angular distribution, say $(0.15+\cos^2\theta)$, would lower the curve at 90° where, however, the data are most reliable.

The most striking feature of the experiments, the forward to back ratio, is independent of the p-p cross section and is consistent with a core phenomenon. (Because the suppression of high momenta might just as well reside in the triton wave function, this experiment might probe the triton if the deuteron wave function were known well enough.) Such a core phenomenon seems indicated by the high-energy scattering[7] and by some forms of pseudoscalar meson theory.[8]

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[3] Fröhlich, Huang, and Sneddon, Proc. Roy. Soc. (London) **191**, 61 (1947). These authors give the parameter α in terms of an old meson mass and employ a different normalization for the wave function.

[4] A. G. Schulz, Jr., University of California Radiation Laboratory Report UCRL-1756 (unpublished).

[5] Durbin, Loar, and Steinberger, Phys. Rev. **84**, 581 (1951).

[6] F. S. Crawford, Jr., and M. L. Stevenson, Phys. Rev. **91**, 468 (1953).

[7] R. Jastrow, Phys. Rev. **79**, 389 (1950).

[8] M. Levy, Phys. Rev. **88**, 725 (1952); A. Klein, Phys. Rev. **90**, 1101 (1953).