Film Flow in Liquid Helium II at Low Level Differences*

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Observations of the flow of the liquid helium II film have been made at very low level differences. To produce the flow a plunger is displaced vertically at an accurately controlled rate inside of a beaker of internal diameter of about one centimeter. By this means the initial behavior when flow is started, the eventual steady-state behavior, and the oscillations around the final equilibrium position after the plunger is stopped, are observed. The results obtained fit the form of equations derived by Atkins if, in addition, the existence of an upper limit to the velocity of flow is postulated. However, the period of the final oscillations is always found to be shorter than that deduced from the initial effects. The final periods also vary more slowly with temperature and more rapidly with the total length of the helium film than do the initial periods. Assuming the film thickness d varies with height h according to a relation $d=D/h^n$, we find from the initial periods that $n\cong\frac{1}{2}$ and from the final oscillations $n\cong 1$. D, the thickness at a height of one centimeter, is approximately 2.5×10^{-6} cm, somewhat smaller values being obtained from the initial periods than from the final periods.

These results are taken as evidence for the existence of a difference between the moving and the stationary film. Various mechanisms for producing the necessary redistribution of the material of the film, if this difference is simply a change in shape, are discussed.

I. INTRODUCTION

AMONG the most striking of the anomalous properties of liquid helium below the lambda temperature are those associated with its unusual transport properties. The best example of the latter is the flow which takes place between a beaker and the surrounding bath if the liquid levels are at different heights. In isothermal conditions, the direction of flow is always toward the region of lower gravitational potential. The first quantitative investigations of the properties of this mass transport were carried out by Daunt and Mendelssohn,[1] who established the existence of a limiting transfer rate, independent of the level difference and varying only with temperature. Their results have been confirmed by many later experimenters,[2–5] some of whom[2,3] have also noted that the critical transfer rate decreases slightly for level differences between 2 and 3 mm.

To date the only experiments at level differences less than 2 mm are the oscillation studies of Atkins.[6] When the beaker level approaches its equilibrium position, the momentum in the film results in an overshoot which produces an oscillation of the beaker level about its equilibrium position. The period of these oscillations depends on the inertia of the film, and this in turn varies with the height of the beaker rim above the bath level. From a study of these oscillations, Atkins deduced a relation for the film thickness as a function of height. The oscillation amplitude, which depends somewhat on the geometry of the apparatus, is only several tenths of a millimeter.

It appeared, therefore, that information on the behavior, and possibly the structure, of the film might be obtained from a study of its flow properties at low level differences, both in the steady state and in the oscillations. To achieve the former condition, the customary capillary beaker was replaced by one with an internal diameter of about one centimeter, inside of which was suspended a plunger, leaving an annular space one millimeter wide. Flow was initiated by moving the plunger up or down.

II. THE APPARATUS

The experiments were performed in a separate chamber suspended in the helium bath. This inner chamber, shown in detail in Fig. 1, is made from a 2-inch copper-glass seal F, soldered at K with Rose's metal to a brass cap D, and sealed at the bottom. Hanging from the rods N is a thin-walled brass cup M which supports the beaker L. This beaker is a $4\frac{3}{4}$-in. length of precision bore tubing of i.d. 0.960 ± 0.003 cm, the bottom of which has been very carefully sealed so that its dimensions were distorted for no more than a half-inch of its length. The top of the beaker was ground to a smooth finish on a carborundum wheel. Before assembly into the apparatus, the beaker was cleaned in hot chromic acid, rinsed with distilled water, and carefully dried.

Suspended in the beaker is the Lucite plunger I which was lapped to a diameter of 0.7625 ± 0.0003 cm along its entire length. It is kept centered in the beaker

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[1] J. G. Daunt and K. Mendelssohn, Proc. Roy. Soc. (London) **A170**, 423, 439 (1939).

[2] K. Mendelssohn and G. K. White, Proc. Phys. Soc. (London) **A63**, 1328 (1950).

[3] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 240 (1950).

[4] J. G. Dash and H. A. Boorse, Proceedings of the National Bureau of Standards Symposium on Low Temperature Physics, 1951, National Bureau of Standards Circular 519 (U. S. Government Printing Office, Washington, D. C., 1952), p. 169.

[5] B. Smith and H. A. Boorse, Phys. Rev. **92**, 505 (1953).

[6] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 119 (1950).

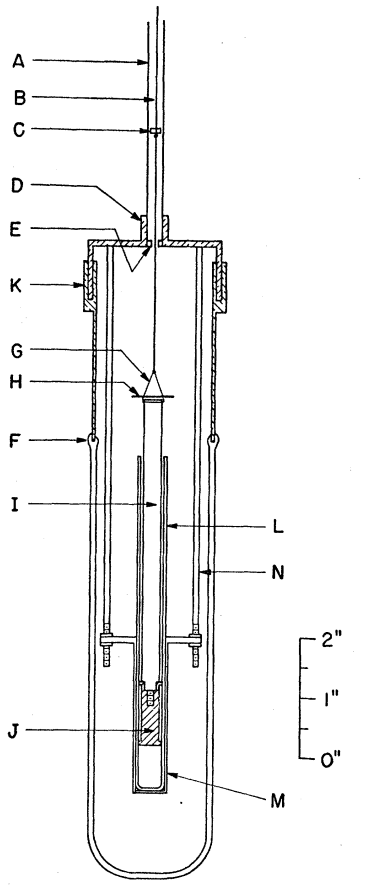

FIG. 1. The inner chamber, showing details of the beaker and plunger.

by the small projections on the brass extension *J*. This brass piece also serves to give the plunger sufficient weight to keep taut the linen string *B* from which it hangs. The plunger does not hand directly from the string, but from a loop *G* of No. 40 Be-Cu wire to which the string is tied. The string passes to the upper part of the cryostat through the $\frac{1}{4}$-inch Monel tube *A* from which the entire inner chamber hangs.

The inset *E* and baffle *C* shield the plunger and beaker from room temperature radiation coming down the Monel tube. The beaker is further protected from vertical radiation by a polished copper disk H mounted on top of the plunger.

At the top of the cryostat the string from which the plunger hangs is attached to a stainless steel shaft which passes vertically through a vacuum seal and then connects with a micrometer screw. The latter is driven vertically by a dc motor through a speed reducer and worm gear. A magnetic amplifier circuit powers the motor and enables control of its speed to within ± 0.5 percent, in the range from 350 rpm to 1550 rpm.

Special care was taken to isolate the cryostat from any sources of vibration to protect the helium levels from mechanical disturbances during an experiment.

The liquid levels were illuminated by a mercury vapor lamp filtered so that only the green 5641A line was passed with its intensity reduced to the lowest level compatible with comfortable observation. A pair of condenser lenses, with their optic axis directed slightly upward so as to make an angle of about three degrees with the horizontal, concentrated the light at the center of the cryostat in such a way that the helium level in the beaker appeared as a bright, sharp line.

The liquid levels were viewed with a telescope mounted on a Gaertner vertical traveling microscope stage. A timer was mounted directly above the microscope drum so that both instruments could be photographed simultaneously by a 35-mm movie camera operated as an automatically advancing single exposure device. The camera was tripped by a solenoid actuated through a relay-operated control circuit by a hand-held microswitch. By means of this circuit, the timer and the motor operating the plunger were not started until the microswitch was tripped to take the first picture. Thereafter the timer ran until the circuit was turned off, but the plunger motor could be stopped or its speed changed at any subsequent operation of the microswitch that the operator chose. Since the camera photographed the timer and microscope drum each time the switch was closed, the times of coincidence of the crosshairs and the helium levels were accurately recorded along with the times of any changes in conditions.

III. PROCEDURE AND ERRORS

A. Procedure During a Run

For at least three days before each helium run, the inner chamber was pumped to a pressure of 3 or 4×10^{-6} mm mercury, thus insuring that the beaker and other parts in the inner chamber were reasonably free of adsorbed gases. The vacuum was maintained until just before liquid helium was transferred into the cryostat, at which time helium gas, specially purified over charcoal at 2000 lb/in.^{2} and at liquid N_2 temperature, was admitted through a charcoal trap at liquid N_2 temperature.

After the transfer was completed, helium was condensed into the inner chamber to the desired level, and the system was closed off completely by means of a stopcock on the vacuum line. The temperature of the helium bath was then adjusted to the desired value and when equilibrium with the inner chamber, as indicated on a differential manometer, had been attained, the actual taking of data was begun.

To measure film transfer rates over large level differences of the order of several millimeters or more, the motor was disengaged from the driving screw and the desired difference in levels was produced by manually raising or lowering the plunger. The subsequent level motion was followed in the telescope and recorded on film.

The data on the behavior of the film transfer at very low level differences were taken with the motor engaged

to drive the plunger. With the telescope lined up on the level in the beaker and the motor control circuits set for the desired speed, the first picture was taken. This recorded the equilibrium position of the level and simultaneously started the timer and the plunger motor. The level was then tracked with the telescope and its position as a function of height was recorded by the camera. When desired, the motor control circuits were switched to the motor off position; thus at the instant of the next photograph, the plunger was stopped, and the process of the level settling back to its equilibrium position was recorded.

B. Flow Rate Measurements

The rate σ, in units of cubic centimeters per second per centimeter perimeter of the beaker, is given by

$$\sigma=\frac{1}{2\pi r_b}(A_b\dot{y}+A_p\dot{x})=\frac{r_b^2-r_p^2}{2r_b}\dot{y}+\frac{r_p^2}{2r_b}\dot{x}, \qquad (1)$$

where r_b is the inner radius of the beaker, A_b is the area of the helium surface in the beaker, A_p is the cross-section area of the plunger, $\dot{y}$ is the velocity of the helium level, and $\dot{x}$ is the velocity of the plunger (see Fig. 2). From the values for r_b and r_p (radius of the plunger) corrected for thermal contraction to liquid helium temperatures, we find $(r_b^2-r_p^2)/2r_b=0.0922\pm0.0019$ cm and $r_p^2/2r_b=0.1474\pm0.0006$ cm. The speed of the driving motor, which determines the plunger velocity $\dot{x}$, could be measured to within 0.5 percent with a General Radio Strobotac that had been calibrated against the ac line frequency. The largest total uncertainty that can result in the term $(r_p^2/2r_b)\dot{x}$ from the combined effects of instability in the motor speed and strobotac calibration and the variations in the radii is ±1.3 percent.

Sighting on the helium levels is a more important source of error. In sighting on the tenth-millimeter divisions of a precision scale, it was found that settings could be reproduced to within 0.002 mm, no matter from which direction they were approached. Since, during a run, extreme care in sighting had to be sacrificed so that readings could be taken more rapidly, it is estimated that helium level positions are accurate to about ±0.005 mm. Consequently, an accuracy of 1 percent can be expected in level velocities measured over level differences of the order of 1 mm. But in the low level difference runs, where flow is observed over intervals less than 0.10 mm, errors as large as ±20 percent can occur.

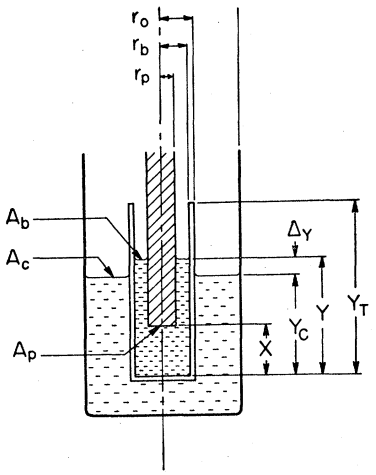

FIG. 2. Schematic representation of the inner chamber.

An experimental check on the validity of these estimates of precision was made by means of runs at a temperature of 2.17°K where the film transfer rate is very small. From Eq. (1) it is evident that the ratio $\dot{y}/\dot{x}$ should equal $r_p^2/(r_b^2-r_p^2)=1.60\pm0.04$ when $\sigma=0$. The observed values of this ratio for five different plunger speeds were 1.58, 1.61, 1.59, 1.60, and 1.61. In addition, these runs indicated that the plunger started moving as soon as the motor was started, and this fact was confirmed by direct observations on the plunger.

C. Film Path Length and Temperature

There are two other parameters which are measured: temperature T and film path length H. By the latter is meant the total length of the helium film measured from the level inside the beaker to the level outside. In the low level difference runs, H is equal to twice the height of the rim of the beaker above the liquid levels.

Vapor pressures read on the oil manometer connected to the helium bath were converted to temperatures on the 1948 scale.[7] The estimated errors in the measured temperatures were ±0.03°K at 1.30°K, ±0.01° at 1.50°K, and ±0.004° at 1.70°K and above. Temperature was controlled with an even greater precision, the maximum observed variation during a flow rate measurement having been 0.004°K. In most cases it was only several ten-thousandths of a degree.

D. Effect of the Illuminating Radiation

Increasing or decreasing the level of illumination by a factor of two produced no observable effect on any of the flow rate measurements. Although the range of intensities covered is not large, the results are in agreement with the findings of other investigators,[3,8] that even the complete absence of radiation does not affect film flow.

IV. EXPERIMENTAL RESULTS

A. Gross Flow Rates

Results of determinations of the temperature dependence of the film flow rate for level differences up to six millimeters are shown in Fig. 3. At each tempera-

[7] H. van Dijk and D. Shoenberg, Nature **164**, 151 (1949).
[8] R. Bowers and K. Mendelssohn, Proc. Phys. Soc. (London) **A63**, 1318 (1950).

ture it was found that the beaker emptied at a rate rougly ten percent faster than it filled; and that the rate of emptying was slightly lower when the level difference was less than 2.5 mm. The filling rate was constant over the range of level differences used. The general form of the temperature dependence of the film transfer rate agrees with that found by previous investigators;[1,2,4] and the decrease in flow when the levels are very close has also been reported.[2,3] Atkins[4] also found a ten percent greater emptying rate, but Mendelssohn and White[2] reported that their emptying rate was 5 percent lower than their filling rate. No satisfactory explanation has been given for these differences in rates under different conditions of flow.

B. Film Flow at Low Level Differences

Figure 4 is an example of a run conducted at low level differences. A better understanding of the significant features of such a run can be had from a consideration of the behavior of the level in two limiting cases.

First, consider a liquid in which there is no film transfer. With the plunger moving, the level inside the beaker would rise at a rate given by $\dot{y}=-(A_p/A_b)\dot{x}$ [see Eq. (1) and Fig. 2] which is the maximum possible level velocity, and is represented in Fig. 4 by the dashed line *a*. In the second case, the film transfer rate is such that the levels inside and outside the beaker are always equal. This represents ideal superfluidity where flow can be maintained at any rate with no difference in level. In this case $\dot{y}=-(A_p/(A_b+A_c))\dot{x}$, and is represented by line *b* in Fig. 4. (A_c is the area of the liquid surface in the inner chamber.)

The following features of the curve actually obtained are the most significant. When the plunger is first started, the level rises very sharply almost along the line *a*, until a peak is reached at a time t_p. Sometimes the peak is broad, as shown in Fig. 4, and at other times it is quite well defined, as can be seen in some of the runs in Fig. 5. After the peak, the level falls, and along this downswing the highest rate of film flow out of the beaker is reached. The transfer rate along this portion of the curve never exceeds that found in the measurements made at large level differences.

After the downswing there are two possible ways in which the curve may progress. One is illustrated in Fig. 4 where the level is seen to rise slowly, following the line *b*, but remaining a small distance above it. The other is shown in the upper runs of Fig. 5, most clearly in No. 7, where the liquid level oscillates about line *b* as an axis. The oscillations are damped, and the later portion of the plot (not shown in Fig. 5) looks very much like that in Fig. 4.

When the plunger is stopped (indicated by the leveling off of line *b* in Fig. 4), a damped oscillation of the level is observed. The period of these oscillations and the rate at which they are damped have also been measured.

It is sometimes possible to discern a step-like structure in the rapid downswing that follows the initial peak at t_p. This effect appears only in runs performed at relatively high plunger speeds where the slope of the downswing is small. Since the scale of the steps is also small, no good quantitative data can be obtained. A somewhat broader, but similar effect can be observed in the gentle rise of the level following the downswing.

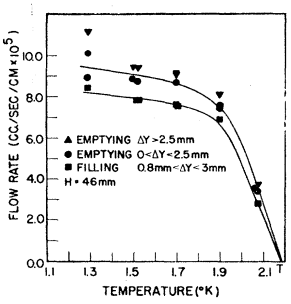

FIG. 3. Film transfer rates for large level differences.

C. Theory

Before trying to interpret the results of the low level difference runs, some discussion of the hydrodynamics

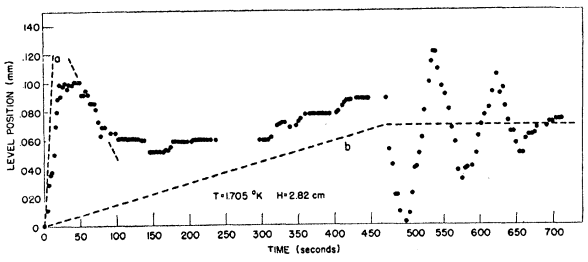

FIG. 4. Results of a typical low level difference run.

of helium II and its application to film flow is necessary. Many authors have given derivations of the hydrodynamic equations of motion of liquid helium, based on the two-fluid concept set forth by Landau and Tisza.[9–11] As a first approximation, liquid helium is treated as a mixture of two noninteracting components. One of these, the superfluid, is assumed to have zero entropy and zero viscosity, and the normal component is treated as a gas of excitations moving through the background of superfluid. The hydrodynamic equations for these two fluids are usually written

$$\rho_s\frac{d\mathbf{v}_s}{dt}=-\frac{\rho_s}{\rho}\nabla p+\rho_s S\nabla T, \tag{2}$$

$$\rho_n\frac{d\mathbf{v}_n}{dt}=-\frac{\rho_n}{\rho}\nabla p-\rho_s S\nabla T+\eta_n(\nabla^2\mathbf{v}_n+\tfrac{1}{3}\nabla\nabla\cdot\mathbf{v}_n). \tag{3}$$

[9] L. Landau, J. Phys. (U.S.S.R.) **5**, 71 (1941).
[10] L. Tisza, Phys. Rev. **72**, 838 (1947).
[11] For general review and other references see R. B. Dingle, Advances in Physics, Phil. Mag. Suppl. **1**, 111 (1952).

ρ_s and ρ_n are the densities of the superfluid and normal fluids, respectively, while $\rho=\rho_s+\rho_n$ is the density of the liquid as a whole. v_s and v_n are the velocities of the two fluids, S is the entropy per unit mass, and p is the pressure. η_n is the viscosity of the normal component.

In the film the viscosity of the normal fluid prevents it from moving so that Eq. (3) may be neglected. Assuming isothermal conditions and irrotational flow, Eq. (2) reduces to

$$\frac{\partial \mathbf{v}_s}{\partial t}+(\mathbf{v}_s\cdot\nabla)\mathbf{v}_s=-\frac{1}{\rho}\nabla p. \tag{4}$$

Atkins[6] used this equation as the starting point to derive expressions describing the oscillations of a liquid helium level about its equilibrium position. His procedure will be applied to find the equations of motion of the beaker level in the present experiments.

Besides those already mentioned, Atkins adds the assumption that the shape of the film is independent of its velocity. The irrotational character of the motion is approximated by assuming that at a given height and time, the velocity of the film is constant throughout its thickness. Dropping the subscript s, let v represent the upward velocity of the superfluid component at some point on the inner wall of the beaker, and let d be the thickness of the film at that point. Then, since σ is negative when the beaker is emptying,

$$\sigma=-(\rho_s/\rho)vd, \tag{5}$$

and so from Eq. (1) and Fig. 2 we have

$$v=-\frac{1}{d}\frac{\rho}{\rho_s}\frac{1}{2\pi r_b}(A_b\dot{y}+A_p\dot{x}). \tag{6}$$

Substituting into Eq. (4) and following Atkins' procedure, we find for the equation of motion of the helium level

$$\ddot{y}+\omega^2 y=\omega^2 bt, \tag{7}$$

where y is the displacement of the level from its equilibrium position,

$$\omega^2=g\left(1+\frac{A_b}{A_c}\right)\Big/\left[\left(1+\frac{r_b}{r_0}\right)\left(\frac{A_b}{2\pi r_b}\right)\left(\frac{\rho}{\rho_s}\right)\int_0^{H/2}\frac{dz}{d}\right], \tag{8}$$

and $b=-\dot{x}A_p/(A_b+A_c)$, the slope of line b in Fig. 4.

The solution of this equation is an undamped harmonic wave about an axis rising with slope b. The simplest way to account for the damping actually observed is to add to the right side of Eq. (4) a retarding term proportional to the velocity. If this term is written as $-2kv$, the equation of motion of the beaker level becomes

$$\ddot{y}+2k\dot{y}+\omega^2 y=\omega^2 bt+2ka, \tag{9}$$

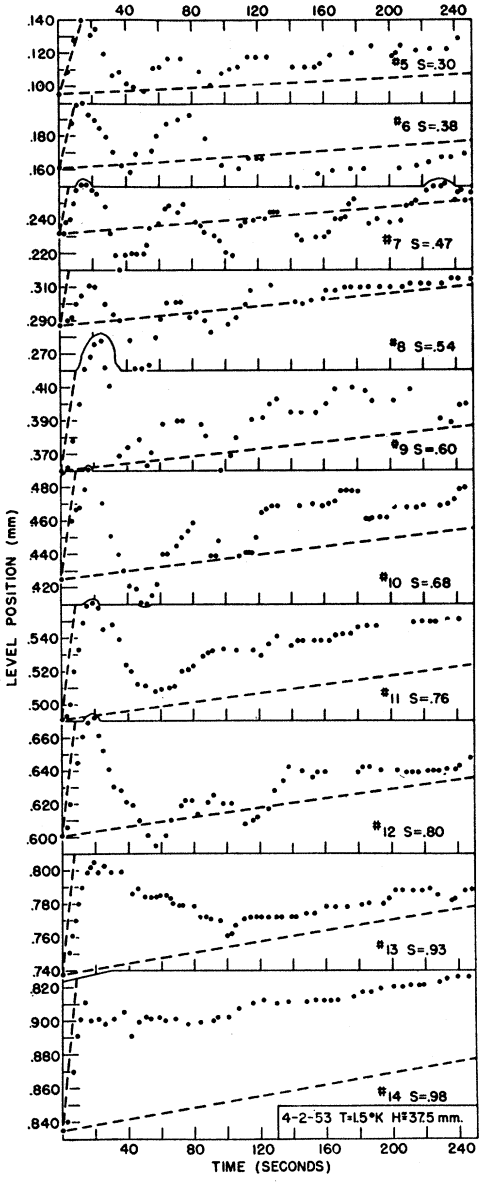

FIG. 5. Example of the variation of low level difference runs with relative plunger speed s.

where $a=-(A_p/A_b)\dot{x}$, the slope of line a in Fig. 4. The existence of a critical flow rate is taken into account by postulating that σ can never exceed σ_c. Since the magnitude of the plunger speed is significant only in terms of the motion it produces, it is represented through the quantity $s=\sigma_b/\sigma_c$. σ_b is the flow rate necessary to maintain a fixed level difference between the beaker and the bath (the slope of line b in Fig. 4).

If at time $t=0$, when the plunger is started downward at some speed $\dot{x}$, the level is stationary in its equilibrium position, its subsequent motion is as follows:

(i) For rates $\dot{x}$ such that the relative plunger speed s is between 0 and $\frac{1}{2}$, the position of the level as a function

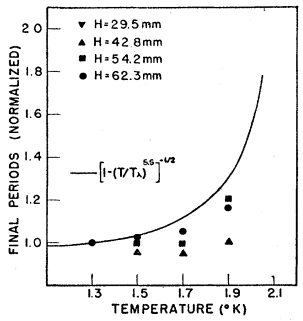

FIG. 6. Dependence of initial peak time t_p on temperature.

of time is given by

$$y=e^{-kt}\frac{a-b}{\omega_0}\sin(\omega_0 t-\delta)+bt+\frac{2k(a-b)}{\omega^2}, \quad (10)$$

where $\omega_0=(\omega^2-k^2)^{\frac{1}{2}}$ and $\tan\delta=2k\omega_0/(\omega_0^2-k^2)$.

(ii) When the plunger motion is fast enough so that $\frac{1}{2}<s<1$, the level rises to its first peak according to Eq. (10) but shortly after the peak the critical flow rate is reached and the level then moves downward at a constant rate until it intersects the line $y=bt+(2k/\omega^2)\times(a-b)$. It then proceeds according to the relation

$$y=e^{-kt}\left(1-\frac{1}{s}\right)\frac{a-b}{\omega_0}\sin(\omega_0 t)+bt+\frac{2k(a-b)}{\omega^2}. \quad (11)$$

The amplitude of the oscillations following the linear downswing becomes smaller as s increases.

In either (i) or (ii) the asymptotic behavior of the level is given by

$$y=bt+2k(a-b)/\omega^2. \quad (12)$$

When this state has been reached, stopping the plunger results in a damped oscillation of the level about its equilibrium position given by

$$y=-e^{-kt}\frac{a-b}{\omega_0}\sin(\omega_0 t-\delta). \quad (13)$$

D. Details of Low Level Difference Runs

(1) *Variation of Level Behavior with s*

The behavior of the helium level in the beaker as a function of time for various rates of plunger motion is, in general form, the same as that predicted by the theory of the preceding section. An example of the variation of the results with relative plunger speed at a particular temperature and film path length is shown in Fig. 5. Within the estimated error, the values of the flow rate calculated from the slope of the downswing following the initial peak increase with relative plunger speed s until the latter reaches a value of ~ 0.6. For larger s the flow is constant at the rate found in runs at large level differences. The difference from the expected value, $s=0.5$, is thought to be due to the fact that the linear portion of the downswing is very short for s only slightly greater than one-half, so that the slope cannot be accurately measured.

Only the initial features of the various plots of level position against time are illustrated in Fig. 5. In most of the runs the plunger was not stopped until at least 600 seconds had passed, and in some cases the slow upward trend of the level was observed for as long as 1400 seconds. In the total of more than seventy runs of this type which were made, there are only three in which the observed level difference falls below the line b. In the rest y_p, the average level difference between the observed points and b is positive and usually of the order of 0.002 mm to 0.020 mm. It has already been noted that in this region the level progresses upward in broad irregular steps and so the instantaneous value of y_p varies over even wider limits. Because of these effects, it is impossible to make an accurate comparison of this level difference with the value to be expected from the last term in Eq. (10). The order of magnitude of this term can be quickly estimated from data obtained from the final oscillations. For example, for $T=1.50$°K and $H=43$ mm, $k=0.009$ sec^{-1} and $\omega=0.107$ sec^{-1}. For $s=0.80$ at this temperature and path length, the plunger speed is such that $a-b$ is about 95×10^{-5} cm/sec. The calculated value of y_p is then 0.015 mm, which is of the same order of magnitude as the observed values.

TABLE I. Comparison of measured and computed values of the differences between initial peak times and final periods. $t_p-t_f/4$ is the measured value and $k/\omega_0^2+0.034/\omega_0$ is the value calculated from the data for the final oscillations only.

H (mm) \ T		1.30°K		1.50°K		1.70°K		1.90°K	
62.3	$t_p-t_f/4$	···	6.3	···	8.3	···	7.7	···	9.3
	$k/\omega_0^2+0.034/\omega_0$	0.80	0.84	1.01	1.32	1.82	2.01	···	5.2
54.2	$t_p-t_f/4$	···	5.6	···	4.2	···	5.5	···	8.8
	$k/\omega_0^2+0.034/\omega_0$	0.66	0.71	···	1.21	1.36	1.44	···	3.03
42.9	$t_p-t_f/4$	···	3.2	···	6.2	···	6.9	···	12.7
	$k/\omega_0^2+0.034/\omega_0$	···	0.68	0.74	1.09	···	1.50	···	···
29.5	$t_p-t_f/4$	···	4.9	···	6.2	···	5.6	···	9.6
	$k/\omega_0^2+0.034/\omega_0$	0.52	0.77	0.67	0.44	0.57	0.51	···	···

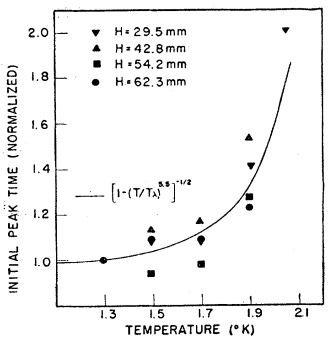

FIG. 7. Dependence of final periods t_f on temperature.

(2) *The Initial Peak and Final Oscillations*

Measurements on the group of experiments from which the results of the preceding section are taken, indicated that the periods of the final oscillations and the initial peak times were not related as the theory predicted. t_p may be calculated from the equation $\dot{y}=0$, which from Eq. (10) is seen to be

$$\cos(\omega_0 t_p-\gamma)=-(\omega_0/\omega)[b/(a-b)]\exp(kt_p), \quad (14)$$

where $\tan\gamma=k/\omega_0$ and the period of the final oscillations is involved through $\omega_0=2\pi/t_f$. From the geometry of the apparatus we have $b/(a-b)=A_b/A_c=0.017$, so that the right side of Eq. (14) is small. For A_c infinite, this term would be zero, and the initial peak time would be $t_p=(\frac{1}{2}\pi+\gamma)/\omega_0$. If we assume that the actual peak differs from this value by only a small interval, τ, Eq. (14) becomes

$$\sin\omega_0\tau=(\omega_0 A_b/\omega A_c)\exp(kt_p). \quad (15)$$

The largest value of k/ω_0 observed was 0.27; in most runs values of the order of 0.10 and less were found. Therefore $\omega_0\cong\omega$, and $\exp(kt_p)$ lies somewhere between 1 and 2. Under these conditions, the right side of Eq. (15) will never be larger than $2A_b/A_c=0.034$. Approximating the sine with its argument, we get $\omega_0\tau<0.034$. In terms of the quantities actually measured from the final oscillations—namely the period $t_f=2\pi/\omega_0$, and the damping coefficient, k—the results of this paragraph may be summarized in the inequality

$$t_p-t_f/4<k/\omega_0^2+0.034/\omega_0. \quad (16)$$

Because the results of the first group of experiments indicated that this relation was not obeyed, a second set was undertaken to check this point specifically. At each temperature and film path length, at least five independent determinations of the initial peak time were made and the final oscillations were observed at the end of two of these runs. The results are presented in Table I. The striking feature of the data presented in this table is that, far from obeying Eq. (16), $t_p-t_f/4$ is actually larger than $(k/\omega_0^2)+0.034/\omega_0$—in a majority of cases by an order of magnitude.

(3) *Temperature Dependence of t_p and t_f*

The dependence of the initial peak times and the final period on temperature are shown in Figs. 6 and 7. The results obtained at various film path lengths are comprised in one graph by using normalized times defined as $t_p(T)/t_p(1.30°\text{K})$ and $t_f(T)/t_f(1.30°\text{K})$.

It is evident from Fig. 6 that t_p rises with temperature much more rapidly than does t_f, and, in fact, follows the scaled reference line corresponding to $[1-(T/T_\lambda)^{5.5}]^{-\frac{1}{2}}$ reasonably well. In Eq. (8) there are two factors to which we may look for the source of the temperature variation of t_p and t_f: namely, ρ/ρ_s and $\int_0^{H/2}dz/d$. If it is assumed that the film thickness d is independent of temperature—there is experimental evidence for this in Bowers' experiments[12]—then the periods determined by Eq. (8) should go as $(\rho/\rho_s)^{\frac{1}{2}}$, which is $[1-(T/T_\lambda)^{5.5}]^{-\frac{1}{2}}$.[13] This suggests that at the start of a run it is the inertia of the superfluid which must be overcome to initiate film flow. The final periods should presumably show the same variation with temperature, unless we admit the possibility that the geometry of the film becomes slightly temperature-dependent when it is moving. Atkins,[6] whose measurements of oscillation frequencies lie within the range of the points in Fig. 6, combined his results with the data for ρ/ρ_s and found a slight increase in film thickness with temperature. This was in agreement with the experiments of Henshaw and Jackson,[14] who used an optical method to measure d.

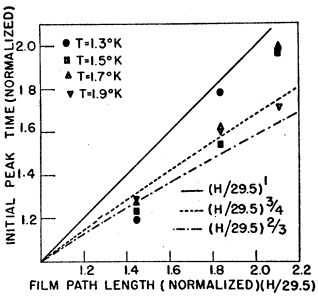

FIG. 8. Dependence of initial peak time t_p on film path length.

(4) *Dependence of t_p and t_f on Film Path Length*

The period of the helium oscillations depends on the shape of the film through the integral in the denominator of Eq. (8). Frenkel[15] and Schiff[16] considered that the shape of the film results from a balance between gravitational forces and the van der Waals forces between the walls and the helium atoms. Film thickness then varies with height according to the relation $d\propto h^{-\frac{1}{3}}$. Bijl, de Boer, and Michels,[17] regarding the quantum-

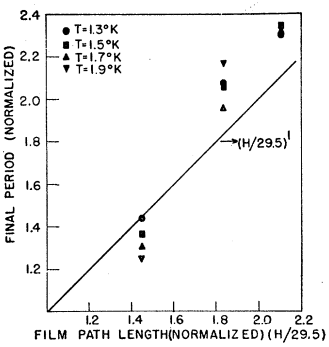

FIG. 9. Dependence of final periods t_f on film path length.

[12] R. Bowers, Phil. Mag. **44**, 1309 (1953).
[13] E. L. Andronikashvili, J. Phys. (U.S S.R.) **10**, 201 (1946).
[14] D. G. Henshaw and L. S. Jackson, Proceedings of the National Bureau of Standards Symposium on Low Temperature Physics, 1951, National Bureau of Standards Circular 519 (U. S. Government Printing Office, Washington, D. C., 1952), p. 183.
[15] J. Frenkel, J. Phys. (U.S.S.R) **2**, 365 (1940).
[16] L. I. Schiff, Phys. Rev. **59**, 838 (1941).
[17] Bijl, de Boer, and Michels, Physica **8**, 655 (1941).

mechanical zero-point energy as the most important factor in film formation, found $d \propto h^{-\frac{1}{2}}$. In the former case, evaluation of the integral in Eq. (8) leads to a period varying as $H^{\frac{3}{2}}$, while in the latter case the variation is as $H^{\frac{1}{2}}$.

In Figs. 8 and 9 the normalized initial peak times and final oscillation periods are plotted against a normalized film path length calculated by dividing H by 29.5, the shortest path length. Despite the wide spread in values corresponding to different temperatures, there is some agreement of t_p with the curve of $(H/29.5)^{\frac{3}{2}}$, whereas t_f lies above the straight line $H/29.5$. Such linear dependence of period on film path length would result from Eq. (8) if film thickness were inversely proportional to height. Probably neither t_p nor t_f obey the simple laws given here because the points at $H=42.8$ mm are in both cases slightly below the indicated curves, while the points for larger film path lengths are somewhat higher.

(5) *The Retardation Coefficient*

The retardation coefficient k was evaluated by plotting the logarithm of the amplitudes of the maxima and minima of the final oscillations against the number of elapsed periods. k is the slope of the line obtained multiplied by the period. The data for the 62.3-mm film path length are shown in Fig. 10. The various plots have been displaced vertically to prevent overlap.

The fact that the decay is exponential does not necessarily mean that the source of the damping lies in the velocity-dependent retardation term introduced into Eq. (9). Robinson,[18] in a detailed analysis of helium superfluid oscillations in narrow channels, has shown that if there is imperfect heat exchange between the bath and beaker, so that conditions are not completely isothermal, an exponential decay will occur. In this situation, the behavior of the level will still be described by an expression similar to Eq. (13), but the finite level difference necessary to maintain a steady subcritical flow—expressed by the last term in that equation—will depend on the lack of complete heat exchange and on the fountain effect. An upper limit to the magnitude of this level difference can be estimated by considering the case where the only means for heat transfer between the inside and outside of the beaker is through the vapor phase. It ranges from 0.009 mm at 1.30°K to 0.126 mm at 1.90°K, corresponding to temperature differences of 10^{-6} °K. For subcritical transfer rates such as are obtained when the relative plunger speed is less than one, these level differences will be proportionately smaller. Since even at the highest temperatures, level differences of this magnitude were not observed, it is concluded that there was good thermal contact through the walls of the glass beaker and its brass supporting cup. However, the fact that a temperature difference of 10^{-7} °K between the contents of the beaker and the bath could produce the observed exponential decay of the oscillations, makes it impossible to draw any definite conclusions concerning the existence of velocity-dependent retardation terms.

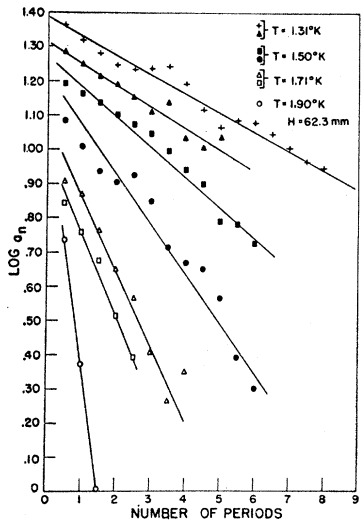

FIG. 10. Decay of the amplitude of final oscillations with time. The lines have been displaced vertically to prevent overlap.

Robinson's analysis indicates that not only does imperfect thermal contact between the bath and beaker introduce damping, but that it also affects the frequency of the oscillations. For the largest value of k/ω_0 observed, the period would be increased by only 2 percent, and for most of the experiments the effect is less than 1 percent and has been neglected.

V. DISCUSSION

The experiments in Sec. IV enabled us to study the motion of the helium film under two different sets of conditions. At the beginning of a run, the film is at rest and in *static* equilibrium with its surroundings. After the oscillations set up by the initiation of the flow have been damped, the level settles into an essentially steady state of motion, indicating that the flowing film has attained *dynamic* equilibrium with its surroundings. The essential result of our experiment is that the behavior of the film is different in the two sets of conditions. While it is true that the equations derived on the basis of the simple theory here presented describe both situations adequately, the most important parameter—the oscillation frequency—and its dependence on temperature and film path length differ for the two cases. The simplest hypothesis which will account for these differences between the first peak and the final oscillations, is that the static film differs from the moving film. At the beginning of a run the necessary changes in the film must take place in addition to the acceleration of the superfluid. From Eq. (8) there are two possible changes that can occur to produce the observed result.

(1) The superfluid content of the moving film might

[18] J. E. Robinson, Phys. Rev. 82, 440 (1951).

increase. If we assume that at a given temperature ρ/ρ_s in the static film is the same as for the bulk liquid, then below 1.80°K (where $\rho/\rho_s=0.70$) the observed change could not be produced even if the moving film became completely superfluid. At higher temperatures, such an increase would imply the existence of an unreasonably large temperature difference between the film and bulk liquid, because of the unique relation between ρ/ρ_s and the temperature.

(2) The film might alter its shape. The exact nature of this change in terms of the assumption that the film shape is represented by $d=D/h^n$ has been discussed in a preceding paragraph. The time necessary for such a change must be of the order of magnitude of the oscillation periods observed, or longer, and would be the interval necessary to redistribute the normal component of the film into the shape corresponding to dynamic equilibrium. This can be accomplished in several ways: flow of the normal fluid; exchange of material with the gas phase through evaporation and condensation; or flow of the superfluid with consequent transitions into or from the normal state to establish the right ratio of ρ/ρ_s.

The drift velocity of the normal fluid in the film is limited by its viscosity to only 10^{-6} that of the superfluid as deduced from the flow rate σ. From this it can be estimated that at least 10^3 seconds would be required to change the film thickness by 1 percent by flow of normal fluid over a distance of 1 cm. Because of the low heat capacities of the film and the substrate, evaporation will be effective only if there is available some other source of the comparatively large latent heat. A possibility is de-excitation of normal fluid into the superfluid state, but in the range of temperatures used, the heat so liberated per particle is less than one-tenth the latent heat of vaporization. Of course, similar considerations apply to condensation and consequent excitation of the normal fluid. To redistribute the film in this manner would therefore require the transfer of at least ten times as much material as would be necessary to accomplish the same effect by superflow and consequent transitions. In addition, the rate of either process would be ultimately limited by the rate of such transitions—i.e., by the interaction between normal and superfluid. Since the experimental results fit the form predicted by the simple two-fluid hydrodynamics which neglects this interaction, it is assumed to be small.

Such a slow change in the film characteristics might be expected to produce a gradual decrease in the periods t_i of the oscillations following the initial downswing. In the series of runs conducted to compare t_p and $t_f/4$, there were only three in which these oscillations were sufficiently well defined so that their periods could be measured accurately. The reason for this is that the factor $(1-1/s)$ in Eq. (11) limits the amplitude, particularly when large s values are used. In the three cases observed, the values found for $4t_p$, t_i, and t_f are given in Table II. In each case the value of t_i is intermediate between $4t_p$ and t_f, just what our considerations would lead us to expect.

TABLE II. Comparison of periods t_p of the observed initial oscillations with the corresponding initial peak times t_p and final oscillations t_f.

T (°K)	H (mm)	$4t_p$	t_i	t_f
1.30	54.2	106.8	91.7	84.3
1.50	54.2	100.8	95.3	84.1
1.50	62.3	128.8	108.5	95.5

There are several theories of the structure of the static film, but none of the mechanism of film flow. It is interesting to speculate on whether modifications introduced into existing theories of the film might lead us to expect an alteration in its shape when it is flowing. Bijl, de Boer, and Michels[17] attributed the existence of the film to a balance between gravitational forces and those resulting from the zero-point energy of the atoms condensed in their lowest state. The atoms are treated essentially as particles in a box, but it is possible that if a wave function adequate to describe their motion when flow is taking place were available, the corresponding zero-point energy would be altered in such a way as to change the stability conditions. Temperley[19] calculates the energy of the bound states of an atom in the van der Waals field of the wall and of the remaining helium atoms, and balances this against the gravitational energy to determine the shape of the film. He attributes film flow to the fact that not all the bound states that exist in the outer layers of the film are occupied. Atoms may move by transitions between these states. In this theory the possibility of an alteration in film shape arises from the perturbing effects on the bound states of the interaction which produces the transitions. The present state of the theory does not permit a calculation of such an effect. Even the more complete quantum-mechanical treatment of the He II problem by Feynman[20] has not yielded any insight into the nature of the film. Generally, from a thermodynamic point of view, the problem is one of minimizing the total free energy of the system. As the kinetic energy of the moving film would contribute to the free energy, a change in shape seems possible.

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[19] H. N. V. Temperley, Proc. Roy. Soc. (London) **A198**, 438 (1949).

[20] R. P. Feynman, Phys. Rev. **91**, 1291, 1301 (1953).