τ_{-} with reasonable accuracy. Further experiments are in progress to evaluate τ_{-} by this method.

The value of τ_{-} in C has been determined in the following way. We chose the thickness of the carbon absorber such that it stopped the same number of mesons as in the case of Al. Hence the same number of positives must have decayed in C as in Al in the same time although the number of decay electrons recorded due to positives alone were smaller in C than in Al, because the solid angle subtended by the D trays with respect to the total thickness of the absorber was smaller in case of C due to its larger thickness. The positive decay curve in Al has been reduced by a factor equal to the ratio of the solid angles and this reduced curve represents the positive decay curve in C as shown in Fig. 2. The mean life of negatives evaluated by subtraction, gives the value 1.93 ± 0.24 µsec.

Only a fraction τ_-/τ_+ of the negative mesons present in the cosmic-ray beam can decay in an absorber. Since the number of negative and positive mesons decaying in the interval t=0 to $t = \infty$ are separately known in the case of C, the ratio of positive to negative mu mesons can be determined from the equation

$$\mu^{+}/\mu^{-} = (n_{0}^{+}\tau_{-})/(n_{0}^{-}\tau_{+}), \qquad (2)$$

where n_0^+ , n_0^- are the number of decay electrons recorded from t=0 to $t=\infty$. Substituting the relevant values from the decay curves of C, the value of this ratio is found to be 0.95 ± 0.16 , agreeing well with the value 1.06 ± 0.03 obtained recently by Morewitz and Shamos.⁶

A detailed report of the experiment will be published in the Transactions of the Bose Research Institute. The authors acknowledge with thanks the advice and encouragement from Professor D. M. Bose, Director, Bose Institute during the course of the experiment.

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Phenomenology of the Pion-Nucleon S Waves*

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 \mathbf{A}^{S} was pointed out by Anderson and Fermi,¹ knowledge of the cross section for photoproduction of π^+ mesons from hydrogen at threshold combined with the π^-/π^+ ratio for photoproduction from deuterium and the ratio of radiative capture to charge exchange capture of π^- mesons in hydrogen as measured by Panofsky² allows one to calculate the difference in slope between the pion-nucleon S phase shifts at zero energy. Calling these phase shifts α_1 and α_3 for the states of I=1/2 and I=3/2, respectively, Bernardini³ reported at the Fourth Rochester Conference that recent photoproduction results give $\alpha_1 - \alpha_3 = (0.16)$ ± 0.04) $p/\mu c$, where p is the center-of-mass momentum of the pion and μ its mass. However, pion-nucleon scattering experiments at 33,4 40,5.6 46,7 and 58 8 Mev all indicate that near 40 Mev α_1 is about 10° while α_3 lies between -1° and -4° . The assumption that 40 Mev is a low enough energy for the pion-nucleon system to insure that the phase shifts below 40 Mev are simply proportional to momentum would therefore predict that $\alpha_1 - \alpha_3 = (0.3)$ $-0.35)p/\mu c$, and hence a zero energy cross section differing by a factor of four from the observed result. Further confirmation of the peculiar behavior of these phase shifts at low energy is given by Lederman's observation³ of five scatterings of 5-Mev $\pi^$ mesons in hydrogen, corresponding to a cross section of 20 to



FIG. 1. S phase shifts vs center-of-mass momentum for models given in Table I. Numbers indicate references for data.

25 mb. In spite of poor statistics, this positive result is by no means trivial, since linear extrapolation from 100 Mey would predict 0.1 mb, and from 40 Mev, 5 mb. The combination of these two results, namely that $\alpha_1 - \alpha_3$ is small and $\alpha_3 + 2\alpha_1$ is large near threshold clearly shows that α_3 and α_1 must have the same sign near threshold and not opposite signs as observed at higher energies.

The purpose of this note is to point out that this is precisely the behavior predicted by the S-wave potential model proposed by Marshak⁹ as extended by Woodruff.¹⁰ The predictions of this model, which uses an attractive exponential for α_1 and a hardcore attractive exponential tail potential for α_3 (see Table I), are compared with experiment^{4-8,11} in Fig. 1. Comparison with the threshold data is given in Table II. Since 20 mb is reached near 5.5 Mev, and the energy in Lederman's experiment is probably not as closely defined as this, the agreement with experiment is clearly very good.

It must be emphasized that the present success of this model does not justify taking it seriously in a quantitative sense. There is no good theoretical reason to expect a static potential model to hold up to energies equal to the rest mass of the pion, and as has already been pointed out in the nucleon-nucleon case,¹² a hard core, attractive tail potential is peculiarly sensitive to kinematic relativistic corrections. Further, until it is shown experimentally that the S phases are positive around 5 Mev, the possibility exists that both are negative, which would imply an energy dependence that could not be readily duplicated by any simple potential model. If this possibility is ignored, the results presented here are rather to be taken to indicate the qualitative features that a successful theory of the meson-nucleon force should exhibit, namely

TABLE I. Parameters for the potentials α_1 : $V = -V_0 \exp(-r/r_0)$; α_3 : $V = \infty$, $r \leq r_c$; $V = -V_0 \exp[-(r-r_c)/r_0]$, $r > r_c$.

Phase shift	Model No.	V ₀ (Mev)	ro (µc)/ħ	r _c (μc)/ħ
α1	T	277	0.336	
	ĨI	105	0.465	
<i>α</i> 3	III	187	0.428	0.386
	IV	163	0.460	0.375

TABLE II. Predictions at low energy.

Model		$\lim_{n \to \infty} (\alpha_1 - \alpha_3) \mu c / p$	σ_{tot} (5 Mev),
α_1	α3	$E \rightarrow 0$	mb
T	III	0.241	15
Ξ. ŤΤ	ĪĪĪ	0.211	12
Ī	ĪV	0.185	17
ĪI	IV	0.154	14
Experiment	Experiment		20-25

a purely attractive force in the S state of isotopic spin $\frac{1}{2}$ and a strong repulsion at short distances surrounded by a somewhat longer-range attractive region for the S state of isotopic spin $\frac{3}{2}$. They should also serve as a guide to experiments in this energy region. In particular, the prediction of a small π^+ cross section which switches from backward to forward scattering around 20 Mev and a greatly enhanced π^- cross section for elastic (rather than charge-exchange) scattering around 5-10 Mev should now be taken rather seriously.

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Shower Structure in the Higher Shower Maxima

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N an attempt to shed more light on the production of showers in connection with the second, third, and fourth maxima of the shower curve, the experimental arrangement as described previously¹ was slightly modified. In Fig. 1, Pb is the lead radiator of variable thickness, I and II are the two crossed counter trays which determine the apparent shower angle, and III is a counter tray that may be shifted between I and Pb. Fivefold coincidences (I, I, II, II, III) as well as anticoincidences (I, I, II, II, -III) were counted as a function of lead thickness Pb. The results, some of which are reproduced in Fig. 2, are clear cut though rather surprising. If tray III is arranged immediately beneath Pb (a=82)cm), the second and third maximum only appear in the anticoincidence curve. The inverse holds if III is lowered to practically a=0 cm. In intermediate positions of III the maxima are divided up between the two curves.

The simplest interpretation of these results is this: The particles emerging from the Pb are neutral ones; a considerable fraction of them decay before reaching tray I, thus giving rise to the





FIG. 2. Shower curves. Full curve: coincidence. Dashed curve: anticoincidence. (1 Torr = 1 mm mercury).

charged showers or pairs recorded by trays I+II. So these neutral particles have at least some of the characteristics of neutral Vparticles.

In the light of these results, a number of discrepancies that seemed to exist between the works of different authors now readily disappear. Since the apex of a shower does not lie in the lead layer but is more or less below, the real shower angle is considerably larger than supposed in our previous work. In fact Broussard and Graves² have observed in a cloud chamber that the second maximum occurs only for rather wide angle showers.

If a counter tray III is arranged beneath Pb and coincidences (I, I, III) or (I, I, II, II, III) are counted, the second and third maximum cannot appear. Likewise the higher maxima must be strongly suppressed, if trapezoidal counter arrangements are used fixing the shower apex within the lead layer. Investigations with both kinds of arrangements have been described and regarded as proof against the existence of higher shower maxima.

This counter work is being continued in combination with a cloud chamber.

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Perturbation Theory with Sommerfeld-Maue Wave Function

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T has been implicitly assumed by a number of authors¹ that the matrix element in the differential cross section for a process is the same whether Sommerfeld-Maue wave functions or non-