Effect of Coulomb Barrier on Meson Production by Nucleon-Nucleus Collisions

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(Received Februrary 19, 1954)

The effect of the Coulomb barrier on charged meson production by nucleon-nucleus collisions is studied by a classical and qualitative treatment of the Coulomb interaction between the emitted meson and the residual nucleus. Especially, the discrepancy in the behavior of positive and negative meson production cross sections as functions of atomic number can be understood qualitatively by this simple argument.

MEASUREMENTS have recently been performed on the variation of the positive and negative meson production cross sections with atomic number which result from the interaction of a 340-Mev proton beam with target nuclei.[1] The targets used are Be, C, Al, Cu, Ag, Pb, and the differential cross section is studied by photographic emulsion exposed at 90° to the direction of the proton beam in the laboratory system. The kinetic energy of the observed mesons ranges from 12.5 to 35 Mev.

A remarkable fact revealed by these observations is that there is a systematic difference in the behavior of the negative and positive production cross sections: the cross section per nucleus for π^- production increases steadily as a function of the atomic number while that for π^+ production shows a tendency to level off or even decrease towards heavier nuclei for all observed meson energies. It would be very hard to understand this without referring to the Coulomb effect, since other effects such as the scattering and absorption of the incident proton and the secondary meson would work in the same direction for both π^+ and π^-.[2] There has, however, been no reliable method which gives the Coulomb effect on the meson production correctly even in a qualitative sense.

As is well-known in the theory of beta decay, a simple way of introducing the Coulomb correction is to multiply the uncorrected cross section by the correction factor $C=2\pi\xi/[1-\exp(-2\pi\xi)]$, $\xi=\pm Ze^2/\hbar v$. This method cannot be used in our case, however, since the nuclear Coulomb field does not belong to a point charge but to an extended charge distribution.[3] In such cases, one uses as the correction factor the absolute square of the regular solution of the meson wave equation in the nuclear Coulomb field evaluated at the nuclear surface. This again does not give a good result because the nuclear size is not smaller than the de Broglie wavelength of the meson considered, which means that one needs a knowledge of the irregular solution of the meson wave equation in addition to the regular one. Finally, even the complete outside solution would not provide a satisfactory estimation of the Coulomb effect since this is essentially determined by the behavior of the meson inside the nucleus which belongs to the yet unknown properties of mesons. At the present stage, it is therefore almost useless to solve the meson wave equation exactly in the region outside of a nucleus. As is mentioned above, this is mostly due to the fact that the nuclear size is large compared to the meson wavelength. This, however, means at the same time that the magnitude of the Coulomb field is small compared to the meson kinetic energy, which leads us to another kind of approximation method.

As is well-known, the Coulomb effect can be understood qualitatively in the following way: it accelerates the π^+ and decelerates the π^-. Hence, the π^+ spectrum has fewer slow particles, and π^- spectrum more slow particles, than they would have in the absence of the Coulomb effect. Now, one may describe the ejection of a meson out of a nucleus classically in our problem, since the Coulomb field is small and slowly varying compared to other quantities. In this sense, the momentum $k_0^{\pm}$ of $\pi^{\pm}$ at the moment of leaving the nucleus would be different from the observed momentum $k^{\pm}$, and it would be rather $k_0^{\pm}$ than $k^{\pm}$ that provides a measure for the initial momentum with which a meson is produced in the nucleon-nucleon collision. For example, a π^+ meson of the observed kinetic energy E, which is produced on the surface of a nucleus with the Coulomb barrier V_c would have an initial kinetic energy $E_0=E-V_c$, while a π^- meson of the same kinetic energy would have $E_0=E+V_c$. This would change the kinematics of the usual treatment which neglects the effect of V_c, and the atomic number dependence of the cross section for fixed E would be considerably different from that of fixed E_0 as one goes towards heavier nuclei if the energy spectrum of the

[1] R. Sagane and W. Dudziak, Phys. Rev. **92**, 212 (1953); University of California Radiation Laboratory Reports UCRL-2284, 2304, 2317, 1953 (unpublished).

[2] Recently, S. Gasiorowicz [Phys. Rev. **93**, 843 (1954)] tried to explain this experiment by taking account of the energy degeneration of the incident proton in the nucleus and the reabsorption of the meson by the nucleus it traverses. His results depend, however, on an unestablished assumption that the protons tend to be confined to a region of somewhat smaller radius compared to the nucleus as a whole. The validity of Gasiorowicz's picture may be examined by considering meson production by *neutron*-nucleus collisions, photomeson production by nuclei, etc.

[3] For a meson of 33 Mev produced by lead, the correction factors C_- and C_+ are about 6.3 and 0.012, respectively. It is to be noticed that use of C_+ for π^+ mesons would be especially meaningless since it describes the case where the momentum of the π^+ meson becomes imaginary at the point nucleus due to the infinitely high Coulomb potential, while in our problem the π^+ meson may preserve its wave character up to the center of the nucleus.

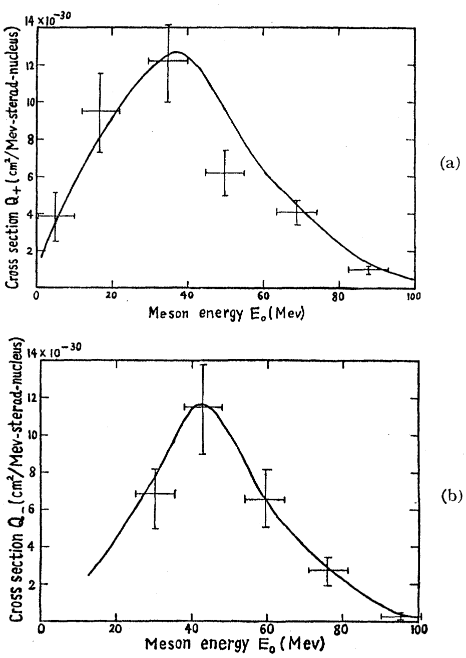


FIG. 1. (a) The energy spectrum Q_+(Pb) as a function of the initial kinetic energy E_0 based on the experimental data of reference 6. The solid curve is a hypothetical energy spectrum which is assumed to be valid for all elements in our analysis. (b) The energy spectrum Q_-(Pb) as a function of the initial kinetic energy E_0 based on the experimental data of reference 6. The solid curve is a hypothetical energy spectrum which is assumed to be valid for all elements in our analysis.

meson is strongly dependent on its energy. We shall show in the following that the principal feature of the above experiments can actually be understood in accordance with this idea by taking $E_0=E\mp V_c$ as the crudest approximation for the initial kinetic energy of the secondary meson.

First, we shall write the production cross section of a meson with the kinetic energy E as

$$d\sigma_\pm(E;Z,N)/dEd\Omega=P_\pm(E;Z,N), \tag{1}$$

where Z and N are the proton and neutron numbers of the target nucleus. This can be rewritten as the cross section for the *initial* kinetic energy E_0:

$$d\sigma_\pm(E_0\pm V_c;Z,N)/dE_0d\Omega=Q_\pm(E_0;Z,N), \tag{2}$$

in conformity with the classical approximation mentioned above.[4] For fixed Z and N, this gives the meson energy spectrum in terms of the initial kinetic energy E_0 and is therefore the one to be compared with the energy spectrum of the ordinary theoretical treatment in which the Coulomb effect is not taken into account.[5] In the same way, the atomic number dependence of Q can be compared with the theoretical expectation. In order to derive the atomic number dependence of Q from the experimental data P, however, one has to know the energy dependence of Q for all elements considered.

The energy spectrum P at 90° has heretofore been measured only for C and Pb.[6] Unfortunately, these are not accurate enough for our purpose. The corresponding spectrum $Q(E_0;\text{Pb})$ is found to be an increasing function of E_0 up to a maximum at ~40 Mev and then to decrease fairly rapidly for larger E_0 (Figs. 1). $Q(E_0;\text{C})$, on the other hand, has a very broad maximum at ~40 Mev. It is expected that the spectrum Q for all heavy elements is not very different from $Q(E_0;\text{Pb})$. Meanwhile, Q is nearly identical with P for light elements irrespective of its energy dependence since V_c is small in these cases. We may therefore assume without serious error that the energy spectrum is proportional to that of Pb for all elements. (The multiplicative factor is not important for our purpose.) We shall assume for simplicity that the energy spectra for all elements are given by the solid curves in Figs. 1, which are drawn more or less arbitrarily out of the multitude of curves which might fit experimental data just as well.

Once the energy spectrum is fixed, one can easily calculate the atomic-number dependence of Q from that of P. In Figs. 2, the spectra Q thus determined are given together with the experimental spectra P. It is seen that the atomic number dependence of Q for both π^+ and π^- can now be approximated by $A^{\frac{2}{3}}$-curves, as is expected from the interaction of the meson with the nucleus.[7] One may thus be lead to the conclusion that the Coulomb barrier is actually the agent which gives rise to the discrepancy between the π^+- and π^--cross sections. One should, however, keep in mind that this is based upon many assumptions and approximations. Some of these points will be discussed in the following.

1. In the first place, it should be stressed that we have treated the effect of the Coulomb barrier in a classical way. Namely, we have treated k and k_0 as simultaneously diagonal operators, and this introduces an error of the order of $[4\alpha Z/(kR)^2]\cdot[(\mu+E)/k]$ for sufficiently large k.[8] This is about 20 percent for a

[4] $Q_\pm$ as a function of E_0 is obtained by shifting $P_\pm$ as a function of E to the left or right by the amount V_c (for π^+ or π^- mesons, respectively).

[5] E. M. Henley, Phys. Rev. **85**, 204 (1952); Passman, Block, and Havens, Phys. Rev. **83**, 167 (1951).

[6] C. Richman and H. Wilcox, Phys. Rev. **78**, 496 (1950); M. Weissbluth, Ph.D. thesis, University of California, 1950 (unpublished); Richman, Weissbluth, and Wilcox, Phys. Rev. **85**, 161 (1952).

[7] Better agreement may be obtained taking account of the collision mean free path of the incident proton and the absorption of the produced meson in the nucleus. Both effects will reduce the production cross section by a factor $A^{-\frac{1}{3}}$. More precisely, the latter is known to multiply the cross section by a factor

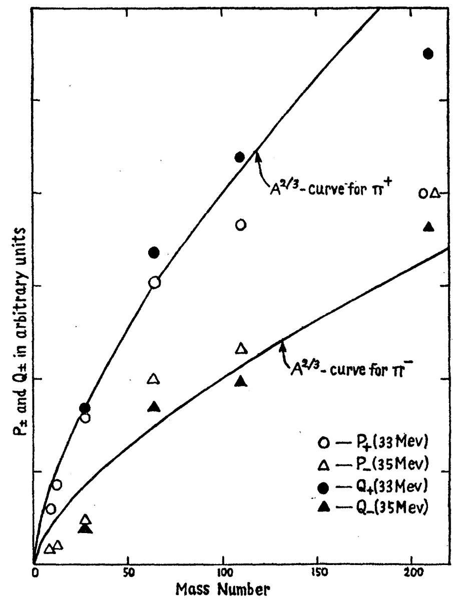
$$3\{\tfrac{1}{2}(1/X)-(1/X^3)+(1/X^3)(1+X)e^{-X}\},$$

where $X=2R/\lambda_a$, $R=a_0A^{\frac{1}{3}}$ being the nuclear radius, and λ_a the absorption mean free path of a meson in the nucleus. See Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).

[8] An estimate of this error is made by evaluating the upper limit for

$$\langle[\Delta,\ (\mu+E\pm\alpha Z/r)^2-\mu^2]_-\rangle/\langle\Delta\rangle\cdot\langle(\mu+E\pm\alpha Z/r)^2-\mu^2\rangle,$$

where $\langle\Delta\rangle$, etc., mean the expectation value of Δ, etc., with respect to the meson wave function.



(a)

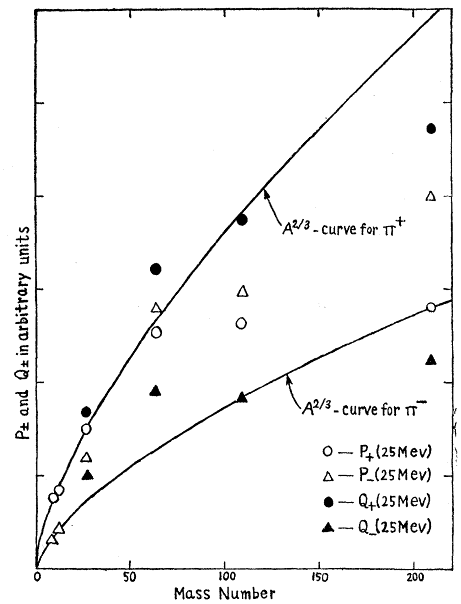


FIG. 2(b)

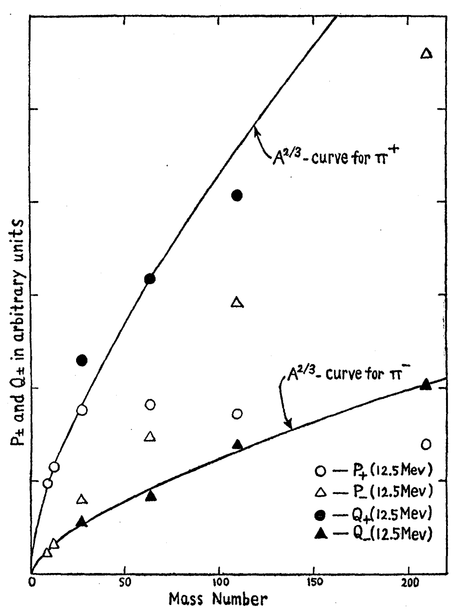


FIG. 2(c)

FIG. 2(a). The variation of Q_+(33 Mev) and Q_-(35 Mev) as well as the experimental data P_+(33 Mev) and P_-(35 Mev) with mass number. The $A^{\frac{2}{3}}$-curves are superimposed on the data for comparison. (b) The variation of $Q_\pm$(25 Mev) as well as the experimental data $P_\pm$(25 Mev) with mass number. The $A^{\frac{2}{3}}$-curves are superimposed on the data for comparison. (c) The variation of $Q_\pm$(12.5 Mev) as well as the experimental data $P_\pm$(12.5 Mev) with mass number. The $A^{\frac{2}{3}}$-curves are superimposed on the data for comparison.

meson of kinetic energy 33 Mev produced by lead. Obviously, the errors become larger for mesons of lower energies and thus our method cannot be applied to very low-energy mesons.

2. We do not know the nuclear radius exactly. Accordingly, we are not sure up to what point the pure Coulomb potential can be used. In this note, we have adopted $R=1.5\times10^{-13}\times A^{\frac{1}{3}}$ cm as the nuclear radius. The result of our analysis, however, would not vary very much for a slight change in the assumed nuclear radius.

3. In the formula (2), we have adopted the surface value $E\pm V_c$ as the initial kinetic energy of the produced meson. Strictly speaking, this is not correct since a meson can be produced at any place within the nucleus and we do not know anything at all about the "meson-nucleus potential." The situation is exactly the same in the case of the ordinary treatment where a meson is assumed to be produced in a hypothetical nucleus which has no surrounding Coulomb field.[5] In so far as one is confined to the analysis of the Coulomb effect in terms of the formulas (1) and (2), one need not worry about the details of the behavior of mesons within the nucleus since they will cancel each other, being common to both (1) and (2).

4. Finally, an objection might be raised against the assumptions we have imposed on the energy spectrum. The present experimental data are very crude so that they cannot determine the energy spectrum with sufficient accuracy. The spectrum used in our discussion is so chosen out of the variety of spectra which fit the data that the Coulomb effect is seen clearly. It is easy to see that another choice would modify our result to a considerable extent although it would not change the qualitative feature of our argument. One should also notice that the energy spectrum is slightly different for different elements. Since this is disregarded in our treatment, some amount of error will be involved systematically in the atomic number dependence of Q. It is therefore highly desirable to have accurate energy spectra for meson production in order to settle these points unambiguously.

Our treatment of the Coulomb effect is merely of a

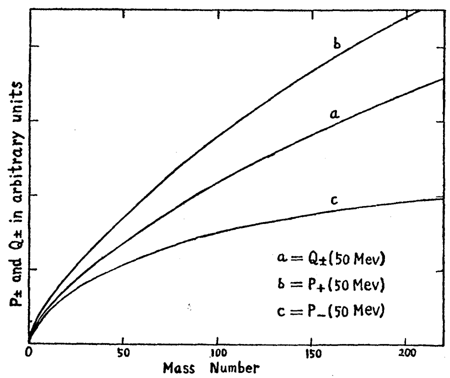


FIG. 3. The predicted variation of P_+ and P_- with mass number at a meson energy $E=50$ Mev. Here $Q_\pm$ are assumed to be proportional to $A^{\frac{2}{3}}$.

provisional character and we must refrain from coming to the final conclusion until more accurate experimental data as well as a more refined theoretical treatment are available. In the following, we shall give some comments which might be useful for checking or establishing our way of approach.

1. If one knows the meson energy spectrum for all elements and if furthermore one knows the atomic number dependence of Q for one meson energy E_0, one can in principle determine the atomic number dependence of Q for any other meson energy. This statement may be useful in checking the consistency of data at various energies. The results given in Figs. 2 are consistent with each other within the accuracy of experiments and theory. Even more interesting will be the prediction of the atomic number dependence of P for $E \gtrsim 50$ Mev, since we are then in the energy region beyond the maximum of the energy spectrum. Curves are shown in Fig. 3 to illustrate what will happen in such cases if one assumes an $A^{\frac{2}{3}}$ variation for $Q_\pm$. It is seen that P_+ will now increase more rapidly than P_- as a function of atomic number, contrary to the low-energy case. This conclusion depends, of course, critically on the shape of the energy spectrum and will be modified considerably by further experimental findings.

2. From the observation of the meson production cross sections at other angles than 90°, one may be able to find the same effect of the Coulomb barrier. The atomic number dependence of the 53-Mev π^+-production cross section at 0° seems to be not inconsistent with our expectation.[9] D. L. Clark measured the relative cross sections for 40-Mev mesons produced by 240-Mev protons in seven elements.[10] The π^+ mesons were observed at $135° \pm 15°$ (in the laboratory) with respect to the beam direction and the π^- mesons at $45° \pm 15°$. These data may be analyzed by our method as soon as the meson energy spectrum is known for this proton energy. Here, we shall only remark that 40 Mev will be close to the maximum kinetic energy of the π^+ mesons at 135° and thus the Coulomb effect will be unimportant in this case.

3. Our method would throw a new light on the energy dependence of the positive-to-negative ratio of mesons produced by nucleon-nucleus collisions. The ratio Q_-/Q_+ calculated from the present data for Pb seems to be slightly inconsistent with the theoretical expectation of Chew and Steinberger[11] since this ratio is too large for high meson energies. In view of the crudeness of the present data, however, it would be premature to draw conclusions about this point.

4. Finally, it may be mentioned that a similar argument will hold in other problems such as the atomic number dependence of the cross section of various nuclei for photomeson production.

The author would like to thank Professor R. Sagane for informing him of the experimental results prior to publication, and Professor Y. Namu and Professor T. Miyazima for many valuable discussions. He would like to express his gratitude to Professor R. Oppenheimer for the hospitality extended to him at the Institute for Advanced Study.

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[9] Hamlin, Jakobson, Merritt, and Schulz, Phys. Rev. **84**, 857 (1951). The meson spectrum in the direction of an incident 341-Mev proton beam has been measured only in the case of a carbon target by Cartwright [University of California thesis, April 16, 1951 (unpublished)]; and by Merritt, Schulz, and Heinz (unpublished).

[10] D. L. Clark, Phys. Rev. **87**, 157 (1952). He also measured the atomic number dependence of 20-Mev π^+ mesons produced by 240-Mev protons in several targets. See D. L. Clark, Phys. Rev. **81**, 313 (1951).

[11] G. Chew and J. Steinberger, Phys. Rev. **78**, 497 (1950).