In summary then, it may be said that the continuum theory may be brought into rather detailed agreement with the measured angular distribution of 3.7-Mev neutrons scattered from bismuth. The theory in the case of cadmium and tin exhibits only the general features of the experimental results and possibly demonstrates a need for a revision of the theory.

ACKNOWLEDGMENT

In conclusion, the authors wish to express their appreciation to Dr. L. Eisenbud for many helpful conversations, to Mr. H. S. Hans for help with the computations, and to Dr. W. F. G. Swann, Director of the Bartol Research Foundation, for his continuing interest in this problem.

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A Study of the Interaction of Neutrons with Moderating Materials

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In the theoretical part of this paper we consider the time dependence of the slowing-down process for certain simple cases: (1) For free nuclei at rest an approximate expression is found for the velocity distribution. (2) The existence of a bond in one direction while the motion is unhindered in other directions is found to have little influence on the slowing-down process. (3) The slowing-down process in a monatomic gas is discussed quantitatively on the assumption of a Maxwellian neutron velocity distribution. The experimental part of the paper reports measurements with a pulsed neutron source and a time analyzer, in which the variations of the neutron velocity spectrum with time were studied by transmission measurements. Measurements on water show that the neutrons approach thermal equilibrium with the moderator at a rate which is in agreement with the theoretical predictions. The equilibrium temperature of the neutrons is found to agree with the moderator temperature over a wide temperature range. At low temperatures evidence is found for the effect of the lattice forces on the slowing-down process. Measurements on the initial slowing-down process for neutrons in heavy water indicate satisfactory agreement with the theory for slowing down by free deuterons at rest. The various factors which determine the rate of decay of the neutron intensity in a moderator are discussed, and the feasibility of using measurements of the decay rate to determine the diffusion and absorption properties of the moderating material as well as the geometrical buckling of complicated moderating systems is considered. In an appendix tables are given for the absorption of thermal neutrons in $1/v$ absorbers.

MODERATING materials are used for the production of thermal neutrons from a fast neutron source, and are required to slow down neutrons to thermal energies in a time which is small compared to the lifetime of the neutrons. The slowing-down time is therefore of considerable practical interest. Since the slowing-down times are of the order of 10 or 100 μsec, the process can be readily studied by the technique of the pulsed neutron source and detector developed mainly for neutron time-of-flight spectroscopy.[1] It is the purpose of the present article to discuss briefly a number of experiments performed with this method, and their theoretical interpretation. A more detailed account of the subject is given in the author's thesis, published elsewhere.[2]

PART I. THEORY

A. General

We consider the case of neutrons produced with the initial velocity v_{in} in an infinite uniform moderator during an infinitely short time interval. The subsequent slowing-down process is in principle uniquely defined by the probability $g(v_0,v)dv$ per unit path for a neutron to suffer a collision in which the velocity is changed from v_0 to the velocity interval dv at v. The velocity distribution $f(v,t)$ at the time t after the production of neutrons is given by the space- and angle-independent Boltzmann equation

$$\frac{\partial f(v,t)}{\partial t}=-\frac{vf(v,t)}{l(v)}+\int v_0 f(v_0,t)g(v_0,v)dv_0 +\delta(v_{in}-v)\delta(t), \quad (1)$$

where $l(v)$ is the total mean free path including absorption of the neutrons in the moderator. In the case when the absorption cross sections are inversely proportional to the relative neutron velocity, the velocity distribution at a given time is the same as for no absorption, since neutrons of all energies are absorbed at the same rate. Since we will in the following consider only this case, we may consider $l(v)$ to be the mean free path for scattering alone.

Except for the problem of solving the integro-differential equation (1) for an arbitrary slowing-down kernel $g(v_0,v)$, the main difficulty lies in the fact that for moderators of practical interest, where in general

[1] Rainwater, Havens, Wu, and Dunning, Phys. Rev. **71**, 65 (1947).
[2] G. F. von Dardel, Trans. Roy. Inst. Technol. Stockholm No. 75 (1954).

account has to be taken of the chemical bonds and thermal excitation in the substance, the expression for the kernel becomes exceedingly complicated. For the theoretical interpretation of the present measurements, the actual moderating substance was therefore replaced by simple idealized structures for which the theory leads to manageable expressions for the slowing-down kernel.

As long as the neutron energy is much higher than the thermal excitation and binding energy of the atoms in the substance, the problem is reduced to the relatively simple case of slowing down against free nuclei at rest, which will be discussed in Sec. B. As the neutron energy decreases, the forces which keep the nuclei together in molecules begin to make themselves noticeable, and the slowing down process will proceed more slowly. Fermi[3] has studied the case of nuclei bound by an isotropic oscillator potential. In this case the energy loss occurs in multiples of the oscillator quanta. In practice the quantum energies of the various degrees of freedom of the molecule may well differ considerably from each other, so that the case of an anisotropic binding is of some interest and will be discussed in Sec. C, neglecting thermal motion.

When the neutron energy has dropped below the value necessary to excite the vibrational and rotational degrees of freedom of polyatomic molecules, the subsequent slowing-down process will occur by energy transfer to the translational degrees of freedom of the molecule as a whole, if the bonds between the molecules can be neglected as is the case in gases and, approximately, in liquids and solids for temperatures above the Debye temperature. A monatomic gas of nuclei having the mass of the molecules should be a good substitute for the actual substance. This case will be discussed in Sec. D.

When the temperature of the moderator is lower than the Debye temperature of the substance, the model of a gas is no longer a good approximation. Kleinman[4] has derived expressions for the differential cross section of a Debye crystal which in principle makes it possible to calculate the slowing-down process.

B. Moderator Containing Free Nuclei at Rest

The time-dependent velocity distribution for this case has been studied by Marshak[5] and Walen.[6] If the initial energy of the neutrons is very high, their velocity distribution can, for dimensional reasons, be written in the form

$$f(v,t)dv=\varphi(x)dx, \tag{2}$$

which depends only on the dimensionless parameter $x=vt/l_0$. l_0 is the mean free path in the moderator, which we assume to be independent of the neutron velocity. It is shown in reference 2 that the velocity distribution at time t, $\varphi(x)$, is related to the time distribution of neutrons with the velocity v, $F(x)$, as introduced by Marshak,[5] by the relation

$$F(x)=\frac{\xi}{2}x\cdot\varphi(x), \tag{3}$$

where $\xi=1+[r^2/(1-r^2)]\log r^2$ is the average logarithmic energy loss per collision, and r is the ratio $(M-m)/(M+m)$, m and M being the masses of the neutron and the scattering nucleus, respectively.

Introducing (2) and the proper kernel into (1), it is easy to obtain expressions for the nth positive and negative moments of $\varphi(x)$,

$$\langle x^{+n}\rangle=\frac{2}{\xi}\prod_{\nu=1}^{n-1}\nu\left\{1-\frac{2(1-r^{\nu+2})}{(\nu+2)(1-r^2)}\right\}^{-1}, \tag{4}$$

$$\langle x^{-n}\rangle=\prod_{\nu=1}^{n}\frac{1}{\nu}\left\{\frac{2(r^{2-\nu}-1)}{(\nu-2)(1-r^2)}-1\right\}. \tag{5}$$

As in Marshak's[5] treatment, it has been attempted to find an approximate expression for $\varphi(x)$ of the form

$$\varphi(x)=Ax^{2/(1-r^2)-1}e^{-x-b/x}. \tag{6}$$

As is shown in Fig. 1, quite good agreement was obtained between the moments of this function and the required moments, Eqs. (4) and (5), with the values for A and b given in Table I. For nuclei of unit mass, Eq. (6) is the exact solution.

The form of the function (6) for best fit in the cases $M/m=2$, 9, and 15 is shown in Fig. 2 and compared to curves derived from Marshak's[5] article.

C. Incompletely Bound Nuclei

As a simple example of the incompletely bound nucleus, we consider the case of a nucleus which is

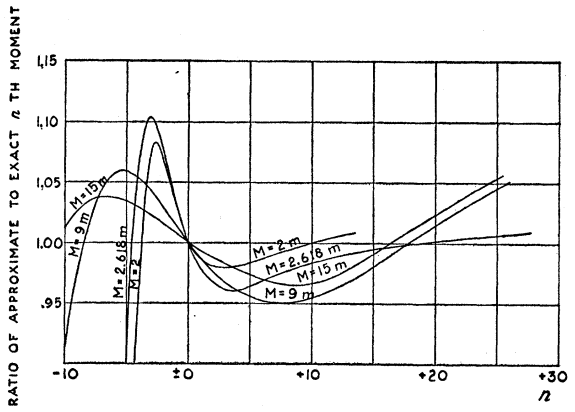

FIG. 1. Ratio of the moments of the trial function (6) to the required moments (4) and (5) of the time dependent neutron velocity distribution in the case of free nuclei at rest.

[3] E. Fermi, Ricerca sci. **7**, 13 (1936).
[4] D. A. Kleinman, Phys. Rev. **90**, 355 (1953).
[5] R. E. Marshak, Revs. Modern Phys. **19**, 185 (1947).
[6] R. Walen, "Recueil de travaux de l'institut de recherches sur la structure de la matière," Belgrad, July 1952 (unpublished).

bound to a plane by a harmonic restoring force but is free to move parallel to the plane. The angular frequency of the harmonic motion perpendicular to the plane is ω.

By integration, over the proper volume in momentum space and over the orientations of the plane, of the matrix element[7] for the energy transfer $n\hbar\omega$ to the motion perpendicular to the plane and $\Delta\hbar\omega$ to the unquantized motion parallel to the plane, we obtain the differential scattering cross section for the energy loss $E_0-E=(n+\Delta)\hbar\omega$

$$\sigma_n(E_0,E)=\frac{1}{n!}\frac{1}{1-r^2}\frac{\sigma_f}{E_0}\int_{\cot\theta'}^{\cot\theta''}(\Delta t^2)^n \times\exp(-\Delta t^2)(1+t^2)^{-\frac{1}{2}}dt. \quad (7)$$

σ_f is the scattering cross section of the free nucleus at rest. The limits of integration are given by

$$\sin\theta'=(\Delta\hbar\omega\cdot M/m)^{\frac{1}{2}}/(E_0^{\frac{1}{2}}-E^{\frac{1}{2}}); \quad \sin\theta''=(\Delta\hbar\omega\cdot M/m)^{\frac{1}{2}}/(E_0^{\frac{1}{2}}+E^{\frac{1}{2}}). \quad (8a,b)$$

The expression (7) has been calculated numerically for initial neutron energies up to $2.2\times\hbar\omega$ and for a number of masses M of the nuclei. Figure 3 shows the result for $M=4m$.

The effect of the incomplete bond on the slowing-down process was studied by the Monte-Carlo process

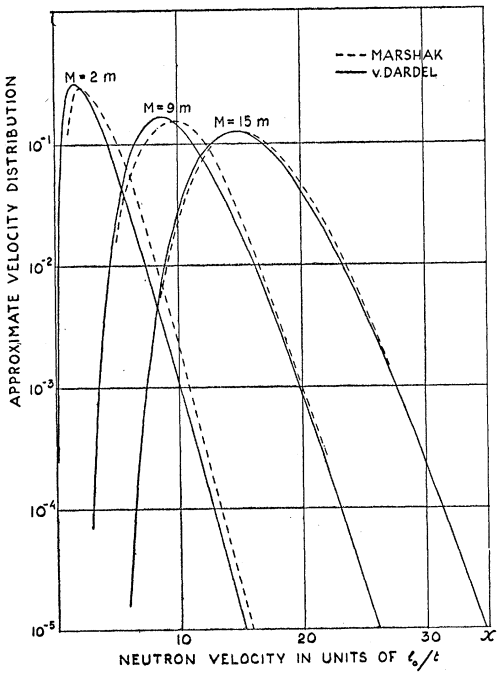

FIG. 2. Approximate neutron velocity distribution as a function of the parameter $x=vt/l_0$ as given by Eq. (6) and as derived from the curves in reference 5.

[7] G. Placzek, Phys. Rev. **86**, 377 (1952).

TABLE I. Optimum parameters in the trial function (6) for best fit to the required moments (4) and (5).

M/m		1	2	2.618	9	15
$2/(1-r^2)$	exact	2.00	2.25	2.50	5.5556	8.533
	approx.	2.00	2.25	2.50	5.500	8.500
$y=2\sqrt{b}$		0	1.9	2.8	12.0	20.5
b		0	0.903	1.96	36.0	105.1
A		1.000	1.528	1.929	3.610	0.681

and gives a rather insignificant decrease in the speed of the process. In consequence, we may neglect the effects of the bonds to a first approximation until the neutron energy is below the quantum energy of the weakest bond.

D. The Slowing Down of Neutrons in a Monatomic Gas

The differential cross section per molecule of a monatomic gas of temperature T for a collision which changes the neutron energy from $E_0=\omega_0^2kT$ to dE at $E=\omega^2kT$ is given by Wigner and Wilkins,[8]

$$\sigma(E_0,E)=\frac{1}{(1-r^2)}\frac{\sigma_f}{E_0}\frac{1}{\sqrt{\pi}}\{\exp(\omega_0^2-\omega^2)\,\mathrm{Erf}(b\omega_0-a\omega) -\mathrm{Erf}(a\omega_0-b\omega)-|\exp(\omega_0^2-\omega^2)\,\mathrm{Erf}(b\omega_0+a\omega) -\mathrm{Erf}(a\omega_0+b\omega)|\}, \quad (9)$$

where

$$a=\tfrac{1}{2}[(M/m)^{\frac{1}{2}}-(m/M)^{\frac{1}{2}}]; \quad b=\tfrac{1}{2}[(M/m)^{\frac{1}{2}}+(m/M)^{\frac{1}{2}}]. \quad (10a,b)$$

A derivation of this formula is also given in reference 2.

It is of interest to calculate the total cross section,

$$\sigma_t(E_0)=\sigma_f\frac{1}{\beta^2\sqrt{\pi}}[\beta\exp(-\beta^2)+(2\beta^2+1)\,\mathrm{Erf}\beta], \quad (11)$$

and the mean energy loss per collision,

$$(E_0-E)_{Av}=-\frac{4kT}{(M/m)+1}+\frac{2kT}{[(M/m)+1]^2} \times\frac{[\beta^3+(5/2)\beta]\exp(-\beta^2)+(2\beta^4+6\beta^2+\frac{3}{2})\,\mathrm{Erf}\beta}{\beta\exp(-\beta^2)+(2\beta^2+1)\,\mathrm{Erf}\beta}, \quad (12)$$

where $\beta=(ME_0/mkT)^{\frac{1}{2}}$. Expression (11) was first derived by Tait.[9]

Since the integral equation (1) cannot be solved explicitly for the kernel corresponding to (9), we have recourse to the following approximate treatment. We assume the neutron velocity spectrum to be Maxwellian at all times, characterized by a single parameter, the neutron temperature T_n, which we expect to de-

[8] E. P. Wigner and J. F. Wilkins, U. S. Atomic Energy Commission Report AECD-2275 (unpublished).

[9] P. G. Tait, Trans. Roy. Soc. (Edinburgh) **33**, 74 (1886).

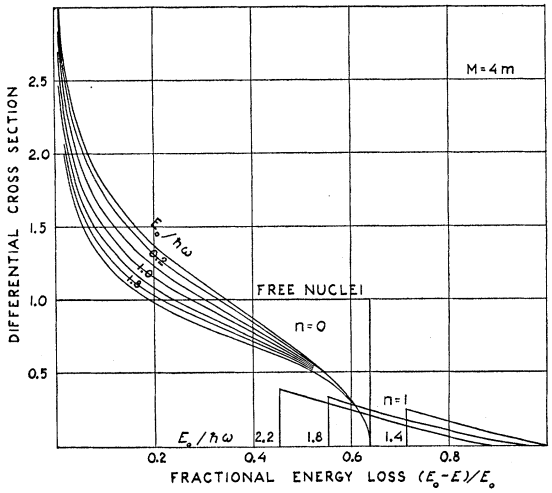

FIG. 3. Average differential cross section as a function of the fractional energy loss for an incompletely bound nucleus of mass $M=4$ m in units of the value for the free nucleus. Parameter: Initial neutron energy in units of the quanta $\hbar\omega$ of the bond. Upper curves: elastic collisions ($n=0$). Lower curves: collisions exciting the nucleus to the first excited state ($n=1$).

crease during the slowing-down process and eventually to approach the temperature T of the moderator.

From (11) and (12) we find for the average rate of energy transfer from the neutrons to the moderating gas the equation:

$$\frac{d\bar{E}_n}{dt}=-\frac{8}{\sqrt{\pi}}\frac{(2kT/m)^{\frac{1}{2}}}{l_b}\cdot\frac{(M/m)^{5/2}}{(M/m+1)^{7/2}}\times\left[1+\frac{T_n-T}{T(1+m/M)}\right]^{\frac{1}{2}}k(T_n-T), \quad (13)$$

where l_b is the mean free path if the nuclei were rigidly bound. In analogy to heat transfer problems, we define as the (asymptotic) heat transfer coefficient between the neutrons and the moderator gas

$$\alpha_0=\frac{8}{\sqrt{\pi}}\frac{(2kT/m)^{\frac{1}{2}}}{l_b}\cdot\frac{(M/m)^{5/2}}{(M/m+1)^{7/2}}k. \quad (14)$$

Since the average neutron energy $\bar{E}_n$ for a Maxwellian spectrum is $\frac{3}{2}kT_n$, Eq. (13) is a differential equation for T_n, with the solution

$$\frac{T_n-T}{T}=\frac{2(1+m/M)}{\cosh(c+\alpha_0 t/\frac{3}{2}k)-1}. \quad (15)$$

If we require the solution to describe correctly the fact that at zero time the neutron energy is infinite, we have to put the integration constant $c=0$.

The result of this simplified argument will be in error if the true velocity distribution during the slowing-down process is markedly different from a Maxwellian spectrum. For neutron energies large compared to thermal energies the velocity distribution is approximately given by Eq. (6), which is definitely not Maxwellian. It is reasonable to assume that the shape of the velocity distribution becomes more and more Maxwellian as the neutrons approach thermal equilibrium with the gas. An upper limit on the errors committed in the simplified theory will therefore be obtained if we compare the time dependence of the average neutron energy as derived from Eq. (15) and, for free nuclei at rest, from Eq. (4). The result is that for energies large compared to kT, Eq. (15) predicts a time scale for the slowing-down process, which in the case of a proton gas is too slow by a factor of 1.3 and for very heavy nuclei too fast by a factor of 0.8, whereas for nuclei of mass four the two models agree. The best picture for the variation of the average neutron energy with time is probably obtained if we use Eq. (4) for the slowing-down process down to an energy where we expect the thermal motion to become of importance, say $5kT$, and join a curve of the form (15) at this point by a proper choice of the constant of integration c.

In order to further check the simplified theory, an approximate solution of the integral equation (1) with the kernel corresponding to the differential cross section (9) has been evaluated for the case $M=m$, by the Monte-Carlo method. From the result the time variation of the mean neutron energy is calculated and compared with the approximate expression (15) in Fig. 4. This comparison confirms the expectation that the initial rate of energy loss predicted by Eq. (15) is too slow but that the subsequent approach to equilibrium with the moderating gas is quite accurately described.

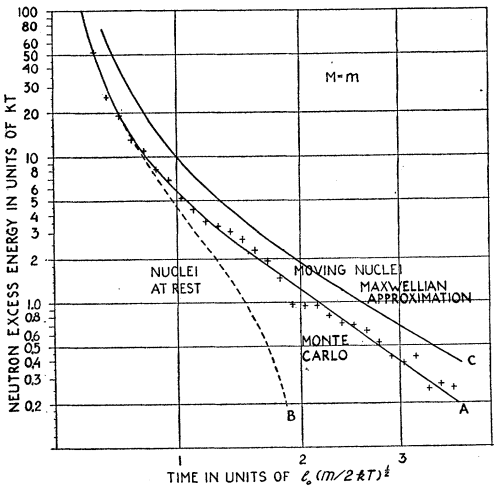

FIG. 4. Average excess energy over the equilibrium value $\frac{3}{2}kT$ of neutrons slowing down in a proton gas of temperature T as a function of time. Curve A: smoothed result of Monte-Carlo calculation. Curve B: calculated neglecting the thermal motion of the protons [Eq. (4)]. Curve C: calculated on the assumption of a Maxwellian neutron velocity distribution [Eq. (15)].

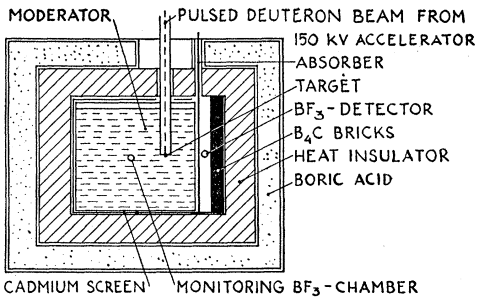

FIG. 5. Experimental arrangement.

E. Effect of the Finite Size of the Moderator

In a finite geometry the time dependence at position $\mathbf{r}$ of the neutron density due to a burst of neutrons at time $t=0$ can be written[10]

$$n(\mathbf{r},t)=Z_0(\mathbf{r})\exp\left(-\kappa_0^2\int Ddt-1/\theta\right)+\sum_{\nu=1}^{\infty}Z_\nu(\mathbf{r})\exp\left(-\kappa_\nu^2\int Ddt-1/\theta\right), \quad (16)$$

where $D(t)$ is the diffusion coefficient of the neutrons at time t, κ_0^2 and κ_ν^2 are the eigenvalues (geometrical buckling) corresponding to the eigenfunctions $Z_0(\mathbf{r})$ and $Z_\nu(\mathbf{r})$ for the fundamental and harmonic modes of the boundary problem, and θ is the mean life for absorption.

The integral $\int Ddt$ is identical to the Fermi age[3] which during the slowing-down process increases from zero to the value L_s^2, L_s being the slowing-down length. After the neutrons have become thermal the age increases linearly with time.

If the dimensions of the geometry are of the same order as the slowing-down length, the harmonic modes will decay almost completely during the initial phase of the slowing-down process, and only the fundamental mode will subsist for the remaining process. We then have for the logarithmic slope of the decay curve at any point in the moderator

$$-\frac{\partial}{\partial t}[\log n(\mathbf{r},t)]=\kappa_0^2D(t)+1/\theta. \quad (17)$$

For a $1/v$ absorber the mean life for absorption θ is a constant, independent of the shape of the neutron spectrum and thus of time. The diffusion coefficient, on the other hand, decreases rapidly with time during the slowing-down process and approaches a constant value when the neutrons become thermal.

The velocity spectrum of the neutrons in a finite geometry is not the same as in the case of an infinite moderator studied in the previous sections, since the leakage probability per unit time depends on the neutron velocity, being larger for neutrons with high velocities. There will thus be a larger drain on the high-energy portion of the neutron spectrum in the moderator, and at a given time the average velocity of the neutrons remaining in the moderator will be lower than if the moderator were infinite. The slowing-down process will thus appear to be somewhat more rapid in a finite moderator, and in the equilibrium state the temperature of the neutrons will be lower than the temperature of the moderator. A quantitative treatment assuming that the neutrons have a Maxwellian velocity distribution indicates that the absolute equilibrium neutron temperature is lower than the temperature of the moderator by a factor of $(1+\frac{1}{2}\kappa_0^2Dk/\alpha_0)^{-1}$. Here κ_0^2D is the leakage probability per unit time, k is Boltzmann's constant, and α_0 is the heat transfer coefficient between the neutrons and the moderator which in the case of a moderating monatomic gas is given by Eq. (14). The effect will be quite important if the amount of moderator is small.

PART II. EXPERIMENT

A. Apparatus

The experimental arrangement is shown in Fig. 5. The moderator is contained in a cubical tank of about 19-cm side, and is surrounded by a double-walled plywood box, the interspace between the walls being filled with boric acid for protection against stray neutrons from outside. For experiments at temperatures other than room temperature, the moderator can be surrounded by a 5-cm thick heat-insulating layer. The moderator is screened against neutrons diffusing in this layer by 0.05-cm cadmium sheets, except at one face of the tank where there is an empty space lined with boron carbide bricks. At this face is positioned a small BF_3 chamber (sensitive volume about 1 cm^3) which measures the flux from the surface. The spectrum of the flux can be investigated by interposing absorbers between the emitting surface and the detector. A list of the boron absorbers used in the experiment is given in Table II. The value k_0 of a boron absorber is defined as the neutron velocity for which the transmission for perpendicular incidence is $1/e$. In the last column is given the corresponding neutron temperature $T_a=mk_0^2/2k$. Measurements were also made with a 0.05-cm cadmium absorber. The measurements were monitored with a monitoring chamber placed in a position where the flux is not influenced by the absorbers, usually in a central channel through the tank.

A block diagram of the apparatus is given in Fig. 6.

A 40-channel time-analyzer described in a separate publication[11] is used for the analysis of the time distri-

[10] S. Glasstone and M. C. Edlund, *The Elements of Nuclear Reactor Theory* (D. Van Nostrand Company, Inc., New York, 1952).

[11] G. F. von Dardel, Appl. Sci. Research **B3**, 209 (1953).

bution of the pulses from the detector. The time-analyzer also emits a reference pulse which is transmitted to the high-voltage side of a small 150-kv accelerator[12] by a radio-frequency transmission link[13] and modulates the ion source of the accelerator. A burst of neutrons is produced by deuteron bombardment of a heavy ice target which is submerged in the moderator to be studied.

B. Methods of Measurement and Treatment of Data

A measurement with the equipment is characterized by the width, c μsec, of the detecting channels, the length, ic μsec, of the pulses which modulate the ion source, and the repetition time, Rc μsec, between pulses. In the following these parameters will be given in the convenient code $(i/R)\cdot c$ μsec. The repetition time was usually 20 channel widths. The shortest channel width of the time analyzer is 2 μsec.

For intensity reasons it is not economical to choose the repetition time so long that all neutrons from the preceding pulse have disappeared before the next is applied. A certain overlap between cycles must be tolerated, and an overlap correction is easily obtained by extrapolation of the measured time distribution exponentially into the next cycle (see Fig. 8).

The procedure in treating the data is to refer the number of counts measured with and without absorbers to a constant number of monitor counts, plot the time distribution on a semilogarithmic plot, determine and apply the overlap correction, and from the corrected values calculate the transmission curves as a function of time after the neutron burst.

The transmission measurements were not sufficiently accurate and extensive to allow the neutron velocity spectrum to be determined by a direct analysis of the transmission curve, as indicated by Eckart.[14] Instead certain assumptions were made concerning the shape of the spectrum, and the calculated transmission compared with the experimental values.

During the first interval of time, which we define as the *epicadmium* period, the energies of most neutrons

TABLE II. Boron absorbers.

No.	Material	g B/cm²	k_0[a] cm/sec	T_a°K
$B1$	Boron carbide	0.0110	0.97×10^5	56
$B2$	"Hysil" glass	0.0244	2.12×10^5	270
$B3$	"Hysil" glass	0.0248	2.16×10^5	279
$B1+B2$		0.0354	3.09×10^5	572
$B2+B3$		0.0492	4.28×10^5	1100

[a] On the basis of $\sigma_B = 710\times10^{-24}$ cm² for a neutron velocity of 2.2×10^5 cm/sec.

[12] E. Blomsjö and G. F. Von Dardel, Appl. Sci. Research **B4**, 1 (1954).
[13] von Dardel, Hellstrand, and Taylor, Appl. Sci. Research **B3**, 35 (1953).
[14] C. Eckart, Phys. Rev. **45**, 851 (1934).

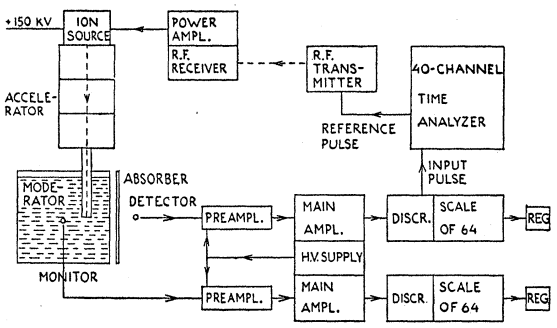

FIG. 6. Block diagram.

are above the cadmium absorption limit. The velocity distribution should be approximately given by the function (6) derived for slowing down against free nuclei at rest, since the chemical bonds and the thermal motion can be neglected. During the subsequent *transition* period to thermal equilibrium, the shape of the velocity distribution is assumed to be Maxwellian, this being the basis for the theory discussed in Part I, Sec. D. In both cases the angular distribution of the neutrons emitted from a surface element is assumed to be proportional to $(\cos\vartheta+\sqrt{3}\cos^2\vartheta)$ as predicted by Fermi.[3] With this law the angular distribution of the neutron flux at the detector would be proportional to $(1+\sqrt{3}\cos\vartheta)$ if the emitting surface were infinite and the flux density uniform. Under the conditions of the experiment some deviations from this angular distribution are expected for neutrons arriving at very oblique angles, which originate near the edge of the surface of the moderator where the neutron flux is lower. These deviations, which will at most result in an increase of the transmission by 6 percent, will, however, be neglected in the treatment.

In order to facilitate the interpretation of the transmission values, tables are given in the appendix for the transmission through $1/v$ absorbers of monoenergetic and Maxwellian neutrons. In the case of the Maxwellian spectrum the transmission is given in terms of the parameter $x=(T_a/T_n)^{\frac{1}{2}}$, where T_a is the characteristic temperature of the absorber as defined above and T_n the neutron temperature. Using these tables, one can interpret a transmission measurement in terms of an effective neutron temperature. For the interpretation of the transmission data through the cadmium absorber, the complicated true energy dependence of the cadmium-absorption cross section was approximated with a constant cross section of 2300 barns below a certain cutoff velocity and a zero cross section above this velocity. In interpreting data for the epicadmium period the cutoff velocity chosen is 9.7×10^5 cm/sec, for which the transmission is 50 percent. In the transition period, where the transmission is low, a lower value 7.5×10^5 cm/sec is used. The measured cadmium transmission values can also be transformed to effective neutron temperatures.

A serious difficulty for the interpretation of the experimental results is the distortion of the velocity spectrum during the time of flight of the neutrons from the emitting surface to the detector. In order to minimize these effects, the volume of the detector was made as small as intensity requirements would allow and positioned as close to the emitting surface as possible. The center of the detector is about 1 cm from the moderator surface, and even over this small distance an appreciable time-of-flight distortion of the spectrum will occur during those time intervals when the emitted spectrum is changing rapidly. The effect is serious chiefly for hydrogenous substances. The flux from a light water moderator measured with the detector at various distances from the surface, the parameters being $(1/20)\times 11$ μsec in our notation, is given in Fig. 7, which shows that time-of-flight effects are appreciable only during the first 10–20 μsec of the slowing-down process. If the velocity spectrum is known it is possible to calculate the expected time-of-flight delay of the various parts of the spectrum, and the apparent transmission. The velocity distribution of neutrons emitted by a hydrogenous moderator has previously been studied by Rainwater and Havens[15] by the time-of-flight method. Whereas this method gives the average distribution over the whole neutron lifetime, the present measurements allow the changes in the velocity distribution during the lifetime to be studied.

C. Measurements on Light Water

Due to the high proton scattering cross section and the large energy loss per collision, the slowing-down process in light water will be very rapid, and it is necessary to use the highest available resolution.

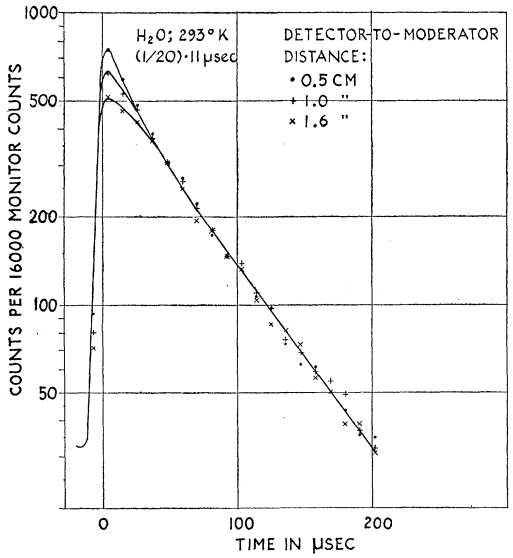

FIG. 7. Neutron flux from light water moderator at room temperature with the detector 0.5, 1.0, and 1.6 cm from the moderator surface.

[15] L. J. Rainwater and W. W. Havens, Jr., Phys. Rev. **70**, 136 (1946).

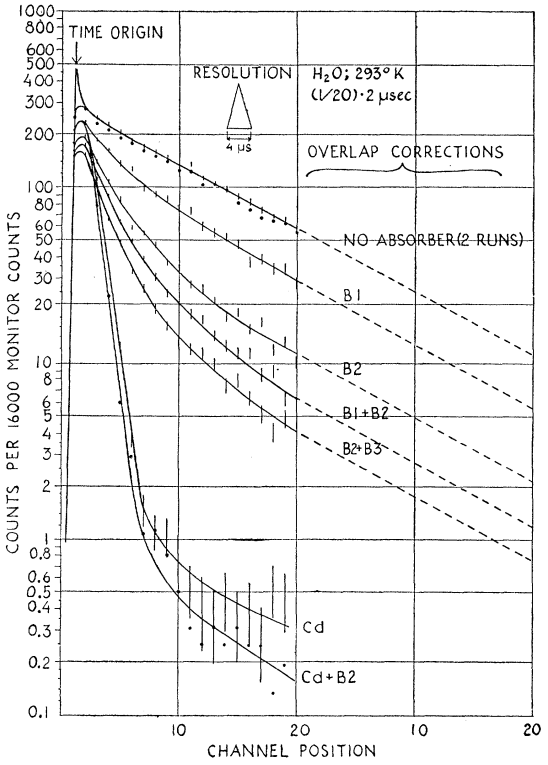

FIG. 8. Neutron flux from ordinary water with 20 g/liter boric acid

In Fig. 8 are reported the results of 2 μsec channel-width measurements of the neutron flux from the tank filled with water at room temperature, and in Fig. 9 the corresponding transmission curves. In order to avoid excessive overlap between periods, about 20 grams of boric acid per liter were dissolved in the water so that the period of the exponential decay was decreased from about 70 μsec to 23 μsec. Measurements could then be repeated with 40 μsec intervals.

When the neutron energy is high and absorption negligible, the transmission is near unity for all curves. During the slowing down process the transmission decreases with neutron energy, and finally approaches a constant value when the neutrons have become thermal.

For the initial, epicadmium, part of the process the experimental curves of Fig. 9 for the cadmium absorber and the boron absorber *B2* should be compared with the theoretical curves I and III, respectively, which are calculated for the velocity distribution given by Eq. (6). The agreement is not very good, mainly due to the time-of-flight distortion of the spectrum. In curves II and IV this effect has been taken into account, and we obtain satisfactory agreement with the initial part of the experimental curves.

During the transition period to thermal equilibrium the neutron energy is too low to excite the vibrations

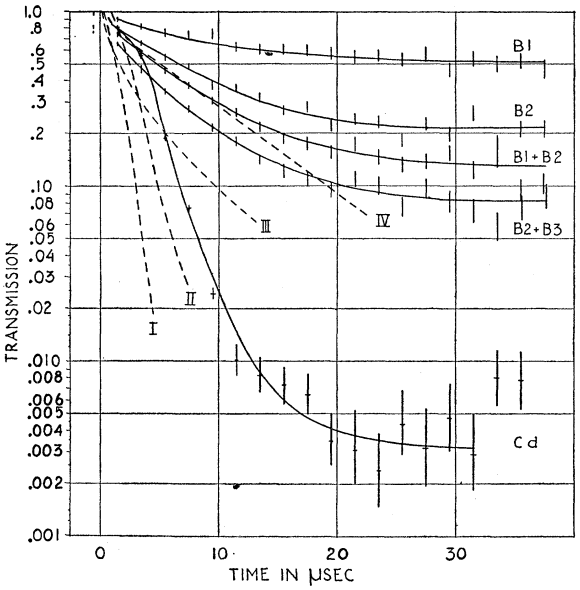

FIG. 9. Transmission of neutrons slowing down in ordinary water with 20 g/liter boric acid. Full curves: experimental transmission through the boron absorbers of Table II and through 0.438 g/cm² cadmium. Broken curves: Calculated transmission through cadmium and through absorber *B2*, neglecting time-of-flight distortion (I and III) and taking time of flight into account (II and IV).

within the water molecule at 0.45, 0.46, and 0.20 ev[16] and the water molecules may therefore be regarded as rigid structures. The neutron will soon also lose ability to excite the hindered rotations of the water molecules with quantum energies of about 0.10 ev,[17] and the main energy transfer is then to the Debye waves in which the water molecules move as rigid units. Since, on the other hand, at room temperature the temperature of the liquid is not small compared to the Debye temperature, which is about 300°K,[17] we may expect the model of a monatomic gas of nuclei having the weight of the water molecules to be a fair approximation. As described in Sec. B, the transmission data of Fig. 9 for the various absorbers were transformed, by means of Table VIII, to the effective neutron temperature values shown in Fig. 10. Except for the thinnest $1/v$ absorber *B1*, the neutron temperature values obtained from measurements with different absorbers are scattered around a single curve, which indicates that the neutron velocity distribution is indeed Maxwellian within the admittedly large experimental uncertainty limits. The time dependence of the experimentally determined temperature variations has the same character as the predicted theoretical curve for a monatomic gas of nuclei of mass 18. The discrepancy is adequately accounted for by the effect of the experimental resolution, time-of-flight effects, and the fact that for water molecules Eq. (15) predicts an initial slowing-down process which is too fast.

The failure of the data for the thin absorber *B1* to conform to the rest of the data would indicate that the true spectrum contains less slow neutrons than a Maxwellian distribution. As is shown by other measurements, this abnormal behavior of the thin absorber data persists even after the neutrons have become thermal. Since thermal neutrons should certainly have a Maxwellian distribution, we attribute the anomaly to some systematic errors in the method to which the thin absorber data would be particularly sensitive, such as departures from the angular distribution $(1+\sqrt{3}\cos\vartheta)$ and errors in the composition of the absorber.

A number of measurements were also made on the effect of temperature on the slowing-down process. Figure 11 shows the transmission through the $1/v$ absorber *B2* of the neutron flux from an ice moderator kept at 98°K and 273°K and from water heated to 354°K. We expect the temperature of the moderator not to influence the initial slowing-down process appreciably when the energy of the neutrons is large compared to thermal energies. The curves for 98°K and 273°K coincide indeed during the initial part, and the failure of that for 354°K to do so can be attributed to an observed erratic behavior of the monitoring chamber at this temperature. We correct for this monitoring error by shifting the curve so that the initial part coincides with the 98°K and 273°K curve (broken

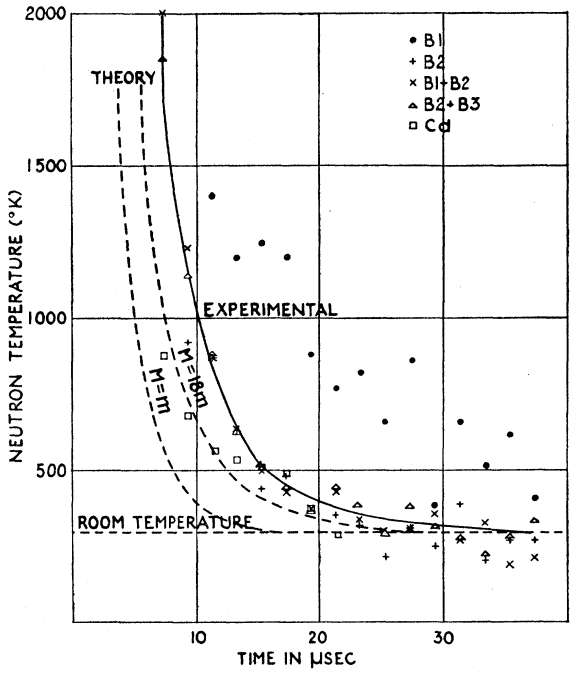

FIG. 10. Effective neutron temperature. Broken curves: theoretical [Eq. (15)]. Full curve drawn through points, calculated from the transmission measurements of Fig. 9.

[16] G. Herzberg, *Infrared and Raman Spectra of Polyatomic Molecules* (D. Van Nostrand Company, Inc., New York, 1947).
[17] N. Bjerrum, Kgl. Danske Videnskab Selskab, Mat.-fys. Medd. **27**, 1 (1951).

curve C'). The curves A, B, and C' should be compared with the theoretical curves D, E, and F, calculated from Eq. (15). For the higher temperatures the resolution is not high enough to resolve the approach to the asymptotic value, but the data are consistent with the conclusion from Fig. 9, that the slowing-down process at room temperature proceeds as if the moderator were a monatomic gas of nuclei of mass 18. The experimental results for 98°K, however, indicate an approach which is substantially slower than what would be predicted for a monatomic gas. This is a strong indication that the model of a monatomic gas is a poor approximation to actual conditions for temperatures which are low compared to the Debye temperature.

The asymptotic transmission values approached after the conclusion of the slowing-down process are in close agreement with the values expected for thermal neutrons in temperature equilibrium with the moderator, and having the angular distribution $(1+\sqrt{3}\cos\vartheta)$. A special investigation was made of the variation of the asymptotic transmission with the moderator temperature over the entire temperature range from 98°K to 354°K. The results are reproduced in Fig. 12 and compared with the curve expected for thermal neutrons in temperature equilibrium with the moderator. The agreement is essentially within the experimental errors.

Manley, Haworth, and Luebke[18] calculated the mean life for absorption θ from decay measurements. In the small geometries, used in the present measurements only the fundamental mode of the neutron distribution will be present, since the harmonics decay during the slowing down process. The logarithmic decay rate is then related to the geometrical buckling of the geometry, the diffusion coefficient, and the mean life for absorption by Eq. (17), so that if two of these are known the third can be determined. Since the mean life for absorption is given by the accurately known absorption cross section and (at least for simple geometries) the geometrical buckling can be calculated from the dimensions of the moderator and an approximate knowledge of the extrapolation distance, the method is probably best suited for the determination of the diffusion coefficient which is not easily determined directly by other methods. The experimental conditions of the present measurements were, however, not chosen with this application in mind, and neither the mean life for absorption nor the geometrical buckling of the moderating system is readily calculated. Only qualitative conclusions can therefore be drawn from the decay-rate data. The measured asymptotic decay rate due to leakage and absorption of neutrons increases markedly with moderator temperature in the range from 98°K to 354°K. Since the mean life for absorption is independent of the temperature of the neutrons and depends only slightly on the temperature of the moderator, the leakage term H_0^2D in Eq. (17) must be responsible for this increase. An attempt was made to estimate the magnitude of the diffusion coefficient at room temperature from the measured decay rate, using rough estimates of the mean life for absorption and geometrical buckling of the moderating geometries. The value obtained was larger by almost a factor of two than the value derived from the diffusion length[19] and the mean life for absorption,[20] but in view of the uncertainty of the estimates this discrepancy may not be significant.

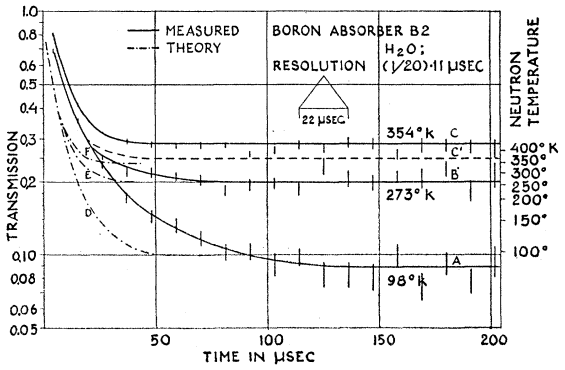

FIG. 11 Transmission through absorber $B2$, Table II, of neutrons slowing down in ice of 98°K and 273°K, and in water of 354°K. Curve C' is the 354° K measurement corrected for a suspected normalization error. Curves D, E, and F are calculated by Eq. (15).

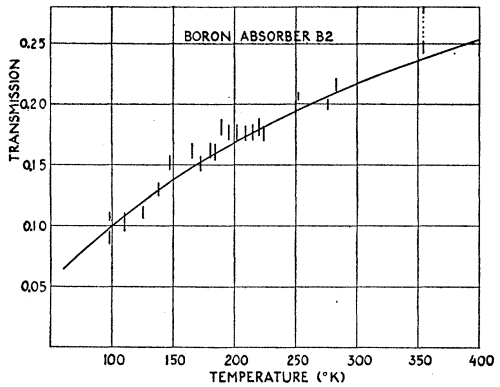

FIG. 12. Measured transmission in absorber $B2$ of neutrons which have spent at least 72 μsec in ice, as a function of the ice temperature. The full curve is calculated for Maxwellian neutrons with the angular distribution $(1+\sqrt{3}\cdot\cos)$ in temperature equilibrium with the moderator.

Note added in proof.—In later more accurate measurements good agreement was found with the expected value for the diffusion correction.

D. Measurements on Heavy Water at Room Temperature

Measurements were made on a quantity of 6.2 liters of heavy water, 99.7 percent pure, in a cubical aluminum vessel with a central cavity. In this small volume only a small fraction of the neutrons will reach low

[18] Manley, Haworth, and Luebke, Phys. Rev. **61**, 152 (1941).

[19] E. Fermi, Atomic Energy Commission Report AECD-2664 (unpublished).

[20] G. F. von Dardel and A. W. Waltner, Phys. Rev. **91**, 1284 (1953).

energies in the heavy water and the intensity of the neutron flux will be much lower than in the case of the light-water measurements. In Fig. 13 is shown the result of measurements on the decay of the neutron flux from the heavy-water moderator, measured with the highest available resolution (channel-width 2 μsec). There is a gap in the curve for times 25–65 μsec, since for greater reliability the 40 channels of the time analyzer were arranged to duplicate 20 time intervals in the most interesting part of the cycle, leaving 20 time intervals unmeasured. It is, however, possible to interpolate the curve in this gap quite accurately, using the fact obtained in another measurement that the continued decay of the neutron intensity is exponential with a period of 84 μsec. A smooth curve through the experimental points is corrected for overlap, whose determination is shown in Fig. 13, and the slope of the corrected curve is plotted in Fig. 14. The theoretical curve is calculated from Eq. (17), using for the diffusion coefficient values calculated from the mean free path and average neutron velocity, obtained from the theory for slowing down by free nuclei at rest. The two curves agree surprisingly well over the initial part of the slowing-down process, where the theory for free nuclei at rest is expected to describe the process well. In the subsequent process the experimental curve approaches a value due to absorption and leakage of thermal neutrons, whereas the theoretical decay constant goes towards a very small value due to absorption alone.

An attempt was made to determine the diffusion coefficient for thermal neutrons in heavy water by a measurement of the asymptotic decay rate. The cavity in the container was filled with heavy water in order to make the moderating geometry more regular, and the target was positioned outside the container, so as not to influence the decay of the neutron distribution in

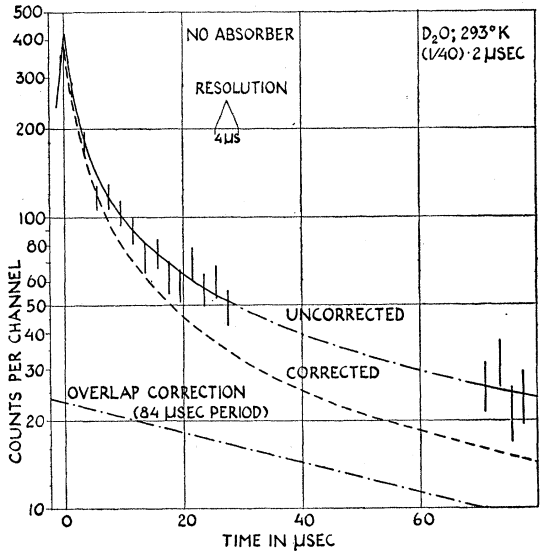

FIG. 13. Neutron flux from heavy water.

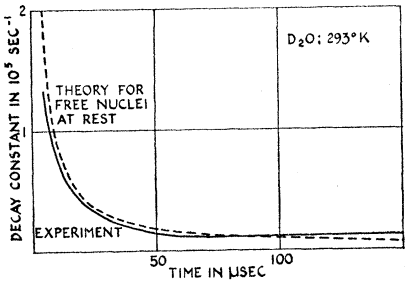

FIG. 14. Theoretical and experimental decay rate of the flux from the heavy water moderator.

the heavy water. The measured decay rate, 10 300±150 sec^{-1}, corrected for an estimated absorption rate of 90 sec^{-1}, and combined with the geometrical buckling, calculated for the volume of the heavy water extended by the extrapolation length of Auger, Munn, and Pontecorvo,[21] yields for the diffusion coefficient of thermal neutrons in 99.7 percent heavy water the value $1.52 \cdot 10^5$ cm^2/sec. If we combine this value with Auger's *et al.*[21] value for the transport mean free path we find a mean neutron velocity of about 2000 m/sec, corresponding to a neutron temperature about 100°K below room temperature. This estimate is rather uncertain since the transport mean free path is energy dependent and the value which relates the diffusion coefficient to the mean neutron velocity may not be the same as the value which results from the measurements of Auger *et al.* The theoretical arguments of Sec. E, Part I, predict that the equilibrium neutron temperature should be about 50°K below the moderator temperature which at least is of the same order of magnitude as the rough experimental value.

It is also possible to measure the neutron temperature by transmission measurements with boron absorbers, as for light water. The result of such measurements while not inconsistent with the above results, were inconclusive due to the presence under the experimental conditions of these measurements of a strong background, which made the interpretation uncertain.

The result of other transmission measurements with cadmium are given in Fig. 15. The solid curve is theoretical, calculated from the velocity distribution Eq. (6) for the slowing down against free deuterons at rest and with the schematic energy dependence of the cadmium absorption cross section discussed in Part II, Sec. B. The experimental points have to be corrected for the effect of resolution, time-of-flight delay, and for the effect of the oxygen atoms. These corrections are small and compensate each other. The agreement between experiment and theory is essentially within the statistical accuracy of the experimental points, indicating that the neutron spectrum above 0.4 ev and its time dependence are not appreciably influenced by the thermal motion and chemical bonds.

[21] Auger, Munn, and Pontecorvo, Can. J. Research A25, 143 (1947).

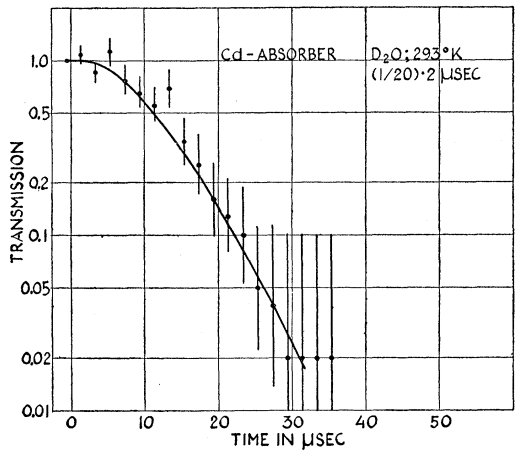

FIG. 15. Transmission through cadmium as a function of time for neutrons slowing down in heavy water. Experimental points normalized to unity for first point. Theoretical curve derived for free deuterons at rest.

CONCLUSION

The pulsed neutron source technique used in the present investigation seems to be capable of giving interesting information concerning the slowing-down process for neutrons in a moderator. The present investigation has mainly been concerned with a rapid survey of the possibilities and limitations of this technique and the accuracy of the experimental results can undoubtedly be improved upon considerably. With a maximum resolution of 2 μsec, the slowing-down process from initial energies to the cadmium absorption limit will not be very well resolved, at least not for hydrogenous materials, and the results will further be invalidated by time-of-flight distortion of the spectrum. For improved measurements designed to study this phase of the slowing-down process, the time scale for the process can be increased by dilution of the moderating substance with a nonmoderating substance and the time-of-flight effects reduced by the use of a thin neutron detector, e.g., of the type described by Alburger.[22] It would, however, appear that more refined measurements in the energy range above the cadmium absorption limit have little of interest to offer for the understanding of the slowing-down process since in this energy range, where the thermal motion and chemical bonds have little effect, the theory is fairly complete. Below the cadmium absorption limit, on the contrary, the complications introduced by the chemical bonds and the thermal motion make a satisfactory theoretical treatment exceedingly difficult, and an experimental study of the slowing-down process in this range by the present method is of considerable interest. A comparison of the slowing-down processes in different hydrogenous substances, as for example the various types of hydrocarbons, may contribute to the understanding of the effect of the chemical bond. Other substances of interest would be crystalline materials of well-defined structure, e.g., beryllium, for which Kleinman[4] has calculated the necessary data to predict theoretically the slowing-down process and approach to thermal equilibrium.

A sufficiently long time after the neutron burst, the neutron spectrum has reached a well-defined equilibrium state which can be studied by transmission measurements. The temperature of these neutrons will depend somewhat on the size of the moderator, but for reasonably large amounts of material the neutron temperature is very close to the moderator temperature and the neutrons are thus thermal neutrons in the true sense of the word.

The diffusion coefficient of these thermal neutrons in the moderator can be determined from the decay rate of the neutron intensity as shown in this investigation, and thermal neutron absorption cross sections can be determined for strong absorbers either by transmission measurements in bad geometry, or by mixing the absorber intimately with the moderator and measuring the increase in decay rate.

The author wishes to thank Dr. G. Placzek for stimulating advice and Dr. S. Eklund for active support.

APPENDIX

The transmission of neutrons with the angular distribution $(1+\sqrt{3}\cos\vartheta)$ through an absorber with the absorption cross section Σ cm^2 per cm^2 area is given by the formula

$$T(\Sigma)=[1-(2\sqrt{3}-3)\Sigma]e^{-\Sigma} -\mathrm{Ei}(-\Sigma)[-(4-2\sqrt{3})\Sigma+(2\sqrt{3}-3)\Sigma^2], \quad (18)$$

where $\mathrm{Ei}(-x)$ is the logarithmic integral. The expression (18) is tabulated in Table III.

TABLE III. Transmission of monoenergetic neutrons having an angular distribution proportional to $(1+\sqrt{3}\cos\vartheta)$ as a function of the macroscopic cross section Σ cm^2/cm^2 of the absorber.

Σ	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	0.9640	0.9358	0.9106	0.8875	0.8659	0.8456	0.8263	0.8080	0.7904
0.	1.0000	0.7736	0.6344	0.5299	0.4475	0.3807	0.3258	0.2800	0.2416	0.2091
1.	0.1814	0.1577	0.1375	0.1200	0.1049	0.0918	0.0805	0.0707	0.0621	0.0546
2.	0.0481	0.0424	0.0374	0.0330	0.0291	0.0257	0.0228	0.0201	0.0178	0.0158
3.	0.0140	0.0124	0.0110	0.00977	0.00867	0.00770	0.00684	0.00608	0.00540	0.00481
4.	0.00427	0.00381	0.00336	0.00302	0.00269	0.00239	0.00214	0.00190	0.00169	0.00151
5.	0.00135	0.00120	0.00108	0.00096	0.00086	0.00076	0.00068	0.00061	0.00055	0.00049

[22] D. E. Alburger, Rev. Sci. Instr. **23**, 769 (1952).

TABLE IV. Transmission of thermal neutrons through a $1/v$ absorber. Case I: Detector of constant efficiency. Isotropic angular distribution. The values for $x \geqslant 3.3$ were determined by extrapolation.

x	0	1	2	3	4	5	6	7	8	9
0.	1.0000	0.8527	0.7395	0.6476	0.5711	0.5064	0.4509	0.4031	0.3615	0.3251
1.	0.2931	0.2649	0.2399	0.2176	0.1978	0.1800	0.1641	0.1499	0.1370	0.1254
2.	0.1149	0.1054	0.0968	0.0890	0.0819	0.0754	0.0695	0.0641	0.0592	0.0547
3.	0.0506	0.0469	0.0435	0.0400	0.0370	0.0343	0.0318	0.0297	0.0277	0.0257
4.	0.0239	0.0222	0.0206	0.0192	0.0179	0.0167	0.0156	0.0145	0.0135	0.0126

TABLE V. Transmission of thermal neutrons through a $1/v$ absorber. Case II: $1/v$ detector. Perpendicular incidence. The values for $x \geqslant 4.0$ were determined by extrapolation.

x	0	1	2	3	4	5	6	7	8	9
0.	1.0000	0.8959	0.8063	0.7281	0.6596	0.5992	0.5450	0.4969	0.4538	0.4151
1.	0.3802	0.3488	0.3204	0.2946	0.2711	0.2498	0.2304	0.2128	0.1965	0.1817
2.	0.1682	0.1558	0.1444	0.1340	0.1243	0.1154	0.1073	0.0985	0.0929	0.0862
3.	0.0804	0.0751	0.0697	0.0651	0.0603	0.0567	0.0529	0.0491	0.0461	0.0431
4.	0.0403	0.0375	0.0352	0.0330	0.0310	0.0290	0.0272	0.0256	0.0240	0.0225

TABLE VI. Transmission of thermal neutrons through a $1/v$ absorber. Case III: Detector of constant efficiency. Perpendicular incidence. The values for $x \geqslant 3.9$ were determined by extrapolation.

x	0	1	2	3	4	5	6	7	8	9
0.	1.0000	0.9161	0.8408	0.7729	0.7114	0.6557	0.6051	0.5591	0.5168	0.4784
1.	0.4432	0.4109	0.3813	0.3540	0.3290	0.3058	0.2847	0.2650	0.2469	0.2301
2.	0.2146	0.2003	0.1870	0.1747	0.1633	0.1527	0.1428	0.1336	0.1251	0.1171
3.	0.1097	0.1029	0.0965	0.0905	0.0849	0.0796	0.0747	0.0700	0.0656	0.0621
4.	0.0585	0.0550	0.0518	0.0485	0.0455	0.0429	0.0403	0.0380	0.0357	0.0337

TABLE VII. Transmission of thermal neutrons through a $1/v$ absorber. Case IV: Detector of constant efficiency. Angular distribution $(1+\sqrt{3}\cos\vartheta)$. The values for $x \geqslant 3.9$ were determined by extrapolation.

x	0	1	2	3	4	5	6	7	8	9
0.	1.0000	0.8671	0.7608	0.6724	0.5974	0.5299	0.4775	0.4285	0.3866	0.3491
1.	0.3160	0.2866	0.2604	0.2370	0.2159	0.1972	0.1802	0.1649	0.1511	0.1386
2.	0.1269	0.1170	0.1076	0.0991	0.0914	0.0843	0.0778	0.0719	0.0665	0.0615
3.	0.0569	0.0528	0.0490	0.0455	0.0420	0.0393	0.0364	0.0337	0.0315	0.0294
4.	0.0275	0.0257	0.0240	0.0223	0.0209	0.0195	0.0183	0.0171	0.0159	0.0149

TABLE VIII. Transmission of thermal neutrons through a $1/v$ absorber. Case V: $1/v$ detector. Angular distribution: $(1+\sqrt{3}\cos\vartheta)$.

x	0	1	2	3	4	5	6	7	8	9
0.	1.0000	0.7652	0.6282	0.5284	0.4508	0.3886	0.3376	0.2952	0.2595	0.2292
1.	0.2031	0.1807	0.1612	0.1442	0.1294	0.1163	0.1048	0.0946	0.0856	0.0774
2.	0.0703	0.0639	0.0581	0.0530	0.0483	0.0441	0.0403	0.0369	0.0338	0.0310
3.	0.0285	0.0261	0.0240	0.0221	0.0204	0.0188	0.0173	0.0160	0.0148	0.0136
4.	0.0126	0.0117	0.0108	0.0100	0.0093	0.0086	0.0080	0.0074	0.0069	0.0064

The transmission of thermal neutrons through a $1/v$ absorber is a function of the dimensionless quantity $x=(T_a/T_n)^{\frac{1}{2}}$, where T_a is the characteristic temperature of the absorber as defined in Part II, Sec. B, and T_n is the neutron temperature. The problem has been studied by Zahn[23] who has given an expansion of the transmission in terms of x, which is useful up to $x=1$, and by Laporte[24] who has given an asymptotic expansion valid for large values of x. In order to fill the gap between these two expressions, the author has extended the use of Zahn's power series, which converges for all values of x, to about $x=4$ by including more terms and recalculating the coefficients with higher accuracy. Laporte's results are then used for extrapolating to larger values of x. The results of the calculations for the 5 cases discussed by Zahn are given in Tables IV–VIII. For the highest values of x the results may be in error by a few units in the last decimal.

[23] C. T. Zahn, Phys. Rev. **52**, 67 (1937).
[24] O. Laporte, Phys. Rev. **52**, 72 (1937).