Angular Distribution of Neutrons Scattered from Cadmium, Tin, and Bismuth*

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The differential cross sections for the scattering of 3.7-Mev neutrons from cadmium, tin, and bismuth have been measured over an angular range between 13 degrees and 140 degrees with an angular resolution of about ±10 degrees. The effects due to higher-order scattering have been removed at each measured angle by extrapolation. Previous measurements of the differential cross sections of aluminum, iron, and lead gave a uniform shift of the position of the first minima in the curve with atomic weight. In the present measurements, the curves for cadmium and tin, while quite similar, do not fall uniformly between those of iron and lead. The curve for bismuth, however, is again quite similar to that for lead. Possible explanations of this behavior are discussed in light of the continuum theory of nuclear reactions.

INTRODUCTION

THE measurement of the differential cross sections for the scattering of 3.7-Mev neutrons from cadmium, tin, and bismuth represents an extension of similar measurements[1] on aluminum, iron, and lead in order to establish the pattern of variation with atomic weight. In addition, the recent continuum theory of nuclear reactions given by Feshbach, Porter, and Weisskopf[2] has been used with a plausible extension to calculate the angular distribution of 3.7-Mev neutrons scattered from nuclei of atomic weight 115 and 209. This should be of value in assessing the possibilities of using the continuum theory in this energy range to give detailed information about neutron scattering.

EXPERIMENTAL

In general, the arrangement used in this experiment is exactly the same as that used in our previous experiment.[1] Neutrons of about 3.7 Mev were produced by bombarding a deuterium gas chamber with 10 μa of deuterons of 0.65-Mev mean energy. A ring geometry was employed in which the angle of scattering was varied both by using different size rings and by an axial movement of each ring.

The previous experiments demonstrated that corrections of considerable magnitude must be made to allow for the higher-order scattering in order to obtain the correct differential cross sections from the apparent differential cross sections observed with a ring of finite thickness. They further demonstrated that, within the experimental accuracy, a linear extrapolation of the apparent differential cross section to zero axial thickness of the scatterer most probably gave the correct differential cross section. The present measurements, therefore, were carried out using only two ring thicknesses. However, unlike the previous experiments in which only a few selected angles were chosen to determine the higher-order scattering correction, in these experiments each ring thickness was used at all the angles of measurement. The result gives the apparent differential cross section as a function of angle for each of the two ring thicknesses used. The true differential cross section now can be found at each angle by a linear extrapolation of the apparent differential cross section to zero ring thickness.

Finally, the neutron flux monitor was changed from that previously used, a butane-filled proportional counter, to a Lucite-zinc sulfide scintillator detector[3] identical with that used in detecting the scattered neutrons. In the previous experiments[1] some difficulty was experienced in maintaining a constant 0°/90° neutron flux ratio. Changing the monitor detector improved this situation somewhat.

DATA

Figure 1 shows the experimental points of the apparent differential cross section for the scattering of 3.7-Mev neutrons incident on the $\frac{3}{8}$-inch and 1-inch thick rings of cadmium. A point by point linear extrapolation to zero ring thickness gives the true differential cross section $\sigma(\theta)$. Figures 2 and 3 present similar information with regard to tin and bismuth. At every angle the apparent differential cross section for the 1-inch rings is larger than the apparent differential cross section for the $\frac{3}{8}$-inch rings, thus demonstrating that at each angle the higher-order scattering adds a positive contribution to the apparent differential cross section.‡ The total cross sections corresponding to $\sigma(\theta)$ have been calculated and are presented together with

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[1] W. D. Whitehead and S. C. Snowdon, Phys. Rev. **92**, 114 (1953).
[2] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953); U. S. Atomic Energy Commission Reports NYO 3076, NDA Report 15B-4 (unpublished); Massachusetts Institute of Technology (Laboratory for Nuclear Science) Technical Report No. 62, 1953 (unpublished).
[3] W. F. Hornyak, Rev. Sci. Instr. **23**, 264 (1952). The authors are indebted to Dr. Hornyak for supplying them with Lucite-zinc sulfide molded buttons.
‡ *Note added in proof.*—This positive contribution of the multiple scattering depends upon the definition of the apparent differential cross section used. It is possible to change the definition of the apparent differential cross section in such a way that multiple scattering does not add any contribution on the average. In this case there will be negative as well as positive contributions to the apparent scattering, depending on the angle.

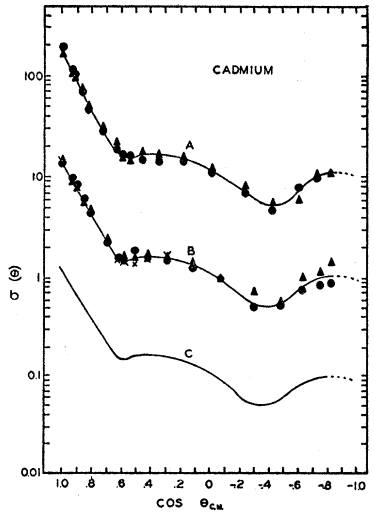

FIG. 1. Angular distribution of 3.7-Mev neutrons scattered from cadmium: (*A*) apparent differential cross section using 1-in. axial thickness ring scatterers; (*B*) apparent differential cross sections using $\frac{3}{8}$-in. axial thickness ring scatterers; (*C*) true differential cross sections for elastic scattering obtained by a point-by-point linear extrapolation of curves (*A*) and (*B*) to zero-thickness scatterer. Ordinates are for curve (*C*) and are in units of barns/steradian. Curve (*B*) ordinate should be reduced by a factor of 10. Curve (*A*) ordinate should be reduced by a factor of 100.

the measured total cross sections in Table I. The measured total cross sections in each case are about 8–10 percent higher than those previously measured at an energy close to 3.7 Mev.[4] However, since both the differential and total cross sections were measured with the same experimental setup, at worst all measurements are systematically from 8–10 percent high which is within our over-all uncertainties of about 15 percent.

As was discussed previously,[1] the integrated cross section corresponding to $\sigma(\theta)$ does not accurately measure the total elastic cross section, since the neutron detector does not discriminate adequately against inelastically scattered neutrons. However, to within about 20 percent, it may be considered to represent this elastic cross section. Subject to this reservation, the difference between the measured total cross section and the integrated cross section corresponding to $\sigma(\theta)$ may be considered to equal the total inelastic cross section. Table I presents a summary of these calculations.

THEORY

Since the neutron energy resolution in this experiment is about 200 kev, the angular distribution of scattered neutrons may be discussed in terms of a theory that averages the differential cross section over an energy interval of this magnitude. Generally, in our energy range there will be many resonance levels within this 200-kev energy interval, hence we may average the differential cross section over many resonance levels. Feshbach and Weisskopf[5] and Feshbach, Porter, and Weisskopf[2] have provided two distinct theories that discuss total neutron cross sections averaged in this manner. The total neutron cross-section measurements of Barschall *et al.*[6] generally show agreement with the theory of Feshbach, Porter, and Weisskopf[2] and are definitely at variance with the theory of Feshbach and Weisskopf.[5] Therefore, we will compare our measurements with the theory of Feshbach, Porter, and Weisskopf suitably formulated to exhibit the angular distribution of neutrons elastically scattered from nuclei.

The theory first will be formulated, assuming that there are no reaction products (i.e., no inelastic scattering, etc.). In general, the differential cross section for elastic scattering is given by[7]

$$\sigma_{sc}(\theta)=\tfrac{1}{4}\lambda\!\!\!^{-2}\sum_l\sum_m(2l+1)(2m+1)(1-\eta_l)\times(1-\eta_m{}^*)P_l(\cos\theta)P_m(\cos\theta), \quad (1)$$

where $\lambda\!\!\!^{-}$ is the deBroglie wavelength of the neutron, divided by 2π, η_l is the phase constant, and $P_l(\cos\theta)$ is the Legendre polynomial. The relation between the phase constant η_l and the corresponding logarithmic derivative f_l is given by[7]

$$\eta_l=\exp(2i\xi_l)\cdot[(f_l-\Delta_l+is_l)/(f_l-\Delta_l-is_l)], \quad (2)$$

where ξ_l is the phase determining the potential scattering, and Δ_l+is_l is the logarithmic derivative of the outgoing wave in the entrance channel.

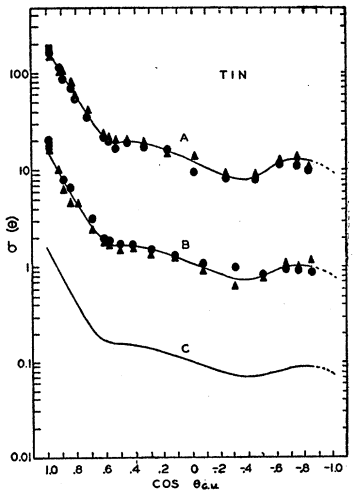

FIG. 2. Angular distribution of 3.7-Mev neutrons scattered from tin: (*A*) apparent differential cross section using 1-in. axial thickness ring scatterers; (*B*) apparent differential cross section using $\frac{3}{8}$-in. axial thickness ring scatterers; (*C*) true differential cross section for elastic scattering obtained by a point-by-point linear extrapolation of curves (*A*) and (*B*) to zero-thickness scatterer. Remarks concerning ordinates are the same as in Fig. 1.

[4] *Neutron Cross Sections*, Atomic Energy Commission Report AECU-2040 (U. S. Office of Technical Services, Department of Commerce, Washington, D. C., 1952); N. Nereson and S. Darden, Phys. Rev. **89**, 775 (1953).

[5] H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949); Feld, Feshbach, Goldberger, Goldstein, and Weisskopf, U. S. Atomic Energy Commission Report NYO-636, 1951 (unpublished).

[6] H. H. Barschall, Phys. Rev. **86**, 431 (1952); Miller, Adair, Bockelman, and Darden, Phys. Rev. **88**, 83 (1952).

[7] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. VIII.

We now average Eq. (1) over an energy interval that includes many resonance levels:

$$\sigma_{sc}(\theta)=\tfrac{1}{4}\lambda\!\!\!^{-2}\sum_l\sum_m(2l+1)(2m+1)[(1-\bar{\eta}_l)(1-\bar{\eta}_m^*)+(\langle\eta_l\eta_m^*\rangle_{Av}-\bar{\eta}_l\bar{\eta}_m^*)]P_l(\cos\theta)P_m(\cos\theta), \quad (3)$$

where all quantities such as $\lambda\!\!\!^{-}$, ξ_l, s_l, and Δ_l that only vary slowly with the energy are treated as constants. The distribution obtained from the term $(1-\bar{\eta}_l)\times(1-\bar{\eta}_m^*)$ is called the "shape elastic" scattering and the distribution obtained from the term $(\langle\eta_l\eta_m^*\rangle_{Av}-\bar{\eta}_l\bar{\eta}_m^*)$ is called the "compound elastic" scattering.[2]

In order to calculate η_l, first it is necessary to choose a model that will provide a logarithmic derivative which includes a description of resonances. Feshbach, Porter, and Weisskopf introduce a simplified version of a nuclear reaction. The zero-order approximate problem is described by a Hamiltonian,

$$H^{(0)}=H_T+H_n, \quad (4)$$

where H_T is the Hamiltonian describing the complicated internal motions of the target nucleus, and H_n describes the relative motion of the neutron and the target and is given by

$$H_n=-(\hbar^2/2m)\nabla^2+V(r), \quad (5)$$

where m is the reduced mass, ∇^2 is the Laplacian in the relative coordinate of the neutron and the target system. $V(r)$ is given by

$$V(r)=-V_0 \text{ for } r<R;\quad 0 \text{ for } r>R, \quad (6)$$

where R is the nuclear radius.

The formation of the compound nucleus is described in the next approximation by adding an interaction operator H' to give for the total Hamiltonian,

$$H=H_T+H_n+H'. \quad (7)$$

For a convenient choice of the matrix elements of H' in the basis determined by the eigenfunctions of H_T, the value of the logarithmic derivative may be found. It is shown then that this rather complicated expression for the logarithmic derivative may be approximated by

$$f_l=w_{1l}+w_{2l}\cot z_l(\epsilon), \quad (8)$$

where w_{1l} and w_{2l} are slowly varying functions of the energy ϵ of the neutron. The phase $z_l(\epsilon)$ is taken as proportional to the energy. If this expression for the logarithmic derivative is inserted into Eq. (2) and averaged over many resonances, the result is

$$\bar{\eta}_l=\exp(2i\xi_l)\cdot[(w_{1l}-iw_{2l}-\Delta_l+is_l)]/(w_{1l}-iw_{2l}-\Delta_l-is_l)]. \quad (9)$$

TABLE I. Total cross sections. The value of $\int\sigma(\theta)d\Omega$, subject to the reservations in the text, is the total elastic cross section.

Element	σ_{tot} in barns This exp.	Ref. 4	Theory	$\int\sigma(\theta)d\Omega$ barns
Cadmium	4.5	~4.0	5.6	2.5
Tin	4.6	~4.2		2.5
Bismuth	8.3	7.6	7.7	6.9

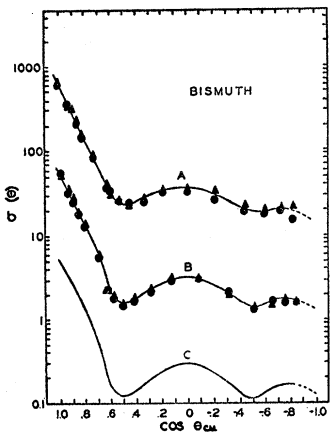

FIG. 3. Angular distribution of 3.7-Mev neutrons scattered from bismuth: (*A*) apparent differential cross section using 1-in. axial thickness ring scatterers; (*B*) apparent differential cross section using $\frac{3}{8}$-in. axial thickness ring scatterers; (*C*) true differential cross section for elastic scattering obtained by a point-to-point linear extrapolation of curves (*A*) and (*B*) to zero-thickness scatterer. Remarks concerning the ordinates are the same as in Fig. 1.

Thus the averaging results in replacing the $\cot z_l$ in Eq. (8) by $-i$. The expressions obtained for w_{1l} and w_{2l} in the perturbation calculation are such that they may be approximated by the real and imaginary parts of the logarithmic derivative of the following non-Hermitean problem in the relative coordinates of the neutron and the target:

$$H=-(\hbar^2/2m)\nabla^2+V(r),$$

where

$$V(r)=-V_0(1+i\xi) \text{ for } r<R;\quad 0 \text{ for } r>R. \quad (10)$$

Thus all quantities in Eq. (3) are provided for except $\langle\eta_l\eta_m^*\rangle_{Av}$. To obtain this, we assume in Eq. (8) that $z(\epsilon)$ is a linear function of the energy,

$$z_l=\pi\epsilon/d_l+\beta_l, \quad (11)$$

where d_l is the average level separation. The product $\eta_l\eta_m^*$ is then averaged with respect to ϵ as in the case of η_l. In particular the result will depend on the unknown parameters β_l. Since we are only interested in statistical results, the average of $\eta_l\eta_m^*$ with respect to ϵ is again averaged with respect to β_l. The result is then

$$\langle\eta_l\eta_m^*\rangle_{Av}=\bar{\eta}_l\bar{\eta}_m^* \text{ for } l\neq m;\quad =1 \text{ for } l=m. \quad (12)$$

Thus the average phase constant in Eq. (9) completely defines an angular distribution of the elastic scattering averaged over many resonances. The non-Hermitean problem defined by Eq. (10), which is used to provide an approximate expression for the average phase, may be solved by a method due to Lax and Feshbach.[8] We

[8] M. Lax and H. Feshbach, J. Acoust. Soc. Am. **20**, 108 (1948); Morse, Lowan, Feshbach, and Lax, "Scattering and Radiation from Circular Cylinders and Spheres" (Reprinted by U. S. Navy Department Office of Research and Inventions, Washington, D. C., 1946).

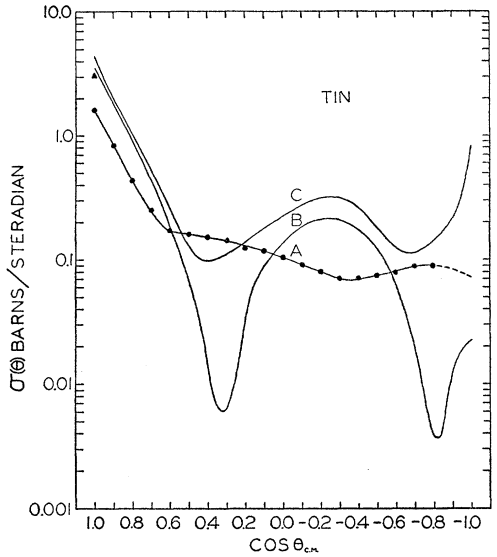

FIG. 4. Comparison of the experimental differential cross section for elastic scattering of 3.7-Mev neutrons from cadmium with the predictions of the continuum model for a nuclear radius of 7.1×10^{-13} cm and a complex potential constant of $-19(1+0.05i)$ Mev: (*A*) experimental results from Fig. 1(*C*); (*B*) "shape elastic" differential cross section; (*C*) "compound elastic" plus "shape elastic" differential cross section using Eq. (12) to determine the "compound elastic" scattering. Solid triangle is $\sigma_{sc}(0)$ from first version of the continuum theory.

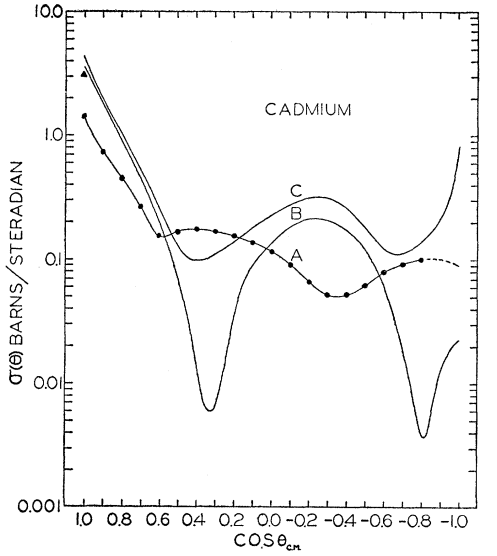

FIG. 5. Comparison of the experimental differential cross section for the elastic scattering of 3.7-Mev neutrons from tin with the predictions of the continuum model for a nuclear radius of 7.1 $\times10^{-13}$ cm and a complex potential constant of $-19(1+0.05i)$ Mev; (*A*) experimental results from Fig. 2(*C*); (*B*) "shape elastic" differential cross section; (*C*) "compound elastic" plus "shape elastic" differential cross section using Eq. (12) to determine the "compound elastic" scattering. Solid triangle is $\sigma_{sc}(0)$ from first version of the continuum theory.

have used for the potential within the nucleus,

$$V=-19(1+0.05i)\text{ Mev};\tag{13}$$

and for the nuclear radius,

$$R=1.45\times10^{-13}A^{\frac{1}{3}}\text{ (cm)}.\tag{14}$$

These values give good agreement with the total cross-section measurements of Barschall.[2,6]

In the preceding theory, it was assumed that no reaction products were present in the decay of the compound nucleus. This restriction may be removed by realizing that the effect of these reaction products upon the entrance channel can be accounted for approximately by introducing an imaginary part to the energy. Thus Eq. (11) is changed to

$$z_l=(\pi/d_l)(\epsilon+\tfrac{1}{2}i\Gamma_r^{(l)})+\beta_l,\tag{15}$$

where $\Gamma_r^{(l)}$ is the reaction width.[2,9] When this is done, it will be found that Eq. (9) for the average phase $\bar{\eta}_l$ is unchanged, and that Eq. (12) for the average phase product $\langle\eta_l\eta_m^*\rangle_{Av}$ is unchanged for $l\neq m$, but that for $l=m$:

$$\langle\eta_l\eta_m^*\rangle_{Av}=1-[\exp(2\pi\Gamma_r^{(l)}/d_l)-1]\times\frac{1-\bar{\eta}_l\bar{\eta}_l^*}{\exp(2\pi\Gamma_r^{(l)}/d_l)-\bar{\eta}_l\bar{\eta}_l^*}.\tag{16}$$

Hence the preceding formulation may be used throughout except for this one change. Table II and Figs. 4–6 present the chief numerical results of the above theory.

TABLE II. σ_{se} is the integrated "shape elastic" cross section. $\bar{\sigma}_c$ is the average cross section for the formation of the compound nucleus. $\bar{\sigma}_{tot}$ is the total neutron cross section. The "compound elastic" cross section plus the average reaction cross section add up to $\bar{\sigma}_c$.

$R(10^{-13}$ cm)	σ_{se}(barns)	$\bar{\sigma}_c$(barns)	$\bar{\sigma}_{tot}$(barns)
7.1 (Cd, Sn)	4.0	1.6	5.6
8.6 (Bi)	6.1	1.6	7.7

[9] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

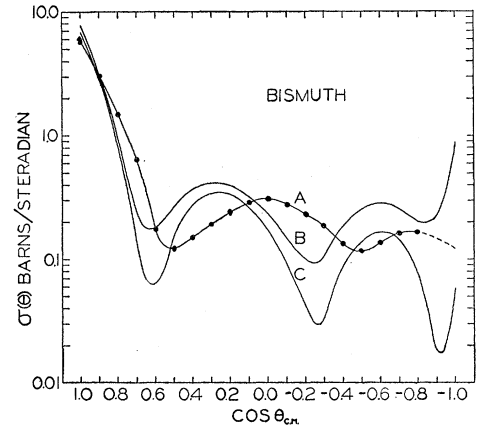

FIG. 6. Comparison of the experimental differential cross section for the elastic scattering of 3.7-Mev neutrons from bismuth with the predictions of the continuum model for a nuclear radius of 8.6×10^{-13} cm and a complex potential constant of $-19(1+0.05i)$ Mev: (*A*) experimental results from Fig. 3(*C*); (*C*) "shape elastic" differential cross section; (*B*) "compound elastic" plus "shape elastic" differential cross section using Eq. (12) to determine the "compound elastic" scattering. Solid triangle is $\sigma_{sc}(0)$ from first version of the continuum theory.

DISCUSSION

A comparison in Figs. 4–6 of the experimental differential cross section for the elastic scattering of 3.7-Mev neutrons from cadmium, tin, and, bismuth with the theoretical predictions of the continuum model shows that the main features of the experimental curves are reproduced by the theory. In general, the theoretical results give a better fit with bismuth than with cadmium or tin. If one were to use Eq. (16) in the determination of the "compound elastic" scattering, there is no choice of the ratio of reaction width to level separation $\Gamma_r^{(l)}/d_l$ that would make the fit much better. The addition of "compound elastic" scattering, however, does adjust the "shape elastic" scattering to a distribution that more nearly resembles the experimental curve than does the "shape elastic" scattering alone. In particular, this "compound elastic" scattering shifts the position of the first minima in the "shape elastic" scattering. Since different nuclei have different amounts of "compound elastic" scattering, this may account for the lack of a uniform shift in the position of the first minima in the experimental elastic scattering distributions as one proceeds from iron to bismuth.

Since the largest discrepancy between theory and experiment occurs in the differential cross section for forward scattering, $\sigma_{sc}(0)$, it is of interest to consider possible adjustments of $\sigma_{sc}(0)$ in the theory. The minimum theoretical value of $\sigma_{sc}(0)$ is given by a theory that does not include "compound elastic" scattering and has the maximum possible cross section for the formation of the compound nucleus. These conditions are met in the first version of the continuum theory.[5] The values of $\sigma_{sc}(0)$ from this theory agree with the experimental value of $\sigma_{sc}(0)$ in the case of bismuth but are too high in the case of cadmium and tin. In the case of bismuth, however, the subsequent form of $\sigma_{sc}(\theta)$ does not offer any possibility of giving a detailed fit. Therefore, it is not possible to obtain a useful value of $\sigma_{sc}(0)$ that agrees with experiment by adjusting only the value of ξ in Eq. (10) and the value of $\Gamma_r^{(l)}/d_l$ in Eq. (16). It can be argued that the double average used to obtain $\langle\eta_l\eta_m^*\rangle_{Av}$ in Eq. (12), strictly speaking, is not a necessary feature of the continuum theory and that one should insert some function of l for the parameter β_l in Eq. (11). To get some idea of the restriction introduced by this average over β_l, the value of $\eta_l\eta_m^*$ was averaged with respect to ϵ with the value of β_l set equal to zero. For bismuth the difference between this method and the double-average method was quite small. For cadmium and tin the difference was somewhat larger but still not large enough to alter appreciably the results of the double average method. Hence, it is not likely that significantly better fits between theory and experiment would be obtained if β_l were known as a function of l.

Since the theoretical value of $\sigma_{sc}(0)$ can be made to agree with experiment by adjusting the nuclear radius, it is of interest to estimate the magnitude of the change necessary to bring about this agreement. The nuclear radius used for bismuth [$A=209$ in Eq. (14)] would have to be lowered by about 6 percent since $\sigma_{sc}(0)$ varies approximately as the fourth power of the radius. This also would move the first theoretical minima closer to the first experimental minima. Since this change in radius would lower the total cross section by approximately 15 percent and since the total cross section is in fair agreement with experiment, it would be necessary to raise the imaginary part of the potential in Eq. (13) by about 15 percent in order to maintain agreement of the theoretical and experimental total cross section. Furthermore, a lowering of the nuclear radius by about 6 percent would be in agreement with the measurements of Coon, Graves, and Barschall,[10] in which they find for 14-Mev neutrons and for elements above barium that the nuclear radius as measured by $(\sigma_{tot}/2\pi)^{\frac{1}{2}}$ falls below the value predicted by $R=1.5\times10^{-13}A^{\frac{1}{3}}$ cm.

In order to adjust the theory to the experimental value of $\sigma_{sc}(0)$ for cadmium and tin it would be necessary to lower the nuclear radius chosen [$A=115$ in Eq. (14)] by about 30 percent. Corresponding to this, it would be necessary to increase the parameter ξ by a large amount to give a sufficiently large total cross section if, indeed, a large change in ξ is capable of adjusting the total cross section by a correspondingly large amount. This large adjustment of the nuclear radius is not very palatable in view of the general experimental agreement of the nuclear radius with the formula in Eq. (14) for 14-Mev neutrons and for values of A near 115.[10] It is conceivable that the theory as it stands would give better agreement if it were applied to an ellipsoidal-shaped target nucleus. However, since only 20–25 percent of the cadmium and tin isotopes have odd neutron numbers and since nuclear eccentricities in the region of $A=115$ are only about 6 percent,[11] it is not likely that the use of an ellipsoidal target nucleus can produce the desired change. Hence, we are reduced to the necessity of a modification of the theory as the only means by which a sufficiently large adjustment can be accomplished. Within the general framework of the theory, it might be possible to choose the matrix elements of H' in Eq. (7) to be a function of the relative coordinate r, or to let the potential in Eq. (6) be a function of the spins of the neutron and the target nucleus.[12] In any case, however, it would be necessary that these changes introduce little effect on the present results in the neighborhood of $A=209$.§

[10] Coon, Graves, and Barschall, Phys. Rev. **88**, 562 (1952).

[11] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), pp. 29, 776.

[12] N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953).

§ *Note added in proof.*—Although the computations of the angular distributions were stated to be found using $R=1.45\times10^{-13}A^{1/3}$ cm, the nuclear radius was inadvertently chosen using $R=1.50\times10^{-13}A^{1/3}$ cm. Since both of these formulas for the nuclear radius have been used in the arguments presented in the discussion, and since the accuracy of the data limits the comparison of the data with theory to within 4 percent in the nuclear radius, it was not considered necessary to change the computations to correspond to a nuclear radius of $R=1.45\times10^{-13}A^{1/3}$ cm. All of the arguments stated in the discussion are essentially valid despite the interchangable use of the two formulas for nuclear radius.

In summary then, it may be said that the continuum theory may be brought into rather detailed agreement with the measured angular distribution of 3.7-Mev neutrons scattered from bismuth. The theory in the case of cadmium and tin exhibits only the general features of the experimental results and possibly demonstrates a need for a revision of the theory.

ACKNOWLEDGMENT

In conclusion, the authors wish to express their appreciation to Dr. L. Eisenbud for many helpful conversations, to Mr. H. S. Hans for help with the computations, and to Dr. W. F. G. Swann, Director of the Bartol Research Foundation, for his continuing interest in this problem.

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A Study of the Interaction of Neutrons with Moderating Materials

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In the theoretical part of this paper we consider the time dependence of the slowing-down process for certain simple cases: (1) For free nuclei at rest an approximate expression is found for the velocity distribution. (2) The existence of a bond in one direction while the motion is unhindered in other directions is found to have little influence on the slowing-down process. (3) The slowing-down process in a monatomic gas is discussed quantitatively on the assumption of a Maxwellian neutron velocity distribution. The experimental part of the paper reports measurements with a pulsed neutron source and a time analyzer, in which the variations of the neutron velocity spectrum with time were studied by transmission measurements. Measurements on water show that the neutrons approach thermal equilibrium with the moderator at a rate which is in agreement with the theoretical predictions. The equilibrium temperature of the neutrons is found to agree with the moderator temperature over a wide temperature range. At low temperatures evidence is found for the effect of the lattice forces on the slowing-down process. Measurements on the initial slowing-down process for neutrons in heavy water indicate satisfactory agreement with the theory for slowing down by free deuterons at rest. The various factors which determine the rate of decay of the neutron intensity in a moderator are discussed, and the feasibility of using measurements of the decay rate to determine the diffusion and absorption properties of the moderating material as well as the geometrical buckling of complicated moderating systems is considered. In an appendix tables are given for the absorption of thermal neutrons in $1/v$ absorbers.

MODERATING materials are used for the production of thermal neutrons from a fast neutron source, and are required to slow down neutrons to thermal energies in a time which is small compared to the lifetime of the neutrons. The slowing-down time is therefore of considerable practical interest. Since the slowing-down times are of the order of 10 or 100 μsec, the process can be readily studied by the technique of the pulsed neutron source and detector developed mainly for neutron time-of-flight spectroscopy.[1] It is the purpose of the present article to discuss briefly a number of experiments performed with this method, and their theoretical interpretation. A more detailed account of the subject is given in the author's thesis, published elsewhere.[2]

PART I. THEORY

A. General

We consider the case of neutrons produced with the initial velocity v_{in} in an infinite uniform moderator during an infinitely short time interval. The subsequent slowing-down process is in principle uniquely defined by the probability $g(v_0,v)dv$ per unit path for a neutron to suffer a collision in which the velocity is changed from v_0 to the velocity interval dv at v. The velocity distribution $f(v,t)$ at the time t after the production of neutrons is given by the space- and angle-independent Boltzmann equation

$$\frac{\partial f(v,t)}{\partial t}=-\frac{vf(v,t)}{l(v)}+\int v_0 f(v_0,t)g(v_0,v)dv_0 +\delta(v_{in}-v)\delta(t), \quad (1)$$

where $l(v)$ is the total mean free path including absorption of the neutrons in the moderator. In the case when the absorption cross sections are inversely proportional to the relative neutron velocity, the velocity distribution at a given time is the same as for no absorption, since neutrons of all energies are absorbed at the same rate. Since we will in the following consider only this case, we may consider $l(v)$ to be the mean free path for scattering alone.

Except for the problem of solving the integro-differential equation (1) for an arbitrary slowing-down kernel $g(v_0,v)$, the main difficulty lies in the fact that for moderators of practical interest, where in general

[1] Rainwater, Havens, Wu, and Dunning, Phys. Rev. **71**, 65 (1947).
[2] G. F. von Dardel, Trans. Roy. Inst. Technol. Stockholm No. 75 (1954).