

Gamma Rays from Li^7 , F^{19} , Ne^{22} , and Na^{22} Produced by Alpha-Particle Bombardment of Lithium and Fluorine*

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The study of the gamma radiation following the inelastic scattering of alpha particles in thin fluorine and lithium targets has led to the discovery of new levels in the compound nuclei Na^{23} and B^{11} . The (α, p) and (α, n) reactions in F^{19} have led to the establishment of the 1.28-Mev level as first-excited state of Ne^{22} , and the discovery of the first-excited state ($T=1$, $I=0^+$) of Na^{23} , respectively. Coulomb excitation is believed to be responsible for the low-energy part of the excitation function of the 196-kev second-excited state in F^{19} , whereas this effect is much weaker for the 113-kev first-excited state of F^{19} . No capture gamma rays exist above the known resonance at 0.958 Mev in the reaction $\text{Li}^7(\alpha, \gamma)\text{B}^{11}$.

I. INTRODUCTION

THE purpose of the present investigation was to examine the energies of the gamma radiation emitted from lithium and fluorine under alpha-particle bombardment and to study the yield of these gamma rays as a function of the incident alpha-particle energy.

Fluorine-19 is known to have two low-lying excited states at about 110 and 190 kev, as found from a study of the alpha-particle groups in the reaction $\text{Ne}^{21}(d, \alpha)\text{F}^{19}$.¹ The gamma rays from these two levels have recently been observed by inelastic proton scattering on fluorine.² One expects to see these gamma rays by the inelastic scattering of alpha particles as well, via the formation of the compound nucleus Na^{23} . Further gamma radiation is to be expected from the reactions $\text{F}^{19}(\alpha, p)\text{Ne}^{22*}$ and $\text{F}^{19}(\alpha, n)\text{Na}^{22*}$ when the residual nuclei are left in excited states. Proton groups from the former reaction have indicated levels in Ne^{22} at 0.4 and 1.4 Mev.³ The latter reaction has a calculated ground-state threshold of 2.325 Mev.

Lithium-7 has its well-known first-excited state at 478 kev, which has been observed in various ways: in the K -capture decay of Be^7 (gamma ray), in the reaction $\text{Li}^6(d, p)\text{Li}^7$ (proton groups and gamma ray), in the reaction $\text{Li}^6(t, d)\text{Li}^7$ (deuteron groups), in the $\text{B}^{10}(n, \alpha)\text{Li}^7$ reaction (alpha groups), in the $\text{Be}^9(d, \alpha)\text{Li}^7$ reaction (alpha groups and gamma ray), as well as in the inelastic scattering of protons, deuterons, and 31-Mev alpha particles.⁴ Alpha-particle capture gamma rays have also been observed from Li^7 , with resonances occurring in the thick target yield of these gamma rays at 0.401, 0.819, and 0.958 Mev;⁵ the decay takes place predominantly via the 4.46-Mev level in B^{11} .

II. EXPERIMENTAL PROCEDURE

The gamma rays were detected by $\text{NaI}(\text{Tl})$ scintillation crystals mounted on a Dumont 6292 photomultiplier tube. We used either a 1-in. \times 1-in. or 1 $\frac{3}{4}$ -in. \times 2-in. crystal, depending on the energy of the gamma ray under study. The output pulses were amplified and fed through a single-channel pulse-height analyzer. The energy calibration of the over-all system was achieved by locating the photopeaks of gamma radiation from known radioactive sources, as follows: ionium (Th^{230}), 68 kev and 142 kev; In^{114} , 190 kev; Ba^{133} , 300 kev and 357 kev;⁶ Cs^{137} , 662 kev; Na^{22} , 511 kev (annihilation radiation) and 1.28 kev; Co^{60} , 1.17 and 1.33 Mev. We found the pulse height to be linear with energy. A 1-volt channel width was used throughout.

Thin targets were prepared by evaporating CaF_2 and Li_2CO_3 on nickel backing. The targets were bombarded by singly ionized helium ions produced in the rf ion source of our pressurized electrostatic generator. The target and NaI crystal were surrounded by a one-inch layer of lead to reduce general background

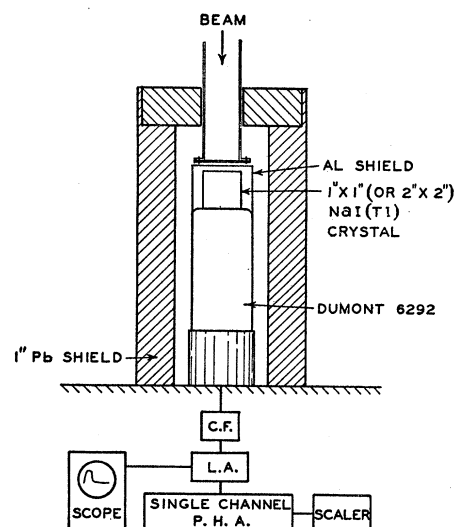


FIG. 1. Experimental arrangement for gamma-ray observation.

* Hayward, Hoppes, and Ernst, Phys. Rev. 93, 916 (1954).

* Preliminary account presented at the American Physical Society New York meeting, Bull. Am. Phys. Soc. 29, No. 1, 11 (1954).

¹ C. Mileikowsky and W. Whaling, Phys. Rev. 88, 1254 (1952).
² Peterson, Barnes, Fowler, and Lauritsen, Phys. Rev. 93, 951 (1954); R. F. Christy (private communication).

³ B. J. Jolley and F. C. Champion, Proc. Phys. Soc. (London) A64, 88 (1951).

⁴ F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 24, 321 (1952).

⁵ Bennett, Roys, and Toppel, Phys. Rev. 82, 20 (1951).

radiation. A schematic of our experimental arrangement is shown in Fig. 1. It may be seen that our crystal subtended a solid angle approaching 2π in the forward direction. The neutrons from the reaction $\text{F}^{19}(\alpha, n)\text{Na}^{22}$ were detected with a B^{10}F_3 counter surrounded by paraffin ("long counter").⁷

III. ALPHA-PARTICLE BOMBARDMENT OF FLUORINE

A. $\text{F}^{19}(\alpha, \alpha'\gamma)\text{F}^{19}$

We observed the gamma rays from the de-excitation of the first- and second-excited states of F^{19} to have energies of 113 ± 2 kev and 196 ± 2 kev, respectively. The photoelectric peaks for these gamma rays, taken at a bombarding energy of 2.55 Mev, are shown in

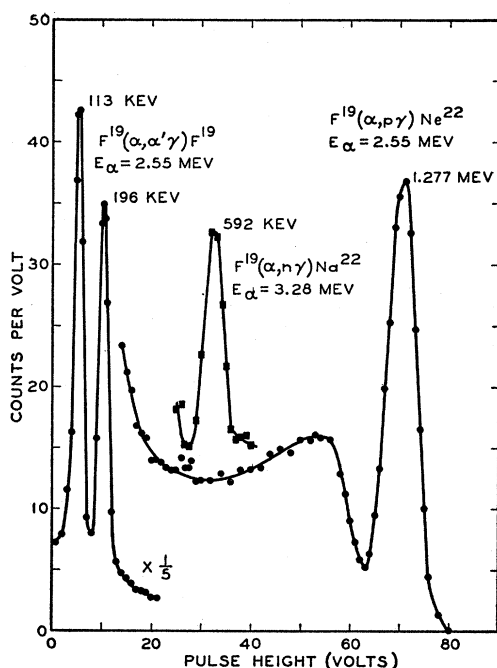


FIG. 2. Gamma-ray pulse-height distributions from alpha-particle reactions in fluorine. Circles refer to 2.55-Mev bombarding energy; squares refer to 3.28-Mev bombarding energy, showing the appearance of the 592-kev gamma ray from Na^{22*} . See Fig. 7.

Fig. 2. The gamma-ray yields as a function of alpha-particle energy for the 113-kev and 196-kev lines are shown in Figs. 3 and 4, respectively. They were obtained by setting the single-channel analyzer on the peak of the gamma ray in question. We made a search for a possible 83-kev gamma ray connecting the two levels by taking a pulse-height distribution at an alpha energy of 2.1 Mev, (Fig. 5), where the ratio of the 196-kev to the 113-kev line is large (6.6). We are able to set an upper limit of 2 percent for the frequency of occurrence of any 83-kev gamma ray compared to the 196-kev line. The apparent peak to the left of the 113-kev peak

⁷ A. O. Hanson and J. L. McKibben, Phys. Rev. **72**, 673 (1947).

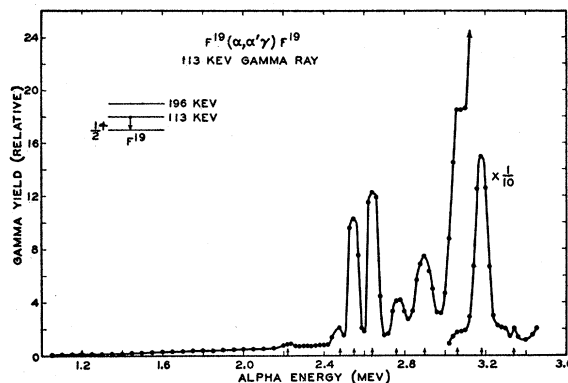


FIG. 3. Excitation curve of the 113-kev gamma ray from the first-excited state of F^{19} . Resonances correspond to levels in Na^{23} . Note slow rise from 1.1 Mev to 2.4 Mev. Thin target.

cannot be the cascade gamma ray, because sufficient amounts of absorber to cut the intensity of such a line by a factor of 10 merely reduced the intensity by about 2. This was the reduction suffered by the 196-kev line. A check with the 190-kev gamma ray from an In^{114} source revealed a peak at about the same pulse height. We conclude that it is produced by the 196-kev gamma ray in the crystal.[†] Hence, for all practical purposes our yield curve for the first-excited state is not contaminated by cascade from the upper state. A rather striking behavior is seen in the yield of the 196-kev line up to about 2.2 Mev. We shall have more to say on this interesting feature in the light of some recent additional evidence. It may be seen that most but not all resonances occur for both gamma rays. The resonance energies, the corresponding excitation energies of the compound

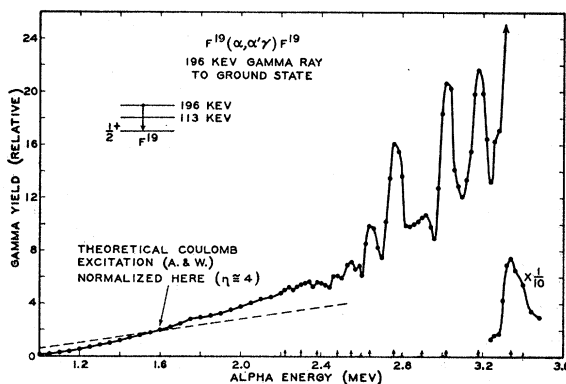


FIG. 4. Excitation curve of the 196-kev gamma ray from the second-excited state of F^{19} . Resonances correspond to levels in Na^{23} . Dashed curve is theoretical expression for Coulomb excitation (Alder and Winther) normalized at 1.6 Mev. $\eta = Z_1 Z_2 e^2 / \hbar v \cong 4$. Note the general rise continuing under the resonances. Thin target. Note added in proof.—Dashed (theoretical) curve incorrectly plotted; correct E_2 expression is in fair agreement with experiment.

[†] Note added in proof.—We have recently found this peak to be completely absent when using 4π geometry (target in well-type NaI crystal).

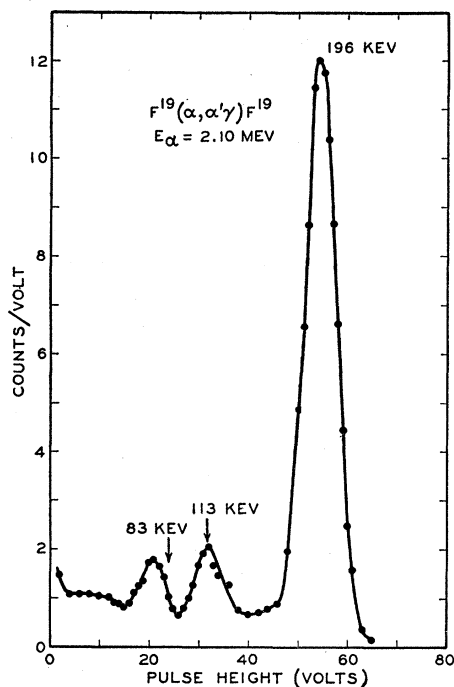


FIG. 5. Pulse-height distribution for low-energy gamma rays from fluorine taken at $E_\alpha = 2.1$ Mev. The first small peak was shown *not* to be an independent gamma ray but is produced in the crystal by the 196-kev line (see text). Arrow indicates where a possible 83-kev cascade between second- and first-excited states of F^{19} would fall. Thin target.

nucleus Na^{23} and relative intensities are summarized in Table I. The rising portion in Fig. 9 was subtracted out for this purpose. No gamma rays were observed between 196 kev and 1.28 Mev below the $F^{19}(\alpha, n\gamma)$ threshold.

B. $F^{19}(\alpha, p\gamma)Ne^{22}$

We found a 1.28-Mev gamma ray, whose thin target yield *vs* alpha energy is shown in Fig. 6. We were able to assign this gamma ray to the known 1.277-Mev level

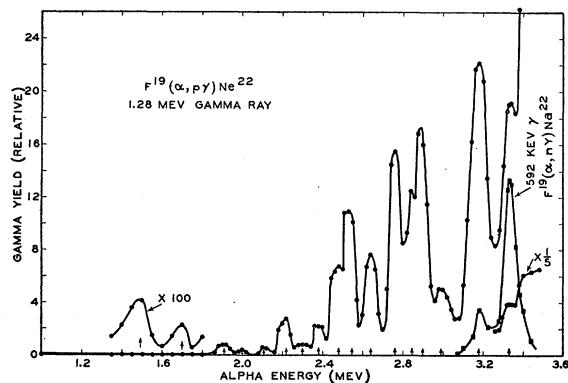


FIG. 6. Excitation curve for the 1.28-Mev gamma ray from the reaction $F^{19}(\alpha, p\gamma)Ne^{22}$. Peaks correspond to levels in Na^{23} . Square points show yield of the 592-kev gamma ray from the first-excited state of Na^{22} , with threshold around 3.05 Mev. Thin target.

in Ne^{22} rather than to the third-excited state of F^{19} at 1.37 Mev by a careful comparison with the energy of the gamma ray from a Na^{22} source which connects the same two states. Furthermore, the photopeaks from Co^{60} at 1.17 and 1.33 Mev were found to bracket the gamma ray. No gamma radiation corresponding to a 400-kev level in Ne^{22} previously reported from a study of proton groups in this same reaction³ was found. Recent observations on the $Ne^{22}(p, p'\gamma)Ne^{22}$ reaction also did not show this gamma line.⁸ We can conclude that the 1.28-Mev level is the first-excited state of Ne^{22} .

C. $F^{19}(\alpha, n\gamma)Na^{22}$

For alpha-particle energies greater than about 3.05 Mev, we noticed the appearance of a gamma ray which we could not detect at any lower energy. The energy of this line was measured to be 592 ± 3 kev. Photographs of

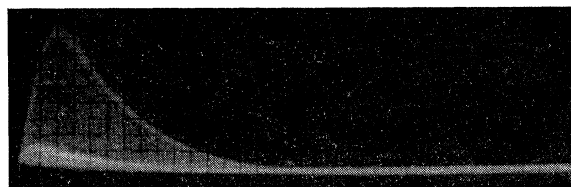
TABLE I. Energy levels in Na^{23} . E_c is the excitation energy of the compound nucleus Na^{23} . Columns 3, 4, 5, and 6 are the relative resonant yields of the various competing outgoing channels. Each gamma-ray yield is independently normalized.

E (lab) (Mev)	E_c (Mev)	Relative intensities of gamma rays			
		113 kev ($\alpha, \alpha'\gamma$)	196 kev ($\alpha, \alpha'\gamma$)	592 kev ($\alpha, n\gamma$)	1.28 Mev ($\alpha, p\gamma$)
1.49	11.73	0.04
1.70	11.91	0.02
1.91	12.08	0.8
2.00	12.15	0.4
2.11	12.24	0.5
2.22	12.32	0.4	0.4	...	2.8
2.30	12.40	...	0.5	...	0.8
2.38	12.47	...	0.5	...	2.2
2.48	12.55	1.3	0.8	...	6.8
2.55	12.60	9.4	1.0	...	11
2.64	12.68	11.4	4	...	7.5
2.76	12.78	3.2	9	...	15.5
2.84	12.85	12
2.90	12.90	6.5	3	...	17
3.02	13.00	...	12	...	5
3.06	13.03	17.5	...	thresh	...
3.18	13.13	149	12	3.5	22
3.34	13.26	19	55	13	20

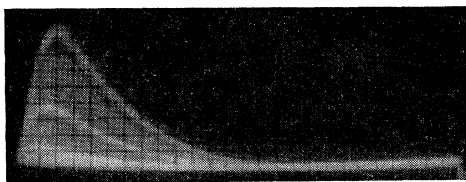
the oscilloscope traces for equal exposures taken at alpha energies of 3.00 Mev and 3.33 Mev, respectively, are shown in Figs. 7(a) and (b). The pulse-height distributions at 2.55 Mev and 3.28 Mev are shown in Fig. 2. The yield curve for the 592-kev gamma ray is shown in Fig. 6. Our suspicion that this gamma ray might come from the hitherto unreported first-excited state of Na^{22} was confirmed by the behavior of the neutron yield for the $F^{19}(\alpha, n)Na^{22}$ reaction. The yield curves for both thick and moderately thin CaF_2 targets are given in Fig. 8. The absolute neutron threshold is seen to occur at about the calculated value of 2.325 Mev. It must be remembered that such a threshold determination is obscured by (a) Coulomb barrier penetration for the incident alpha particles, (b) the presence of fairly well-defined resonances of the compound nucleus Na^{23} , and (c) the fact that the ground-state spin of Na^{22} is

⁸ Cox, vanLoef, and Lind, Phys. Rev. **93**, 925 (1954).

3^+ , inhibiting emission of threshold (*s*-wave) neutrons. Nevertheless, the agreement of observed and calculated values is quite good. A rather pronounced additional rise in neutron yield can be seen around 3.05 Mev, probably corresponding to a new neutron group leading to the first-excited state of Na^{22} and corresponding closely to the "threshold" for the 592-keV gamma radiation. Since this level has presumably spin 0^+ (see following) threshold neutrons can emerge quite easily. The calculated gamma-ray threshold for the 592-keV radiation turns out to be at 3.04 Mev, in excellent agreement with our observations. A crude check on the angular distribution of the 592-keV gamma ray is not inconsistent with isotropy. The lifetime of the transition could be seen to be less than ~ 0.1 second. A measurement of this lifetime is in progress.[†]



(a)



(b)

FIG. 7. Photograph of oscilloscope pulses produced by gamma radiation from fluorine in a $1\frac{1}{2}$ -in. \times 2-in. NaI(Tl) crystal. (a) $E_\alpha = 3.00$ Mev, showing 1.28-Mev line from $\text{F}^{19}(\alpha, \beta\gamma)\text{Ne}^{22}$; (b) $E_\alpha = 3.33$ Mev, showing appearance of the 592-keV gamma ray from the first-excited state of Na^{22} .

IV. ALPHA PARTICLE BOMBARDMENT OF LITHIUM

A. $\text{Li}^7(\alpha, \alpha'\gamma)\text{Li}^7$

The thin target yield curve for alpha-particle excitation of the 478-keV level in Li^7 is shown in Fig. 9. Three prominent resonances at 1.91, 2.46, and 3.06 Mev are present, corresponding to excitation energies in B^{11} at 9.88, 10.24, and 10.62 Mev. Some evidence of weak resonances at 2.60 and 2.70 Mev may also be discerned. This curve is in quite good agreement with some later work at the California Institute of Technology carried up to 2.8 Mev.⁹

B. $\text{Li}^7(\alpha, \gamma)\text{B}^{11}$

We also investigated the high-energy gamma rays resulting from alpha-particle capture in a thin Li_2CO_3

[†] Note added in proof.—From a motion picture record of the counting rate after beam was stopped we conclude that the lifetime is < 0.01 second.

⁹ C. W. Li (private communication).

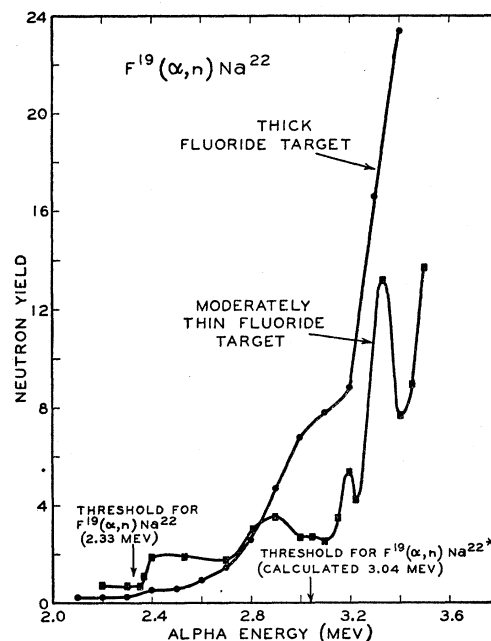


FIG. 8. Neutron yield for the reaction $\text{F}^{19}(\alpha, n)\text{Na}^{22}$ as a function of alpha-particle energy. Circles are for thick target; squares are for moderately thin target. Arrows indicate calculated thresholds for ground state and first-excited state neutrons (assuming 592-keV level separation).

target. The integral bias on our scintillation detector was set so as to accept gamma rays above 4.4 Mev, since it is known that the capture resonances lead to levels in B^{11} decaying via the 4.46-Mev state in that nucleus.⁵ The thin target yield for the 0.82-Mev and 0.96-Mev resonances are shown in the insert of Fig. 9. Their widths were found to be about 4 keV and 7 keV in the center-of-mass system, respectively. No further capture resonances were detected up to 2.5 Mev, the yield remaining essentially at background level.

V. DISCUSSION OF RESULTS

A. Energy Levels of F^{19}

We undertook this study of gamma rays from fluorine during our survey of Coulomb excitation of many nuclei by alpha particles,^{10,11} so that we surmised immediately¹⁰ that the peculiar linear rise in the yield curve (Fig. 4) for the 196-keV gamma ray might be Coulomb excitation without compound nucleus formation. The near absence of a similar effect in the yield of the 113-keV line (Fig. 2) was quite striking. Now the simple semiclassical theory of Coulomb excitation by electric quadrupole excitation given by Alder and Winther¹² is not expected to hold for this extremely light nucleus. The dotted line in Fig. 4 represents the

¹⁰ G. M. Temmer and N. P. Heydenburg, Phys. Rev. **93**, 351 (1954).

¹¹ N. P. Heydenburg and G. M. Temmer, Phys. Rev. **93**, 906 (1954).

¹² K. Alder and A. Winther, Phys. Rev. **91**, 1578 (1953).

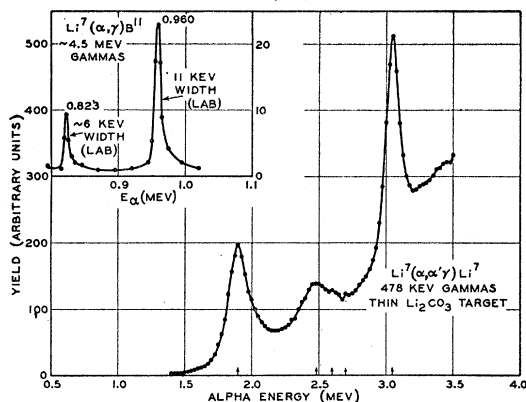


FIG. 9. Excitation curve for the inelastic excitation of the 478-keV gamma ray from Li^7 . Resonances are excited states in B^{11} . Insert shows yield of capture gamma rays ($E_\gamma > 4.5$ Mev). No further capture resonances observed up to 2.5 Mev. Thin target.

theoretical curve, normalized to the data at 1.5 Mev. The experimental yield is seen to rise considerably more rapidly. Some of this discrepancy may be blamed on a small amount of compound nucleus formation which contributes to the yield.[§] From the yield for the $\text{F}^{19}(\alpha, p\gamma)\text{Ne}^{22}$ reaction (Fig. 5), we know that there are levels of Na^{23} below 2.2-Mev bombarding energy. The Coulomb barrier of fluorine for alpha particles is about 5 Mev, so that one might expect the bombarding particle to get *closer* than is considered in the semi-classical treatment, and hence to produce more excitation than predicted. We have evidence for a similar effect in Na^{23} , where the deviations from the theoretical prediction are considerably smaller. The general trend of the rise up to 2.2 Mev in F^{19} can be seen to continue under the compound nucleus resonances to higher energies. This trend is very much weaker in the 113-keV excitation. Since the ground state of F^{19} has spin $\frac{1}{2}^+$, a simple explanation of the smallness of this effect for the lowest level would be a $\frac{1}{2}$ assignment, since a quadrupole transition could not connect these two states. A spin of $\frac{3}{2}^+$ or $\frac{5}{2}^+$ would be indicated for the 196-keV level. We have just been informed of some important work on these levels using proton bombardment¹³ and a recently reported method¹⁴ for measuring lifetimes of excited states, yielding values of $\sim 10^{-7}$ second for the 196-keV level and $\sim 10^{-9}$ for the 113-keV state. These authors assign spins of $\frac{5}{2}^+$ and $\frac{1}{2}^-$ to these states on the basis of the lifetime values, their failure to observe any cascade gamma ray connecting the two levels, and an isotropic angular distribution of the 113-keV gamma ray. Our results thus find a very satisfactory explanation. It is interesting to note that this Coulomb excitation effect in light nuclei could not be clearly observed with protons because of the lower

[§] Note added in proof.—See, note added in proof, caption of Fig. 4.

¹³ R. W. Peterson and R. F. Christy (private communications). We are grateful to Professor Christy for informative discussion.

¹⁴ J. Thirion and V. L. Telegdi, Phys. Rev. **92**, 1253 (1953).

barrier of the nuclei and the consequent presence of strong compound nucleus resonances extending to practically zero bombarding energy.

The first-excited state of F^{19} possibly constitutes the only clearcut example among the 72 nuclei we have studied where a low-lying level is accessible *only* by an electric dipole transition.

B. Energy Levels of Ne^{22}

Since we do not find any evidence for any gamma ray of energy less than 1.28 Mev, (except for the 113-keV and 196-keV levels of F^{19}), we confirm the fact that the latter gamma ray comes from the first-excited state of the even-even nucleus Ne^{22} . In accordance with the systematics of these nuclei¹⁵ this should be the level having spin 2 and even parity.

C. Energy Levels of Na^{22}

In view of the evidence presented in Sec. III (C), we believe to have found the first-excited state of the odd-odd nucleus Na^{22} . In a recent survey of striking regularities in the isotopic spin singlets and triplets of odd-odd nuclei, extending from H^2 to K^{38} , Staehelin^{16,17} predicted a level in Na^{22} located between 600 and 700 keV ($T=1$, $I=0^+$), a prediction which is now believed to be confirmed. The Coulomb energy

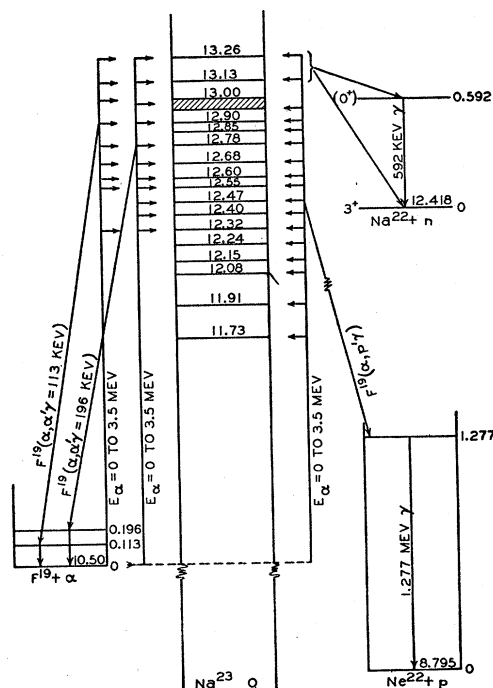


FIG. 10. Composite level diagram illustrating the various reactions investigated by observing gamma radiation from fluorine targets bombarded by alpha particles.

¹⁵ G. Scharff-Goldhaber, Phys. Rev. **90**, 587 (1953).

¹⁶ P. Staehelin, Helv. Phys. Acta **26**, 691 (1953).

¹⁷ P. Staehelin, Phys. Rev. **92**, 1076 (1953).

difference between this 0^+ level and the ground state (0^+) of Ne^{22} leads to a value $r_0 = 1.40 \times 10^{-13}$ cm, in good agreement with those obtained from the other members of this series.¹⁷

The 592-keV transition connecting the 0^+ state with the $I = 3^+$ ground state of Na^{22} (pure $M3$) has an expected lifetime¹⁸ of 0.06 second. The internal conversion coefficient gives a negligible correction to this value.¹⁹ Our upper limit of ~ 0.1 second is consistent with this prediction.¹¹ Considering the direct positron decay from the 592-keV state in Na^{22} to the ground state of $\text{Ne}^{22}(0^+ \rightarrow 0^+)$, a lifetime of ~ 15 seconds is calculated on the basis of $\log ft = 3.44$.¹⁶ We see therefore

¹⁸ M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 906 (1951).

¹⁹ Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **83**, 79 (1951).

¹¹ Note added in proof.—See, however, note added in proof, galley 8.

that a theoretical branching ratio of about $190 \div 1$ in favor of the electromagnetic transition should obtain.

Another missing level in this series occurs in P^{30} . Our attempts to detect this level in the reaction $\text{Al}^{27}(\alpha, n)\text{P}^{30*}$ have not been conclusive to date, although we were able to detect the 2.5-minute positron activity from the ground state. These experiments are continuing.

D. Energy Levels of Na^{23} and B^{11}

The energy levels in the compound nucleus Na^{23} are listed in Table I. A level diagram illustrating all states reached in the alpha-particle bombardment of F^{19} is shown in Fig. 10. No levels in this region of excitation in Na^{23} have been previously reported. New excited states in B^{11} at 9.88, 10.24, and 10.62 MeV have been found by the inelastic scattering of alpha particles in lithium, as discussed in Sec. IV (A).

Many-Particle Configurations in a Central Field*

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Closed formulas are obtained for the fractional parentage coefficients of j - j coupled configurations of three and four equivalent particles. In states of low seniority, these formulas can be used to simplify the calculation of familiar types of matrix elements. Some extension is made to the study of more complex configurations. In particular, it is shown that near the ground state the energy level spectrum of an even-even nucleus should be independent of the number of particles in the unfilled shell.

INTRODUCTION

THERE are two well-known ways of describing an antisymmetric configuration of several equivalent particles bound in a central field. The first¹ is the permutational construction in which the wave functions of the several particles with the several sets of quantum numbers are arrayed in a Slater determinant to give directly an antisymmetric form. However, this structure proves to be quite unwieldy when one tries to calculate matrix elements of various operators, since a great many cross terms come into the expression.

The second^{2,3} is the method of fractional parentage in which one considers just one particle of the configuration separated from all the others. The antisymmetry requirement is satisfied by leaving unspecified all the quantum numbers pertaining to the combination of the

"other" particles and coupling the angular momentum of the one separated particle to some particular linear combination of all the allowed states of the "others." The fractional parentage coefficients which determine this particular linear combination for some given configuration are usually found as the solutions to a set of simultaneous algebraic equations and are tabulated for any particular problem.

However, it is clear that a direct (though formal) determination of the coefficients can be achieved by equating the wave function as written in the fractional parentage way to the permutational form mentioned first. The solution generally gives the fractional parentage coefficient in terms of the transformations which interrelate all the different vector coupling schemes of the several angular momenta. Using the techniques of Racah,⁴ we have obtained formulas for the fractional parentage coefficients for configurations of three and four particles.

Using recursion relations, first derived by Racah³ and extended in our Appendix, which are developed around the seniority concept, we can deduce formulas for the fractional parentage coefficients for some states of con-

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† Gulf Oil Corporation Fellow.

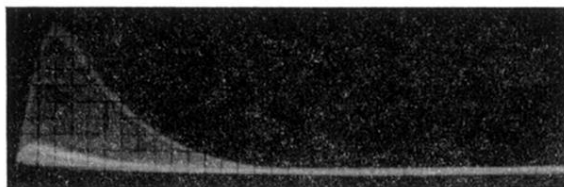
‡ Now at the Weizmann Institute of Science, Rehovoth, Israel.

¹ E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1951), p. 162 et seq.

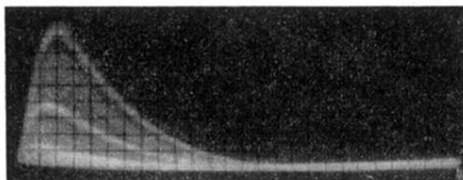
² S. Goudsmit and R. F. Bacher, Phys. Rev. **46**, 948 (1934).

³ G. Racah, Phys. Rev. **63**, 367 (1943).

⁴ G. Racah, Phys. Rev. **62**, 438 (1942).



(a)



(b)

FIG. 7. Photograph of oscilloscope pulses produced by gamma radiation from fluorine in a $1\frac{3}{4}$ -in. \times 2-in. NaI(Tl) crystal. (a) $E_\alpha=3.00$ Mev, showing 1.28-Mev line from $F^{19}(\alpha,p\gamma)Ne^{22}$; (b) $E_\alpha=3.33$ Mev, showing appearance of the 592-kev gamma ray from the first-excited state of Na^{22} .