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Proton-Proton Scattering Near the Interference Minimum*

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The differential proton-proton scattering cross section at 45° in the laboratory system has been measured in the energy region 350–420 kev. The interference minimum occurs at 383.9±1.5 kev. The minimum in the ratio to the Mott cross section thus occurs at 382.8±1.5 kev, and at the latter energy the S-wave phase shift is $\delta_0=0.2527\pm0.0011$ radians, in good agreement with the most recent data in this energy region. The combination of our point with the Wisconsin data alone agrees with the previous analysis of data from a wide range of energies.

I. INTRODUCTION

ONE of the time-honored[1] purposes of proton-proton scattering measurements[2] has been to determine the nuclear S-wave phase shift as a function of energy. Such measurements give a detailed description of the force field between two protons with antiparallel spins and no relative angular momentum.

It has been emphasized[3] that two parameters, the effective range and the scattering length, are sufficient alone to predict remarkably well the S-wave phase shift over a large energy range. Unfortunately, force fields derived from static potentials of widely different functional dependence on the separation between the protons, or even velocity-dependent forces, may well have the same best two-parameter fit to the low-energy data. The first information about the "shape" is given only when a third parameter becomes necessary to fit the data.

In order to choose, for example, between Yukawa potentials with and without a repulsive core the energy range of experiments must be wide and the low-energy data must be particularly accurate. High-energy measurements give large effects but are more difficult to interpret because they may have in them appreciable amounts of higher partial waves than S wave, and because a comparison with low-energy data relies heavily on the assumption of a known velocity-dependence of the forces. On the other hand, the accuracy required in low-energy measurements necessitates the most painstaking work,[4] and the interpretation may still involve the velocity-dependence of the forces.

Proton-proton scattering experiments are particularly difficult at energies appreciably below 1 Mev, where the scattering and stopping powers of thin windows and of the scattering gas are relatively large. Because there is no bound state, because no experiments at thermal energies are possible, and because total cross section measurements would give negligibly little nuclear information even if possible technically, the measurements at energies below 1 Mev, as at higher energies, must be differential scattering experiments. Such experiments necessitate in general absolute measurements of the energy and numbers of incident protons, density, and volume of target protons, mean angle and solid angle subtended by the detector of scattered protons, and efficiency of the detector.

In practice accurate phase shifts may be obtained in this energy region by measuring[5] the relative yield of scattered protons as a function of angle. This requires absolute measurements of only energy and mean angle of scattering, but also requires considerable knowledge of the relative efficiency of the detector as a function of scattered proton energy, since the energy changes rapidly with angle.

There is in addition one set of exceptional energies and angles at which it is necessary to have absolute values for only the energy and mean angle, without an angular survey, and to correct if necessary for the change in the relative efficiency of detector and current monitor over a narrow energy interval only. These are combinations of energy and angle at which the repulsive electrostatic force[6] cancels out a part or all of the effect of the attractive S-wave nuclear force.[7] The destructive interference is most marked at an angle of 90° in the cm system. The energy of this minimum is determined in the present experiment by studying the relative

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[1] Breit, Thaxton, and Eisenbud, Phys. Rev. **55**, 1018 (1939).

[2] Herb, Kerst, Parkinson, and Plain, Phys. Rev. **55**, 998 (1939).

[3] L. Landau and J. Smorodinsky, J. Phys. (U.S.S.R.) **8**, 154 (1944); Julian Schwinger, Phys. Rev. **72**, 742 (1947); H. A. Bethe, Phys. Rev. **76**, 38 (1949); and J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).

[4] Worthington, McGruer, and Findley, Phys. Rev. **90**, 899 (1953).

[5] Yovits, Smith, Hull, Bengston, and Breit, Phys. Rev. **85**, 540 (1952) survey and interpret all the data and quote unpublished results of a thorough investigation by Heydenburg and Little in the low-energy regions.

[6] The dipole-dipole magnetic interaction has been treated as a perturbation on the nuclear interaction by J. Schwinger, Phys. Rev. **78**, 135 (1950).

[7] A rough optical analog to this experiment is the following: A spherically symmetric cloud of free electrons, of known density as a function of radius, is irradiated by light of wavelength comparable with the mean radius of the charge distribution. The scattered light is observed at a distant point at some fixed angle. Now a very small dielectric ball of scattering power comparable to that of the charge cloud is introduced at its center. The wavelength of the light is varied until the scattered intensity is a minimum at the angle of observation. The S-wave phase shift by the ball at one wavelength, and hence a single measure of the scattering properties of the dielectric ball, is thus determined.

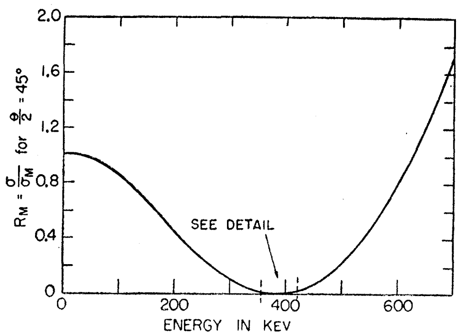

FIG. 1. Ratio of proton-proton cross section to Mott cross section at 45° laboratory angle.

yield of scattered protons at 90° c.m. (45° lab) as a function of energy from approximately 350 to 420 kev.

The laboratory cross section at 45° in the 0 to 1-Mev region may be predicted from the best fit to the combined data at higher energies.[5] For simplicity we chose to compute the cross section from the two-parameter ("shape independent") fit to the latest Wisconsin data[4] only. The quantity plotted in Fig. 1 is actually the ratio of the p-p cross section computed in this way to the Mott cross section for protons with nothing but a pure Coulomb force between them. To get the differential cross section in millibarns per steradian in the laboratory, the ordinate in Fig. 1 must be multiplied by $20.73/E^2$, where E is the bombarding energy in Mev.

An accurate determination of the cross-section minimum is not as difficult as it appears from Fig. 1. Plotted on a more appropriate scale in Fig. 2, the detail of

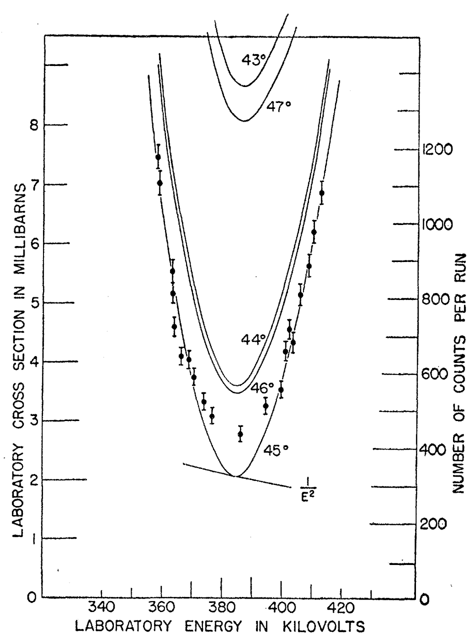

FIG. 2. Detailed proton-proton cross section at laboratory angles near 45°.

the laboratory cross section shows a sharp minimum. We were preparing to do an accurate absolute cross section measurement until we observed experimentally this rapid variation of cross section with energy.

An order of magnitude estimate of the inaccuracies in the experiment and in its interpretation may be given as follows: At the 45° minimum the nuclear phase shift is approximately equal to $e^2/\hbar v$, where v is the relative velocity of the two protons at the energy of the minimum. We determined this energy to be 383.9 ± 1.5 kev, i.e., with 0.4 percent accuracy in v^2. Thus the nuclear phase shift at the minimum is obtained to about 0.2 percent. The 2- to 4-Mev data give phase shifts accurate at those energies to the order of 0.1 percent. When these data are used to predict the phase shift expected at the minimum the differences due to choice between popular static potentials are of the order of only a few tenths of a percent. A slightly velocity dependent force field—for example a nuclear "potential" the apparent depth of which changes by $\Delta v^2/c^2$ on going from one velocity to another—will also give comparable effects of a few tenths of a percent in phase shift in going over this energy range.

II. EXPERIMENTAL DETAILS

A. Machine

The proton beam was provided by a pressurized Van De Graaff generator[8] built with this experiment in mind. The beam from the radio-frequency ion source was focused in the conventional way and analyzed outside the machine by deflection through about 15° using a permanent magnet with an adjustable flux path to vary the magnetic field. Additional focusing of the mass-1 beam alone by an electrostatic alternating-gradient lens provided up to 5 microamperes through the collimator and to the scattering chamber (Fig. 3).

Voltage stabilization was provided by corona control operating on a signal from overlapping slits placed in the deflected mass-2 beam.

B. Scattering Chamber

The fixed-angle scattering chamber was designed as small as possible to keep the path length of incident and scattered protons to a minimum in the scattering gas. Differential pumping was used at the beam entrance to give as little stopping power as possible in the entrance collimators. The counters were operated using the purified hydrogen scattering gas at the scattering chamber pressure with no windows to impede the scattered protons.

The proton beam entered the scattering chamber through a series of collimators which also served as impedances for gas flow in the differential pumping

[8] Cooper, Frisch, Storrs, and Strumski, Phys. Rev. **79**, 708 (1950). A letter describing the hydrogen supply for the ion source and the strong focusing electrostatic lens is being submitted to *Review of Scientific Instruments.*

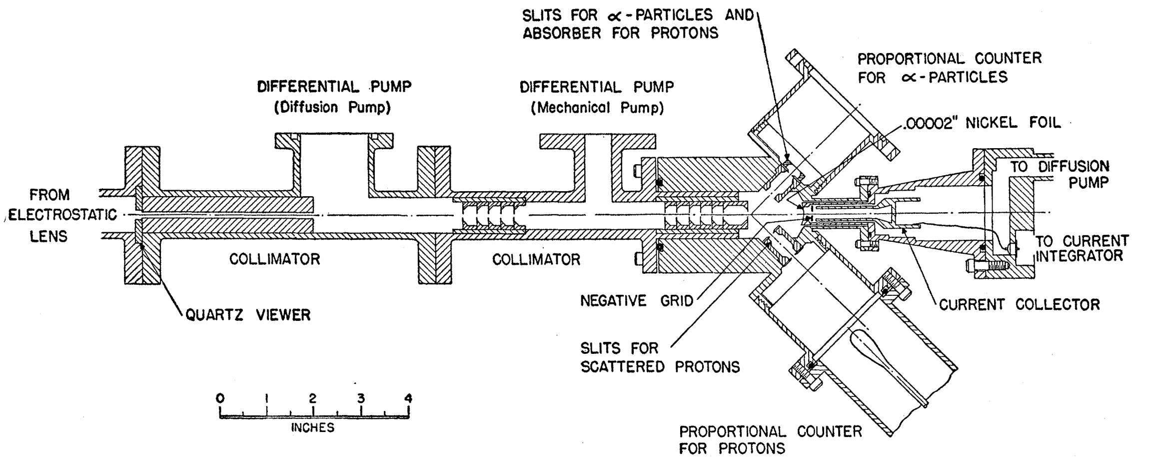

FIG. 3. Scattering chamber.

system. The first collimator was a brass tube 0.125 in. in diameter and four inches long. The next two consisted of a series of 0.040-in. holes in steel disks less than 0.001 in. thick at the inside edge, spaced $\frac{1}{4}$ in. apart. The angular spread thus defined was $\pm 0.5°$ about the central axis, or a maximum difference of angles of 1.0°. That there was no detrimental degradation of the energy of the protons by the walls of the collimator was indicated by the good functional form of the observed minimum in the *p-p* cross section.

The current collector cup was kept at ground potential with a magnetic field to prevent secondary electrons from leaving the cup and a negative grid (a ring) to prevent secondary electrons from the nickel foil from entering the cup. The charge collected was measured with an electronic current integrator. By observing the Rutherford scattering of protons from argon, the rate and energy dependence of the current collecting system and of effects from the varying chamber pressure could be studied. Also in this way the correct value for the magnetic field ánd shielding voltage was determined.

The geometry was such that one could count the protons scattered through an angle of 45° on either side of the beam in each of two perpendicular planes. Coincidences between the two protons emerging from a single scattering were thus possible in each plane, and were used at first to check the identification by pulse height of the pulses coming from the counter.

Slits were provided in front of each of the counters such that only protons scattered through a narrow range of angles centered on 45° could enter. The first slit was located 0.625 in. from the beam along the line of the scattered protons; the second slit was 0.625 in. beyond the first. Typical slit dimensions were 0.012 in.×0.200 in. and 0.018 in.×0.200 in., respectively. There were no foils at this point; the counters operated in the scattering chamber gas at the scattering chamber pressure. The counters were especially designed for this (see Fig. 4); the center wire was $\frac{1}{8}$-inch drill rod 12 inches long cantilevered from a large Kovar-glass lead through and joined smoothly at the free end to a $\frac{3}{8}$-inch diameter rounded section. The operating voltage was in the vicinity of 1200 volts. When running the experiment with pure H_2 at about 1.5-cm Hg, typical pulses were 15 times noise with a pulse-height distribution that at best had a width at half-maximum about $\frac{1}{5}$ the most probable pulse height. The counters were adequately proportional so that the protons scattered from protons could be separated from protons scattered by heavier nuclei.

The counter output was amplified by a Model 100 amplifier and the pulses recorded on a 10-channel pulse-height analyzer in parallel with integral scalers Model 210. The availability of the 10-channel analyzer was of a far greater importance than would be indicated merely by the large statistical gain it gave over integral or even single differential channel analyzers.

C. Gas Handling and Purification

At its minimum the 45° *p-p* scattering cross section is only 1/1760 times that for the scattering of the protons by oxygen, the principal contaminant of the tank hydrogen used. Commercially available H_2 must be purified by more than a factor of 10 before the *p-p* yield is comparable to the *p*-O yield. To purify the scattering gas, we used activated charcoal at liquid nitrogen temperature, passing the hydrogen through

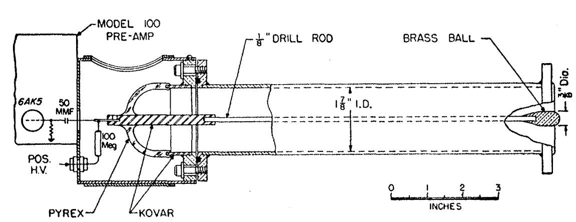

FIG. 4. Low-pressure proportional counter.

a charcoal-filled stainless steel cylinder at pressures above 1000 psi. This purification was carried out by batch, one purification yielding enough gas for several scattering runs. There seemed to be no appreciable increase in contamination while stored in the conventional metal cylinders which were well evacuated before filling with purified gas.

The gas thus purified was fed through a reduction valve and a needle valve into the scattering chamber. It was pumped out through the collimators by the differential pumps. When the pressure in the chamber was 1.5 cm of mercury, the pressure was 1.0 mm at the first differential port and 10^{-3} mm at the second differential pump. The chamber pressure and the pressure at the first differential port were measured by oil monometers. Since we were interested primarily in relative cross sections we did not need to know the pressure absolutely, although small corrections involving the ratio of pressures were made assuming the zero to be correct.

When large currents were passed through the collimators, the pressure in the chamber would rise significantly (approximately 10 percent for a current change from 1 to 5 microamperes) presumably because of the increase in viscosity of the hydrogen gas in the collimator. This effect is probably caused by the temperature rise from the additional energy lost in the gas by the more intense incident beam. It was therefore necessary to monitor the pressure to a few percent, and to keep the current fairly constant during a run.

D. Energy Measurement

Originally it was intended to measure the energy of the beam by a time-of-flight method and to control by modulating an electron beam in a separate accelerating tube. We found, however, that with corona control operating from the mass-2 beam the generating voltmeter readings were remarkably reproducible, and that the fluctuations in beam energy about the average energy were not more than a few kilovolts (evidenced by the sharpness of the proton induced resonances described below). The average generating voltmeter galvanometer reading could be kept to about ± 300 volts. Hence we felt no need at all of more rapid control equipment and no acute need of a more stable system. The energy measurement was therefore based entirely on the generating voltmeter readings, and no new absolute measurement of energy was made in this experiment.

The generating voltmeter rotor was driven by a synchronous motor whose speed was monitored during the runs with a 60-cycle vibrating reed frequency standard. Corrections of the order of 0.2 percent due to the variations in the line frequency were applied to the data.

The signal was taken from a stator whose geometry corresponded to that of the rotor, which was grounded. The signal was rectified by 6H6 diodes and the voltage drop across a 10K temperature-controlled resistor measured with a precise potentiometer. The resistors in the measuring circuit were such that the time constant was about a millisecond.

The machine temperature and insulating gas pressure were kept constant over the period of time necessary to complete a set of scattering runs and to calibrate the generating voltmeter.

To calibrate the generating voltmeter we used the well-established points on the nuclear energy scale[9] given in Table I. Two kinds of fluorine-containing targets were used, each for a separate purpose. By filling the scattering chamber with a trace of difluoroethane ($C_2H_4F_2$) in hydrogen at the usual pressure, the alpha particles from the reaction $F^{19}(p,\gamma\alpha)O^{16}$ may be observed in one of the proportional counters. A thin nickel window placed in front of the alpha-counter slit system absorbs the scattered protons, but allows the alpha particles from the 340.2-kev fluorine resonance to pass through. This scheme is not possible at the 483.1-kev resonance because the elastically scattered protons penetrate the foil.

Since the beam collimators were filled with essentially the same gas as they were during the actual proton-scattering run, this measurement provided a direct connection between the 340-kev proton energy at the scattering volume with the generating voltmeter reading.

To find the similar connection for the other proton energies we bombarded a LiF rotating target evaporated

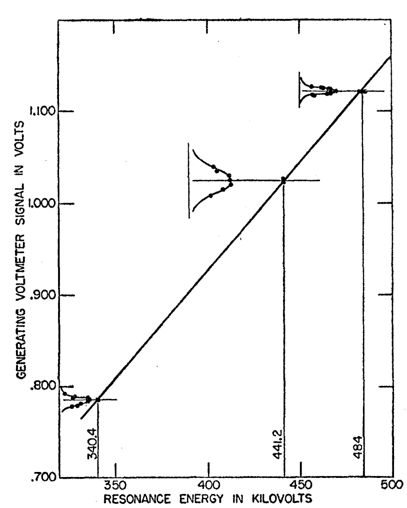

FIG. 5. Calibration of generating voltmeter using fluorine and lithium resonances. The values shown for the resonance energies are not the recent ones used in the calculation. Using 340.2 kev for the lower fluorine resonance and 441.2 kev for the lithium, we observe 482±2 kev for the energy of the upper fluorine resonance, as compared with the 483.1±0.5 kev in Table I. Not shown is a higher fluorine resonance at 873.5±0.8 kev, which also indicated a linear calibration.

[9] A summary of the various measurements of these resonances is given by S. E. Hunt, Proc. Phys. Soc. (London) **A65**, 982 (1952).

in place at the scattering volume below the same beam collimators as were used during the proton scattering runs. The γ rays from the three reactions in Table I were detected with a Victoreen Geiger tube. In this way the linearity of the generating voltmeter reading was checked. This target was not used for an absolute calibration because it was felt that we could not rely on a solid target being completely free from coatings when bombarded with high proton current densities. Actually, freshly evaporated targets gave the same calibration at 340.2 kev as did the gas target.

The solid target was also used to investigate the rate of energy loss in the various gases used in the scattering experiment by varying the position of the target along the beam. The shift in the 340.2- and 483.1-kev resonances was observed as the chamber was alternately evacuated or filled with H_2 gas or filled with various concentrations of $H_2-C_2H_4F_2$ mixture at various pressures.

Figure 5 shows the generating voltmeter readings plotted *versus* the published values of the resonances. These readings have all been corrected for energy loss in the gas or in the target coating. Because the generating voltmeter appears to be linear within the errors in Table I, we will express the energy of the minimum in terms of the energies of the two fluorine resonances so that possible future changes in their energies may be applied directly to our result. The lower resonance has the dominant effect in determining the energy of the nearby minimum.

E. Experimental Procedure

After the current collector was calibrated by scattering from an argon-hydrogen mixture, the machine energy was calibrated and the p-p scattering data taken according to the following procedure.

The chamber was filled with a 1 percent by volume mixture of $C_2H_4F_2$ in H_2 and the generating voltmeter was calibrated by observing the $F(p,\gamma\alpha)$ alphas from fluorine at 340.2 kev in the scattering volume. The chamber was then pumped out and pure hydrogen allowed to flow through. The yield of scattered protons was observed at each energy for two approximately five-minute intervals. Then the machine energy was shifted to an energy roughly symmetric about the minimum. After about five hours the generating voltmeter calibration was repeated with the fluorine-containing gas.

Immediately after this the solid target chamber was put in place of the scattering chamber below the collimator and the energy loss of the protons and generating voltmeter linearity were measured.

III. RESULTS AND REDUCTION OF DATA

The points in Fig. 2 are our best run of p-p laboratory cross section plotted against energy. Both ordinate and abscissa are absolute measurements, and the curves are computed from a best shape-independent fit to the Wisconsin data.[4] Three previous runs had been taken, all giving this same value for the minimum,[10] but since the experiment was not understood nearly so well in the earlier runs we will base our conclusions entirely on the data in Fig. 2.[11]

TABLE I. Established points on the nuclear energy scale.

Reaction	E_r in kev	Width in kev
$F(p;\gamma\alpha)$	340.2 ± 0.5	2.9
$Li(p,\gamma)$	441.2 ± 0.5	12.2
$F(p;\gamma\alpha)$	483.1 ± 0.5	2.2

Our finite angular resolution should give an effective angle of about 44.4° for about half the protons and 45.6° for the other half. The effect of scattering in the hydrogen gas should make the deviation of the observed counts from the 45° curve even greater. The observed points thus lie too low by the order of 15 percent.

The value of the energy of the minimum is determined by folding the 45° curve to get the line of symmetry. A renormalization of absolute cross sections by a 15 percent increase would change the energy of the minimum by only about 0.2 kev, since the curves are almost symmetric about a line that is almost vertical.

A renormalization of 15 percent is outside the 8 percent or so we would have estimated as the absolute accuracy, except for a possible enrichment of the argon used in calibrating the current integrator by selective diffusion through the collimator of the hydrogen with which it was mixed. However, the comparison of our data on p-d scattering[10] with that of Freier *et al.*[12] makes more than a 10 percent error in calibrations unlikely. Also if the points are all raised by the same factor the shape of the curve is not quite right; the resultant curve is too narrow at the top, touching the 46° curve at about 35 kev above and below the minimum. A better estimate for the correction for finite solid angle would make this distortion worse, and the energy resolution observed with the resonances seems too good (about 3 kev) to affect the curve appreciably.

This apparent difficulty with the absolute value of the observed cross section was not investigated thoroughly, and it may well all be instrumental. For it to be genuine the 45° theoretical p-p cross section would have to be too high by roughly three-quarters of a millibarn over this energy region. For example, an improbable positive nuclear D-wave phase

[10] Cooper, Frisch, and Zimmerman, Phys. Rev. **90**, 339 (1953).

[11] The run plotted could have had five times as many counts in the same time with almost the same angular resolution. Unfortunately, it was done with slits which, for a study of pulse group separation, had been made unnecessarily narrow in the direction perpendicular to the plane of scattering.

[12] Freier, Stratton, Brown, Holmgren, and Yarnell, Phys. Rev. **86**, 593 (1952).

shift of the order of 10^{-3} radian in this energy region would bring the data and theory into better agreement.

The 45° curve as plotted is a fairly good match to the data for S-wave purposes. The curve could not be displaced by more than about a kilovolt up in energy or one and a half kilovolts down without being noticeably off the best fit to the points. A reasonable estimate of the best fit, combined with a 15 percent renormalization of the ordinates of the points, gives the minimum of the observed curve as about 0.4 kev lower than the minimum of the computed 45° curve. We assign an over-all error to the experimental value of about 1.5 kev, giving $E_{\min}(45°)=383.9\pm1.5$ kev. In terms of possible different values E_1 and E_3 of the fluorine resonances in the future, $E_{\min}=383.9+(99.2/142.9)(E_1-340.2)+(43.7/142.9)(E_3-483.1)$.

The minimum in ratio to Mott is less by 1.05 kev, giving 382.8 kev. An approximate expression for the phase shift at the energy of the minimum in ratio to Mott, is $\delta=\eta-0.156\eta^3$, where $\eta=e^2/v=0.2557$. Therefore, $\delta_0=0.2527\pm0.0011$ at 382.8 kev.

TABLE II. Values of K at 382.8 kev, and the energy of the minimum in ratio to Mott.

	K at 382.8 kev	$E_{\min}$
Yukawa	3.905	381.6
Observed	3.917±0.016	382.8±1.5
Shape independent	3.924	383.4
Square	3.939	384.7

Table II gives the values of K at 382.8 kev and the energy of the minimum in ratio to Mott which we calculate from the best fits to the Wisconsin data[13] for various assumptions of well shape.

Our determination of the energy of the minimum, in terms of the published *best* fits to the Wisconsin data, seems to exclude the possibility that the p-p interaction might be described by a static square well, or Yukawa plus a very large repulsive core.[14] Less good fits to the Wisconsin data, but still within reason, give for any particular shape a spread in the values of the predicted $E_{\min}$ comparable with the spread in Table II. Hence, one cannot with certainty exclude a square well on the basis of these data alone.

Considered with all the data from a wide energy region, our datum fits in well with the conclusion, already indicated by Yovits *et al.*,[5] that the force field cannot be described by a potential shaped like a square well, or a Yukawa well with a large repulsive core.

We are particularly indebted to Peter Rose for considerable help with the experiment and to C. J. Strumski for a great part of the construction and operation of the machine.

[13] H. H. Hall and J. L. Powell, Phys. Rev. **90**, 912 (1953).
[14] Hafner, Hornyak, Falk, Snow, and Coor, Phys. Rev. **89**, 204 (1953).