A β-Decay Energy Systematics

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A β-decay energy systematics is presented which exhibits linear relations for disintegration energies between atoms (Z,N) and $(Z+1, N-1)$ when plotted as a function of N keeping Z constant. Separate plots are necessary for odd and even values of A. The disintegration energy lines show marked discontinuities for $N=50$, 82, and 126. There is also considerable indication of a discontinuity at $N=28$, and in the *region* of $N=20$ sharp changes in slope are observed. Proton magic numbers at 50, 82, and possibly 28 are indicated by gaps in the spacing of the disintegration energy lines. No definite submagic numbers are at present identifiable. Changes in the even-odd differences of neutron and proton binding energies show up as changes in the line separation pattern. The most striking pattern changes can be interpreted as due to decreases in the even-odd differences for both neutrons and protons after their respective magic numbers. The systematics is useful in estimating disintegration energies in cases where measurements have not yet been made. Part of a systematics for double β decay is also shown.

1. INTRODUCTION

THE continued study in many laboratories of β disintegration decay schemes by means of spectrometers and coincidence techniques has resulted in the fairly accurate determination of total disintegration energies for about 450 radioactive nuclei. It is therefore now possible to give a general presentation of the results, to make an over-all comparison of the experimental values with those given by the semiempirical mass formula,[1] and to discuss implications for nuclear structure.

Although the semiempirical formula has proved helpful in portraying the general features of the mass surface, it has been known for some time that it does not represent the experimental facts in detail. Nevertheless, in searching for over-all regularities in the data, it is natural to look to this familiar generalization for guidance and suggestions.

The semiempirical formula implies three simple relations between total β disintegration energies[2] and nuclear parameters. These relations, one linear and the other two approximately so, are illustrated in Fig. 1 where mass differences between *atoms* (Z,N) and $(Z+1, N-1)$ are plotted as a function of the neutron number of (Z,N) for A odd. Z is as usual the proton number, N the neutron number, and A or $N+Z$ the total number of nucleons in the nucleus. The values used were taken from the table of Metropolis and Reitwiesner.[3] In such a plot mass differences corresponding to β^- emission appear as positive quantities and those corresponding to electron capture or β^+ emission as negative quantities. The energy zero is that for zero kinetic energy of a β^- particle. (Z,A) will be called hereafter the "disintegrating atom" no matter whether the disintegration actually proceeds from (Z,N) to $(Z+1, N-1)$ or *vice versa*.

The most familiar of the three relationships is probably the strictly linear one which follows from the parabolic expression for the mass surface which the semiempirical formula gives for constant A [Eq. (8); Appendix]. If such parabolas actually exist, the disintegration energies for constant values of odd A will give precisely straight lines when plotted against the neutron number, as shown by the dashed lines of Fig. 1.

A second relationship, almost linear, is implied between disintegration energies for which the disintegrating atoms have constant values of $I=N-Z$. Such energies are seen in Fig. 1 to lie on lines which are only slightly convex toward the N axis.

Still a third relation is to be found for decay energies of disintegrating atoms of the same Z. These energies lie along lines of positive slope which are just slightly concave toward the N axis. Other approximately

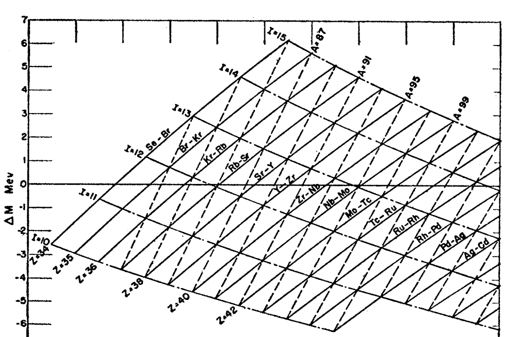

FIG. 1. Odd-A atomic mass differences or total disintegration energies between (Z,N) and $(Z+1, N-1)$ as predicted by the semiempirical mass formula (see reference 1). Values for integral N and Z from the calculations of Metropolis and Reitwiesner (see reference 3) have been connected by three sets of lines: dashed lines for constant A, full lines for constant Z, and long and short dashes for constant $I=N-Z$.

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[1] See Appendix for expression and references.

[2] The term "β disintegration" is used here to cover disintegration by emission of β^- and β^+ particles and by capture of electrons. By "total disintegration energy" is meant the energy released in transitions from ground state to ground state of the *atoms* involved.

[3] N. Metropolis and G. Reitwiesner, *Table of Atomic Masses*, reproduced by Argonne National Laboratory through the courtesy of the authors, March, 1950 (unpublished).

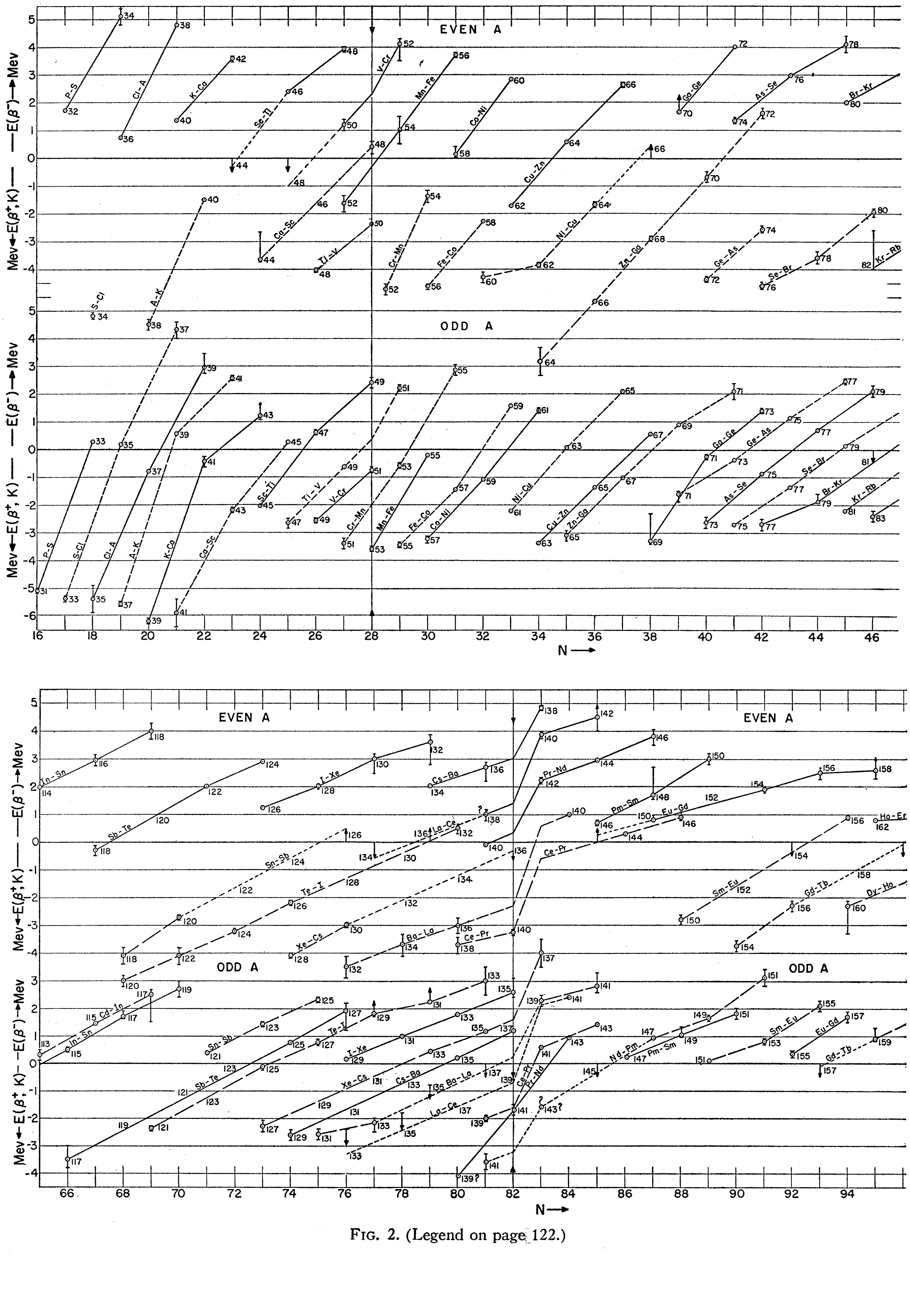
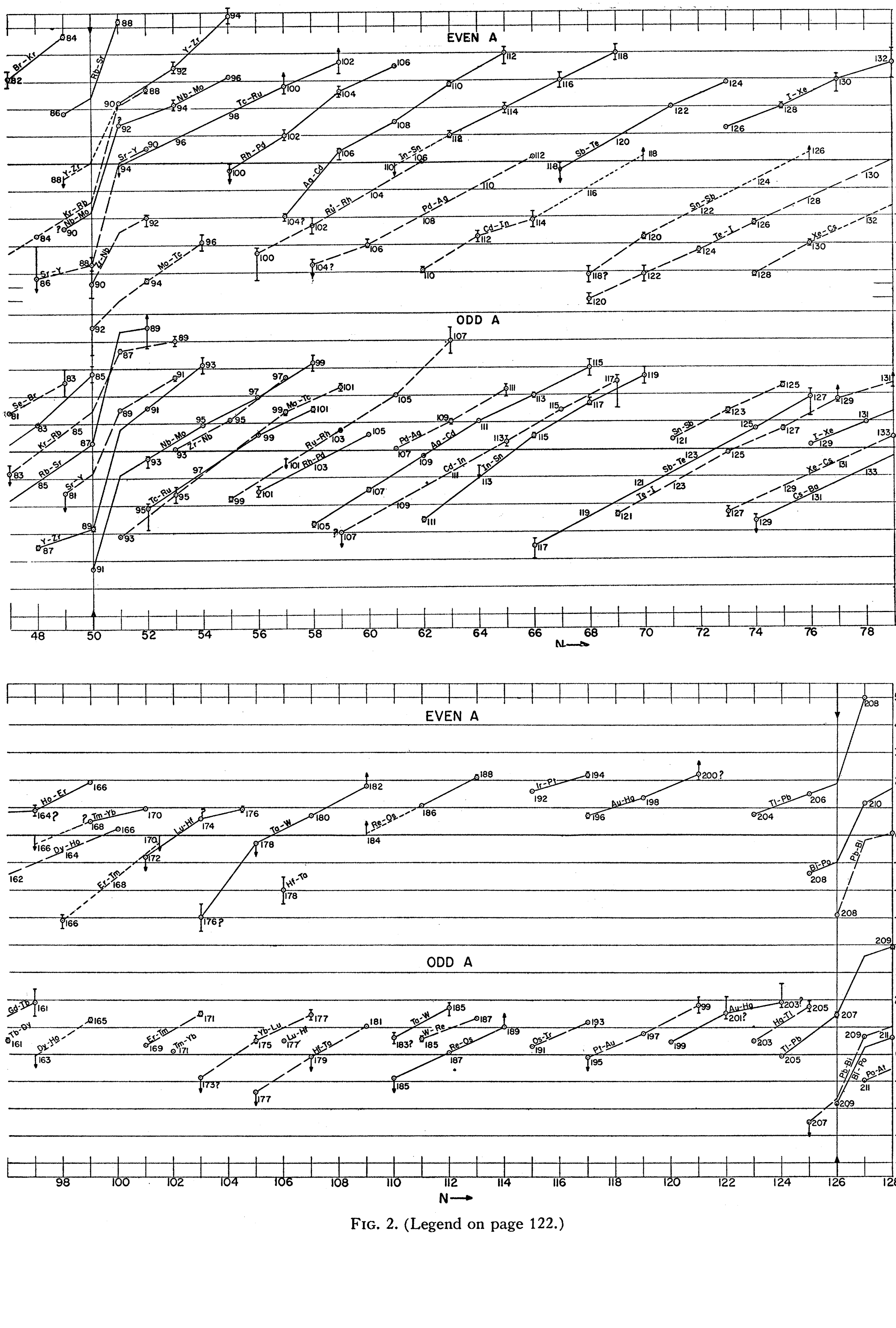


FIG. 2. (Legend on page 122.)



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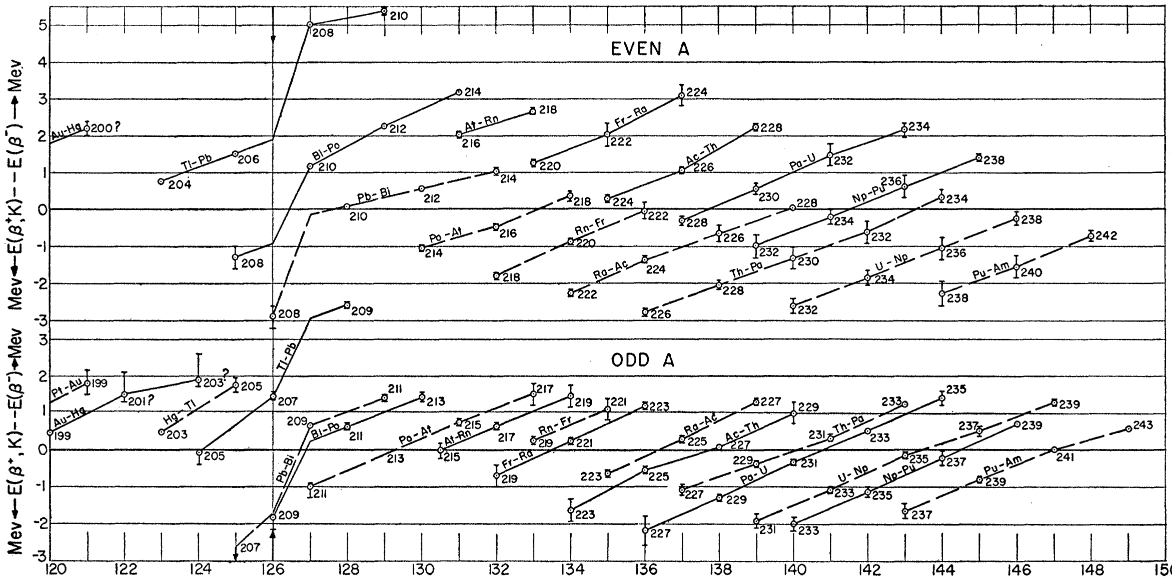

FIG. 2. Observed total disintegration energy, E, for $Z,N \rightleftarrows (Z+1, N-1)$ as a function of neutron number N. Points are connected along constant Z of the "disintegrating" atom. Limits of error are those assigned by the present authors. Φ indicates that it is possible to assign both upper and lower limits. Large asymmetry in the limits usually indicates doubt as to whether or not a single γ should be considered in cascade with the β. The replacement of one bar of Φ by an arrow indicates that it is possible to estimate the limit of error of a point in one direction but not another (e.g., there may be several γ rays whose places in the decay scheme are unknown). If, further, the open circle is deleted and the arrow points downward from the bar, this indicates that it is possible to say only that the energy is less than a certain value. When the arrow starts at the $E=0$ line, it means that there is evidence for the instability of one member of the pair concerned with respect to the other. Question marks near points indicate doubt as to whether a β particle was actually observed. Question marks near mass number show doubt about the mass assignment. Points are joined by line segments, full lines for odd Z and long dashes for even Z. When only an arrow is available, or other reasons for extreme doubt exist, short dashed lines in conformity with the general pattern have been drawn for both odd and even Z.

Note added in proof.—Drafting errors: Ca-Sc^{44} error limits should be reversed; Cr-Mn^{52} abscissa should be $N=28$.

Changed values: Mn-Fe^{52} -1.6 to $\geqslant -2.0$; Ni-Cu^{60} -4.3 to -6.2; Zn-Ga^{71} 2.1 to 2.9; Ag-Cd^{108} 1.5 to 1.8; Cd-In^{112} -2.7 to -2.5; In-Sn^{112} 1.0 to 0.7.

Additional values: Ca-Sc^{47} 2.1; Co-Ni^{56} <-1.8; Ga-Ge^{67} ~ -4.4; Br-Kr^{87} 8.0; Kr-Rb^{85} 0.7; Ru-Rh^{98}? -5.0; Sn-Sb^{116} -4.7; Cs-Ba^{130} 0.4; Ce-Pr^{145} ≥ 2.0; Pr-Nd^{145} 1.7.

linear regularities are implicit in the semiempirical formula as is easily seen in Fig. 1.

It has been known for some time that the experimental disintegration energy data in the magic number regions show large deviations from the first-mentioned regularity, namely that connected with the mass parabolas for constant A. The observations of Glueckauf,[4] Suess,[5] and Kohman[6] that smaller deviations also exist in nonmagic number regions have made it evident that straight lines for decay energies for values of constant A are far from the general rule. However, because of the great usefulness and familiarity of the parabolic mass picture, Coryell[7] has worked out tables of empirical corrections to the parabolic constants of the semiempirical formula. With the help of these corrections approximately correct β-decay energies can be computed in the framework of the parabolic picture.

Suess and Jensen[8] showed in 1951, on the basis of the data then available, that for odd A, except at the magic numbers, a nearly linear relationship existed for decay energies of disintegrating atoms with constant values of I. The linear relationship was especially good between disintegrating atoms with Z even, values for odd Z lying either below or above the even-Z line. These odd-Z deviations were discussed by these authors in an illuminating way in connection with "pairing" energy.

2. DESCRIPTION OF PRESENT SYSTEMATICS

The present authors searched independently for regularities in β-decay energies and noticed that nearly linear relationships are to be found for decay energies of disintegrating atoms with constant Z. (In Fig. 1 these energies are to be found on the slightly concave lines of positive slope.) When all the experimental data were reviewed, it turned out that for constant Z two sets of nearly straight lines can be drawn through the disintegration energies plotted as function of N in the manner of Fig. 1, one set for odd-A values of the disintegrating atom and another set for even-A values. Marked discontinuities occur only at the magic neutron numbers, as might be expected since the lines are functions of only one kind of nucleon.

[4] E. Glueckauf, Proc. Phys. Soc. (London) **61**, 25 (1948).
[5] H. E. Suess, Phys. Rev. **81**, 1071 (1951).
[6] T. P. Kohman, Phys. Rev. **85**, 530 (1952).
[7] C. D. Coryell, Ann. Rev. Nuclear Sci. **2**, 305 (1953).
[8] H. E. Suess and J. H. D. Jensen, Arkiv Fysik **3**, 577 (1952).

This fact gives the present systematics an advantage over the Suess-Jensen type where irregularities can be caused by proton as well as neutron magic numbers and by changes in pairing energy with both Z and N. The Suess-Jensen systematics has, however, the advantage that more experimental points are available for a given line.

Actually, if only one of the three approximately linear regularities mentioned is to hold without the other two, the shell model suggests that the one fos constant Z is the most likely to do so. According to thir model the greatest smoothness in masses is to be expected between shell edges when the number of one kind of nucleon is kept constant and pairs of nucleons of the other kind are added. The near-linearity of the β-decay energies plotted as function of N is evidence for the smoothness of neutron binding energies when the proton number is kept constant. One would also expect the converse to hold and therefore to find a similar linear pattern in disintegration energies if plotted as a function of Z and connected along lines of constant N. However, there are not enough data to make more than a cursory check of this expectation.

The present "best" values[9] of decay energies plotted as function of N and connected along lines of constant Z are presented in Fig. 2. In this figure the results for odd and even A have been separated to avoid crowding. The same abscissa, N, the neutron number of the disintegrating atom, is used in both cases so the two sets of lines are drawn one just above the other, each with a separate set of ordinates. Points for which the disintegrating atoms have odd Z are connected by full lines which will be called odd-Z lines. Even-Z lines are drawn with long dashes. Very short dashes indicate extreme doubt as to the line pattern in either case.

In Fig. 2 limits of error as estimated by the present authors are indicated. Although single straight lines could in most cases be drawn within these limits, it was decided to connect the "best" value points with line segments in order to show clearly the existing state of the data. This treatment emphasizes the possible real deviations from linearity and suggests in some cases subshell effects.

The points in Fig. 2 can also be connected along lines of constant A or constant I of the disintegrating atoms. However, when this is done, less regularity is found than that resulting from connections along constant Z. It is believed, therefore, that interpolation or extrapolation in the pattern of Fig. 2 should give the best possible present estimate of disintegration energies not as yet measured. It is obvious, however, that in many regions there are not yet enough data to indicate the trend clearly. Future results may also indicate breaks or changes of slope which are not now suggested.

It may turn out that all the lines of Fig. 2 will be found to bend down slightly. The experimental data are still too inaccurate for one to say whether or not they exhibit in general a concavity toward the N axis as do the values from the semiempirical formula in Fig. 1. In some cases they definitely seem to do so, but in these cases the results can also be interpreted as a sudden change in the line slope at a particular neutron number, due possibly to the closing of subshells or even to configuration differences within a shell.

Figure 3 shows comparisons between three of the longest lines from Fig. 2 and the corresponding curves of the semiempirical formula. It illustrates the different types of disagreement which exist between the actual data and the semiempirical predictions in nonmagic number regions.

3. STABILITY PREDICTIONS FOR SPECIFIC NUCLEI

In the systematics of Fig. 2 the criterion for β stability is that the same nucleus should appear as daughter product in both a positive and negative disintegration, i.e., as the end product of both a β^- and K capture disintegration. Thus the fact that Te^{125} is stable is shown by a positive disintegration energy at 125 on the Sb–Te line and a negative disintegration energy at 125 on the Te–I line.

Using this criterion and interpolating and extrapolating where necessary, it is easily seen that it is quite improbable that there should be any β-stable Tc or Pm isotopes except for Tc^{97} and Pm^{145}. For both these nuclei the present line pattern suggests instability, but very slight changes in the data would alter the prediction. Evidence for long-lived electron capture in Pm^{145} has recently been found by Butement.[10] At^{215} is another

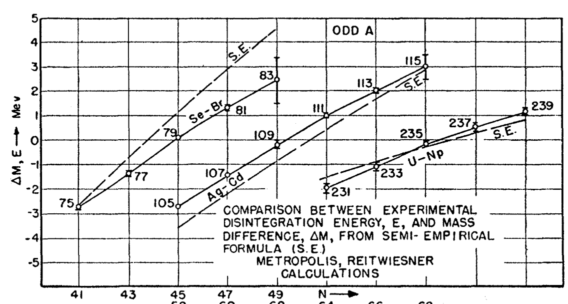

FIG. 3. Observed total disintegration energies (points connected by full lines) compared with values from the semiempirical formula (dashed lines) for three different values of the Z of the "disintegrating" atom.

[9] Values selected were based on data summarized in National Bureau of Standards Circular 499 (U. S. Government Printing Office, Washington, D. C., 1950), its three supplements, and the New Nuclear Data lists of Nuclear Science Abstracts. In the heavy element region a number of values, not directly measured, were estimated with the help of the α systematics of Perlman, Ghiorso, and Seaborg, Phys. Rev. **77**, 26 (1950) and Meinke, Ghiorso, and Seaborg, Phys. Rev. **81**, 782 (1951). A check which these authors were kind enough to make with us showed almost identical interpretations of the data. Considerable help in the construction of many decay schemes was derived from R. W. King's "Beta Decay Schemes and Nuclear Structure Assignments" (unpublished).

[10] F. D. S. Butement, Nature **167**, 400 (1951); M. L. Pool and D. N. Kundu, Phys. Rev. **88**, 171 (1952), assign a 16-day positron activity to Pm^{145} which does not fit the systematics at all.

borderline case but Fr^{219} seems certainly β stable. The short α half-lives of the nuclei which would be the β parents of Fr^{219} undoubtedly have prevented the observation of their β decay. Po^{215} and Po^{218} are clearly β-unstable in agreement with the experimental evidence of Karlik and Bernert,[11] but Po^{216} seems almost certainly β-stable contrary to the observations of these authors.

The evidence from the decay scheme[12] of the 113 chain that Cd^{113} is unstable with respect to In^{113} is so convincing that a positive point was put on the Cd−In line at 113. However, even without this datum the line pattern strongly suggests that Cd^{113} is unstable with respect to In^{113} by 250–350 kev.

Another interesting case is that of Sb^{123}. According to the present pattern, this nucleus is stable with respect to Te^{123}. There should be K capture from Te^{123} to Sb^{123} with a disintegration energy of 250–350 kev. The spin change between the ground states is known to be 3 and according to the shell model there is no change of parity. If the half-life is $>2\times10^{14}$ years[13] the lower limit of log ft is $\sim$21. This value seems large in comparison with the Rb^{87} log ft of 17.6[14] which is presumably for a $\Delta I=3$, yes transition, but the effect of the appropriate forbidden correction factors to f_0 may reduce the difference. A good number for the disintegration energy of Sb^{127} would help to make the systematics prediction for the $Sb^{123}-Te^{123}$ disintegration energy more precise.

4. CONNECTION OF SYSTEMATICS WITH NEUTRON AND PROTON BINDING ENERGIES

Consideration of the relation of neutron and proton binding energies to the slopes and separations of the lines of Fig. 2 gives insight into the implications of the present systematics. Let $B_n{}^{ij}$, $B_p{}^{ij}$ represent the neutron or proton binding energy *to* a nucleus with $Z+i$ protons and $N+j$ neutrons and E^{ij} the β-decay energy between $(Z+i, N+j+1)$ and $(Z+i+1, N+j)$ as indicated in Fig. 4. Then as is seen from this figure, the slope, $K(Z)$, of one of the constant Z lines of Fig. 2 is equal to $(E^{02}-E^{00})/2$.

In terms of the binding energies:

$$\left.\begin{aligned}2K(Z)=E^{02}-E^{00}&=(B_p{}^{02}-B_p{}^{00})-(B_n{}^{02}-B_n{}^{00})\\&\cong[2\partial B_p/\partial N]_z-[2\partial B_n/\partial N]_z\end{aligned}\right\}, \quad (1a)$$

$$\left.\begin{aligned}&=(B_n{}^{10}+B_n{}^{11})-(B_n{}^{01}+B_n{}^{02})\\&\cong[\partial B_{2n}/\partial Z]_N-[2\partial B_n/\partial N]_z\end{aligned}\right\}. \quad (1b)$$

[11] B. Karlik and T. Bernert, Naturwiss. **31**, 298 (1943); Z. Physik **123**, 51 (1944); Naturwiss. **32**, 44 (1944).

[12] Apparently there is no observable γ ray from the 3.5-year isomeric state to the ground state of Cd^{113} in agreement with the original report of Gum, Thompson, and Pool, Phys. Rev. **76**, 184 (1949). The γ transition should be an $E5$. According to the empirical $E5$ energy-lifetime relationship of M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 906 (1951), if the partial half-life is greater than 10^{10} sec, the energy must be less than 300 kev.

[13] L. I. Rousinow and J. M. Igelnitzky, Compt. rend. acad. sci. U.R.S.S. **49**, 343 (1945).

[14] Curran, Dixon, and Wilson, Phys. Rev. **84**, 151 (1951).

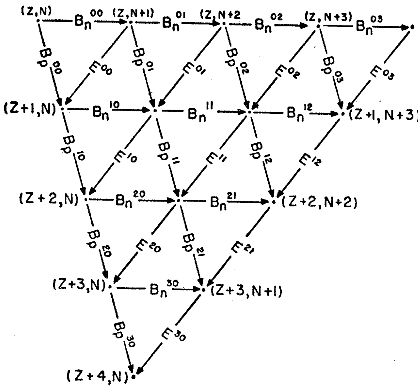

FIG. 4. Schematic diagram showing relations between total beta disintegration energies E, neutron binding energies B_n, and proton binding energies B_p. Any closed loop is an "energy cycle" for which the sum of the vector quantities around the path must vanish if the neutron-proton mass difference is subtracted from E. Examples:

Loop (Z,N), $(Z, N+1)$, $(Z+1, N)$, (Z,N) requires:

$$B_n{}^{00}+E^{00}-B_p{}^{00}-(n\text{-}p)=0;$$

Loop $(Z, N+1)$, $(Z, N+2)$, $(Z+1, N+1)$, $(Z+1, N)$, $(Z, N+1)$ requires:

$$B_n{}^{01}+E^{01}-B_n{}^{10}-E^{00}=0.$$

According to Eq. (1a) there will be a sudden increase in line slope if either the binding energy of the last proton is markedly increased by the addition of two neutrons or the binding energy of the last neutron is markedly decreased by such addition. Such marked changes are found in Fig. 2 at the magic neutron numbers for all values of Z and so are presumably due predominantly to the latter effect. Equation (1b) expresses the slope in an alternative way in terms of changes in B_n only.

The "separation" of two consecutive lines in Fig. 2, e.g., As−Se and Se−Br (such lines are adjacent on the odd-A plot but not on that for even A), will be defined as $E^{00}-E^{11}$ in the notation of Fig. 4 and denoted by $D(Z+1)$ where $Z+1$ is the charge of the atom common to both lines, i.e., Se in the example above. Thus on the odd-A plot the "separation" between the As−Se and Se−Br lines is equal to the difference between the β-disintegration energy of As^{77} and that of Se^{79} or 0.5 Mev, and is designated as $D(34)$. Obviously a slightly different value for the separation results if another pair of points, such as the disintegration energies of As^{79} and Se^{81}, is chosen for its evaluation. However, in spite of this ambiguity it turns out that the definition is adequate for the discussion of general trends.

The advantage of the definition is that it makes it possible immediately to relate the line pattern of Fig. 2 to neutron and proton binding energies. In the notation of Fig. 4,

$$D(Z+1)=E^{00}-E^{11}=(B_n{}^{11}-B_n{}^{10})-(B_p{}^{11}-B_p{}^{01}). \quad (2)$$

Clearly the magnitude and even the sign of D depend upon which of the two neutrons and protons is odd and which even. The four different possible cases are easily distinguished with the help of subscripts e and o to denote even and odd particles, respectively, and primes to indicate that the even particle of one of the pairs designated in Eq. (2) has entered the nucelus before the odd one and so is not its "partner." They are:

for odd-A lines:

$$D_1 \equiv E[(Z)_e, (N)_o] - E[(Z+1)_o, (N+1)_e] = (B_{ne}-B_{no}) - (B_{pe}-B_{po}),$$

$$D_3 \equiv E[(Z)_o, (N)_e] - E[(Z+1)_e, (N+1)_o] = (B_{pe}'-B_{po}) - (B_{ne}'-B_{no}); \quad (2a)$$

for even-A lines:

$$D_2 \equiv E[(Z)_e, (N)_e] - E[(Z+1)_o, (N+1)_o] = -(B_{ne}'-B_{no}) - (B_{pe}-B_{po}),$$

$$D_4 \equiv E[(Z)_o, (N)_o] - E[(Z+1)_e, (N+1)_e] = (B_{ne}-B_{no}) + (B_{pe}'-B_{po}). \quad (2b)$$

$B_{ne}-B_{no}$ and $B_{pe}-B_{po}$ will hereafter be called briefly the even-odd neutron and proton differences. A study of the line separations, or D's, then gives information about the relative magnitudes of these differences.

Values of the line separation, D, as found from differences between the most reliable points near the zero-energy line in Fig. 2 are shown in Fig. 5. Care was taken not to use values on opposite sides of magic neutron numbers, so "direct" effects of such numbers are eliminated. "Direct" effects of proton magic numbers, that is, effects due to sudden changes in the general magnitude of the proton binding energies would, however, be expected to show up in D_4 and D_3. These D's involve proton pairs which are not "partners" and thus particles which can be on either side of a magic number. Marked peaks in the plots for these D's are indeed observed at $Z=50$ and $Z=82$ while a small peak is to be seen at $Z=28$ on D_4 but not on D_3.

Discontinuities in the D's are found in a number of places. The most striking ones can be interpreted consistently by assuming that there is a decrease in the even-odd difference for neutrons (or protons) after a neutron (or proton) magic number (with 28 considered as magic). Just such a decrease is to be expected from the calculations of M. G. Mayer[15] on the interaction energy of a number of identical nucleons in the same shell, assuming jj coupling and short-range attractive forces. These calculations give:

$$E(\text{even}) - E(\text{odd}) = C(2j+1)/A, \quad (3)$$

where C depends on the strength of the interaction. This difference is not expected to be identical with the even-odd binding energy difference since the calculation took no account of interaction with unlike nucleons in the same shell or with the whole nuclear "core." Nevertheless, from (3) one would expect decreases in the even-odd differences with decreases in j.

[15] M. G. Mayer, Phys. Rev. **78**, 22 (1950).

It seems odd at first that such decreases occur chiefly at the magic numbers where according to the shell model the decrease in j is not so large as in other regions. The explanation may be in the suggestion made by Mayer that in regions where high- and low-j levels are close together an odd nucleon may go into the low-j state while a pair enters the high-j state because of the higher pairing energy predicted by (3). A further note on the data on pairing is in preparation.

The difference expressed in Eq. (3) should be more nearly equal to the quantities 2ν and 2π, now generally called the neutron and proton pairing energies, respectively. Following Coryell,[7] we define ν and π through the mass expressions:

$$M(Z_e, N_o) = f(Z,A) + \nu, \quad M(Z_o, N_e) = f(Z,A) + \pi, \quad (4)$$

where $f(Z,A)$ is the function of Eq. (5) in the Appendix. If $\nu = \pi$, there is a single mass parabola for a given odd A. This is also the condition for approximately equal line separations on the odd-A plot of Fig. 2. If the separation between an even-Z line and the following odd-$(Z+1)$ line is greater than that between the odd-$(Z+1)$ line and the next even-$(Z+2)$ one, ν is greater than π. If the reverse is true, π is greater than ν. Coryell[7] has used changes in line spacing to estimate values of $\pi - \nu$.

The line spacing pattern gives clear-cut results about the relative values of ν and π. However, no simple relationship exists between the π, ν picture and the even-odd difference picture as can be seen at once from

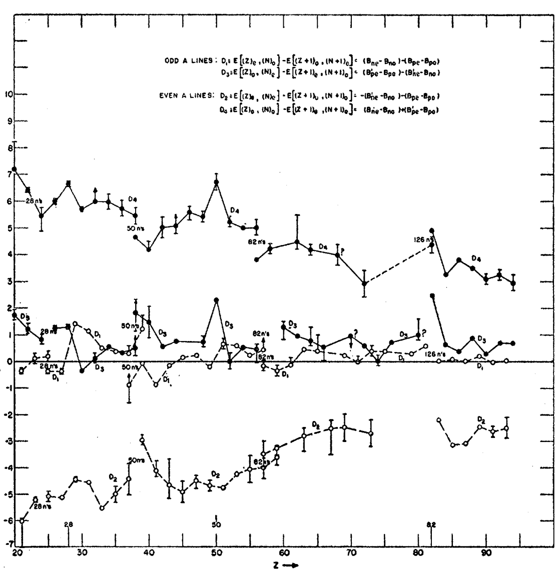

FIG. 5. Line separations, D, as defined by Eqs. (2a) and (2b), evaluated from best points of Fig. 2 near $E=0$. Example:
$$D_1(Z=29) = E(\text{Ni}^{63}) - E(\text{Cu}^{65}) = 0.07 + 1.34 = 1.41 = [B(36\text{th}n) - B(35\text{th}n)]_{Z=29} - [B(30\text{th}p) - B(29\text{th}p)]_{N=35}.$$
Odd—A line separations: D_1 lower middle, D_3 upper middle. Even—A line separations: D_2 bottom, D_4 top curve.

Eqs. (2a) and (2b). If $D_1=D_3$, $\nu=\pi$, but the corresponding relation between the binding energies is quite complicated.

5. CONCLUSIONS ABOUT NUCLEAR STRUCTURE

1. *There are sudden large decreases in neutron binding energies just after neutron numbers 50, 82, 126, and probably 28.* The sharp discontinuities in the disintegration lines of Fig. 2 could be attributed either to changes in neutron or proton binding energies according to Eq. (1). However, for several Z values direct evidence for decreases in neutron binding energies after magic neutron numbers 28, 50, and 126 has already been found by means of Q values for (d,p),[16] (γ,n),[17] and (n,γ)[18] processes. Information has been lacking at $N=82$. The systematics of Fig. 2 indicates that such discontinuities exist for all appropriate proton numbers at $N=50$, 82, and 126. At $N=28$ sudden jumps occur on the V−Cr even-A line and the Ti−V odd-A line and apparently not on the Mn−Fe even-A and the Cr−Mn odd-A lines. However, the data are still rather meager in this region.

TABLE I. Magnitudes in Mev of discontinuities at magic neutron numbers.[a]

Disintegration line	Odd N ΔB_n	Even N $\Delta B_n+\Delta B_p$
	27, 29	*28, 30*
Ti−V	−1.0	
Cr−Mn	−0.0?	
Mn−Fe	−0.9	
	49, 51	*50, 52*
Kr−Rb	−1.5	∼−2.9
Rb−Sr	∼−2.3	−2.9
Sr−Y	−1.7	∼−3.8
Y−Zr		−3.7
Zr−Nb		
Nb−Mo		∼−3.5
	81, 83	*82, 84*
Xe−Cs	−2.1	
Cs−Ba	−1.3	
Ba−La	∼−2.2	∼−2.1
La−Ce	−1.8	
Ce−Pr	∼−1.8	
Pr−Nd	−1.4	
	125, 127	*126, 128*
Tl−Pb	−2.8	∼−1.0
Pb−Bi	−1.9	∼−2.4
Bi−Po	∼−1.8	

[a] Values tabulated were found by extrapolating the lines of Fig. 2 from the low sides of the magic neutron numbers, N_m, to abscissa values of (N_m+1) and (N_m+2) in the respective cases of odd and even neutrons in the disintegrating nuclei and then subtracting the extrapolated values from the experimental ones. For odd N this gives the abnormal change, ΔB_n, apart from the regular decrease, in binding of the (N_m+1) neutron over the (N_m-1) neutron, for even N the abnormal change in binding of the (N_m+2) neutron over the N_m neutron plus the abnormal change, ΔB_p, in the binding of the last proton when a neutron has been added to the new shell. Decreases in the even-odd neutron differences on crossing shell edges make ΔB_n larger for even than for odd N. The magnitudes of such decreases are not sufficiently well known to allow estimates of ΔB_p.

[16] J. A. Harvey, Phys. Rev. **81**, 353 (1951); G. F. Pieper, Phys. Rev. **88**, 1299 (1952).

[17] Sher, Halpern, and Mann, Phys. Rev. **84**, 387 (1951). In addition to their own results, these authors list the results of earlier experiments with references.

[18] Kinsey, Bartholomew, and Walker, Phys. Rev. **82**, 380 (1951).

Estimates of the magnitudes of the neutron binding energy changes implied by the present systematics are given in Table I. They are of the same general magnitude as those found in other ways. No effect similar to that seen for the above-mentioned neutron numbers is found at $N=20$.

2. *There are sudden large decreases in proton binding energies just after proton numbers 50, 82, and possibly 28.* The evidence for these discontinuities is from the separations of the lines of Fig. 2 which are plotted in Fig. 5. As mentioned above, the sharp peaks in D_3 and D_4 for $Z=50$ and $Z=82$ are to be attributed to sudden decreases in the binding of the 51st and 83rd protons. At $Z=28$ there is a small peak on D_4 but not D_3. There are not sufficient data to draw conclusions about effects at $Z=20$.

One would expect the heights above background of the D_3 and D_4 peaks at the proton magic numbers to be approximately equal to the values of the discontinuities in the proton binding energies at these points. These values, ∼1.5 Mev at $Z=50$ and $Z=82$ are in agreement with what is known directly about these discontinuities from actual proton binding energies. In the Pb region, which is that of 82 protons and 126 neutrons, a number of binding energies in addition to those directly measured can easily be found with the help of α- and β-decay energies so that here the picture of both neutron and proton binding energies is quite clear.[19]

3. *Very little, if any, effects attributable to sudden binding energy changes at possible "submagic" numbers such as 40, 58, 64 are evident at present.* Figure 2 shows possible breaks in some but not all of the disintegration energy lines in the neighborhood of $N=58$ and $N=64$. However, as the disintegration energy data become more accurate they may prove very helpful in showing how neutron shells are filled for different values of Z.

4. *The difference between the binding energies of neighboring even and odd like nucleons decreases after a magic number.* Figure 5 shows graphs of the D's on which this statement is based. In this plot one sees, for example, that after $N=50$, D_1 and D_4 suddenly decrease while D_2 and D_3 take sudden jumps upward. These changes can be accounted for only if it is assumed that the even-odd neutron difference after $N=50$ is smaller than before $N=50$ by about 1 Mev.

At $Z=50$, peaks in D_3 and D_4 are observed due to the proton magic number. It is seen that in addition both D_3 and D_4 are lower after $Z=50$ than before, while D_1 and D_2 are slightly higher, a phenomenon which can be accounted for only by a drop of about 0.5 Mev in the even-odd proton difference after the magic proton number. Similar and even greater changes in the D's are observed in the region of $Z=28$ although this

[19] N. Feather, Phil. Mag. **2**, 141 (1953); A. H. Wapstra, thesis, Amsterdam, 1953 (unpublished); K. Way and M. Wood, Phys. Rev. **86**, 608 (1952).

proton number is not shown up very clearly as magic by means of a peak.

5. *The slopes of the β disintegration energy lines of Fig. 2 do not agree with those given by the semiempirical formula.* Figure 6 shows a comparison of the observed slopes and those calculated from the semiempirical formula values of Metropolis and Reitwiesner. The observed slopes were taken from the best (by eye) straight lines given by points near $E=0$ and the semiempirical formula slopes were evaluated at the bottom of the potential valley. The observed slopes are greater than these semiempirical slopes for the very heavy elements ($Z>70$) and markedly less for elements in the regions $Z=20$, $Z=34$, and $Z=54$.

6. *N_Z, the value of N for which a given β disintegration energy line crosses the axis, shows large positive and negative deviations from the N_Z given by the semiempirical formula.* This value of N, in general a nonintegral one, is that for which $M(Z,N_Z)$ is just equal to $M(Z+1, N_Z-1)$. Figure 7 compares the experimental values of

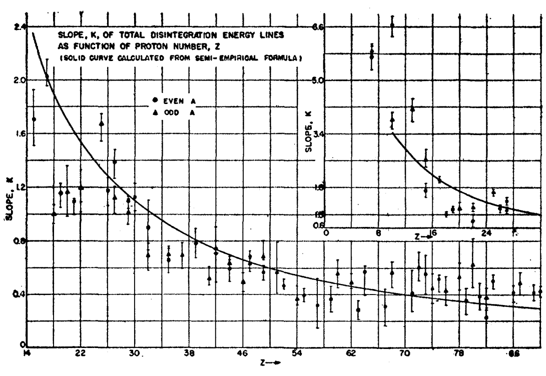

FIG. 6. Comparison of slopes of disintegration energy lines of Fig. 2 in the vicinity of $E=0$ with slopes of analogous lines as given by the semiempirical formula.

N_Z with those calculated from the semiempirical formula. The experimental values jump from one side of the calculated curve to the other showing that the semiempirical formula gives only the general location of the bottom of the valley. Coryell, Brightsen, and Pappas[20] have noted a similar discontinuous behavior for Z_A, the most stable charge for a given A [see Eq. (8)].

7. *The energy release to be expected in double β decay shows even greater regularity than that of single β decay as shown by Fig. 8.* Here odd-A and even-A values lie approximately on the same lines. The condition that they do so for a line such as Zr−Mo is that the even-odd neutron difference in Zr should be equal to the even-odd neutron difference in Mo. Such an equality can exist but is not general as implied by Fig. 8. Nevertheless the over-all simplicity of the double β-decay energy pattern may point the way to a convenient empirical mass formula. The double β-decay regularity was first pointed out by H. E. Duckworth.

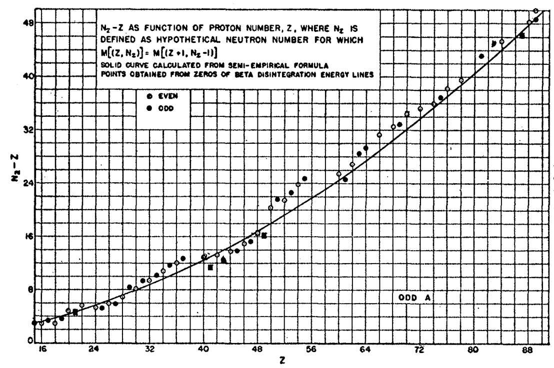

FIG. 7. Comparison of observed values of N_Z-Z with those found from semiempirical formula. N_Z is the generally nonintegral value of N for which $M(Z,N_Z)=M(Z+1, N_Z-1)$.

The authors are indebted to Gladys H. Fuller for assistance in the comparison with the semiempirical formula. They have profited greatly from discussions with Professor Charles D. Coryell and Dr. C. L. McGinnis.

APPENDIX

A simplified form of the original Weizsäcker[21] semiempirical mass formula, proposed by Bethe and Bacher[22] in 1936, has been in general use since that time. It is:

$$M(Z,N)=NM_n+ZM_p-k_v(N+Z)+k_s(N+Z)^{\frac{2}{3}} +[k_p(N-Z)^2/(N+Z)]+[k_e Z^2/(N+Z)^{\frac{1}{3}}]. \quad (5)$$

Metropolis and Reitwiesner[3] have expressed $M(Z,N)$ in the following form:

$$M(Z,A)=1.01464A+0.014A^{\frac{2}{3}}-0.041905Z_A +0.041905(Z-Z_A)^2/Z_A+0.036\lambda A^{-\frac{1}{3}}, \quad (6)$$

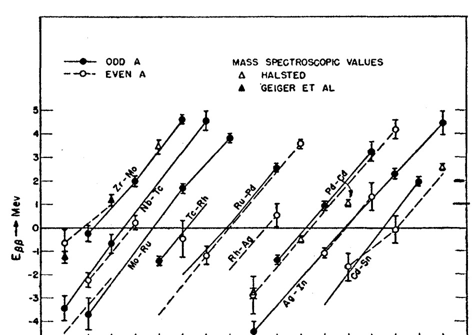

FIG. 8. Energies of two successive disintegrations, or energy differences in double β decay, plotted in the manner of Fig. 2. Points shown are from sums of appropriate points on Fig. 2. Where no points are indicated, values have been read from lines on Fig. 2. Mass spectroscopic values are from Geiger, Hogg, Duckworth, and Dewdney, Phys. Rev. **89**, 621 (1953) and R. E. Halsted, Phys. Rev. **86**, 408 (1952).

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[20] Coryell, Brightsen, and Pappas, Phys. Rev. **85**, 732 (1952).

[21] C. von Weizsäcker, Z. Physik **96**, 431 (1935).

[22] H. A. Bethe and R. F. Bacher, Revs. Modern Phys. **8**, 165 (1936).

where $Z_A=A/(1.980670+0.0149624A^{2/3})$ and $\lambda=+1$ for A even, Z odd; $\lambda=-1$ for A even, Z even; and $\lambda=0$ for A odd. Equation (6) is equivalent to Eq. (5) with the addition of the pairing term, $0.036\lambda A^{-3/4}$, and with the following values of the k's in mass units:

$$k_v=0.01507, \qquad k_p=0.02075,$$
$$k_s=0.014, \qquad k_e=0.000627.$$

In considering the predictions of the semiempirical formula for neutron and proton binding energies the following derivatives are convenient:

$$\partial^2M/\partial N^2=(2/9)k_sA^{-4/3}-8k_pZ^2A^{-3}-(4/9)k_eA^{-7/3},$$
$$\partial^2M/\partial Z^2=(2/9)k_sA^{-4/3}-8k_pN^2A^{-3}-\tfrac{2}{3}k_e[3A^{-1/3}-2ZA^{-4/3}+\tfrac{2}{3}Z^2A^{-7/3}], \quad (7)$$
$$\partial^2M/\partial N\partial Z=(2/9)k_sA^{-4/3}+8k_pNZA^{-3}+\tfrac{2}{3}k_e[ZA^{-4/3}-\tfrac{2}{3}Z^2A^{-7/3}].$$

Bohr and Wheeler[23] expressed the semiempirical formula in the form:

$$M(Z,A)=A(1+f_A)+\tfrac{1}{2}B_A(Z-Z_A)^2+\lambda\delta_A, \quad (8)$$

where f_A is the packing fraction at the bottom of the mass valley, Z_A is a generally nonintegral value of Z for the "most stable" nucleus of given A, B_A is analogous to k_p/A and $\lambda\delta_A$ is again a pairing term. Empirical values for B_A, Z_A, and δ_A have been given by Bohr and Wheeler,[23] Coryell,[7] Suess,[8] Kohman,[6] and others.

[23] N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).

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The Ranges of Fragments from High-Energy Fission of Uranium*

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The ranges in aluminum of several fragments from the fission of U^{238} induced by 18-Mev deuterons and by 335-Mev protons have been measured by a radiochemical method. The ranges found are of the same order of magnitude as those reported for slow-neutron-induced fission. The difference in the forward and backward recoil ranges in the deuteron (18-Mev) case is consistent with the momentum corresponding to compound nucleus formation. The ranges found in the proton (high-energy) case are shorter than those of the deuteron case, the differences being greater for the lighter fragments. These differences are explained by the change in mass of the complementary fragments due to evaporation of neutrons prior to fission in the proton case, which causes the observed fragment to receive a smaller fraction of the total kinetic energy.

I. INTRODUCTION

THE ranges associated with the fission fragments from slow-neutron fission have been studied by a number of experimenters. This work is adequately reviewed by Katcoff, Miskel, and Stanley,[1] who themselves have made an extensive study of the ranges in air of the fragments from plutonium fission. The characteristics of their range study have been found to be consistent with the kinetic-energy distribution of the fission fragments as determined by ionization-chamber measurements.[2] In the field of high-energy fission, Jungerman and Wright[3] have studied the kinetic energies of fission fragments produced by 45-Mev and 90-Mev neutrons. No range measurements for high-energy fission have been reported. Such range measurements were undertaken by a radiochemical method because it makes possible the independent study of fragments of various identities. Because the production of sufficient radioactivity was a problem, the experiments were restricted to fission induced by charged particles. Originally 18-Mev deuterons were used, because of the high beam currents available. An improved technique made possible experiments with 335-Mev protons.

II. EXPERIMENTAL METHOD

Fission was induced in a thin uranium source by 18-Mev deuterons and 335-Mev protons in the 60-in. and 184-in. Berkeley cyclotrons. Adjacent to this source during the irradiation was a stack of aluminum foils. After the irradiation, the relative amount of a certain radioactive fission product in each foil was determined by radiochemical methods. From these data and the known thicknesses of the foils, the mean range of that particular kind of fission fragment was calculated.

The geometrical arrangement is indicated in Fig. 1. If the fission recoils are isotorpic, half of them will enter

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[1] Katcoff, Miskel, and Stanley, Phys. Rev. 74, 631 (1948).
[2] D. C. Bruton and W. B. Thompson, Phys. Rev. 76, 848 (1949); Can. J. Research A28, 498 (1950).
[3] J. Jungerman and S. C. Wright, Phys. Rev. 76, 1112 (1949).