PHYSICAL REVIEW VOLUME 94, NUMBER 5 JUNE 1, 1954

Energy and Temperature Dependence of the Secondary Emission of MgO†

A. J. Dekker
Department of Electrical Engineering, University of Minnesota, Minneapolis, Minnesota

(Received January 18, 1954)

It is shown that the data on the energy and temperature dependence of the secondary yield of magnesium-oxide single crystals, recently published by Johnson and McKay, can be understood on the basis of a simple model. In fact, quantitative agreement is obtained for the temperature coefficient of the yield. The importance of measurements far away from either side of the maximum of the yield curve is stressed.

1. INTRODUCTION

A NUMBER of accurate measurements on the secondary emission coefficient of magnesium-oxide single crystals has recently been published by Johnson and McKay[1] in his journal. Their measurements of the secondary yield δ extend over a primary range up to 5 kev and over a temperature range between room temperature and 740°C. In view of the fact that the yield curves have been measured far beyond the maximum, it seems of interest to attempt to explain these observations on the basis of a simple model and, as we shall see below, this is indeed possible. The reasons why in secondary emission the regions far away on either side from the maximum are of particular interest in setting up suitable models for the secondary emission process will be made clear below.

The fact that the specimens used by Johnson and McKay showed variations in $\delta_{\max}$ between about 5.5 and 7.1 is probably of no consequence for the considerations given in the present paper. Johnson and McKay ascribe these variations to possible changes in the physical structure or in the work function resulting from the treatment after cleavage. Another possible factor may be sought in variations in the concentration of excess oxygen in the crystals.[2] Whatever be the cause, experience in this laboratory indicates that vacuum heating of MgO to a temperature of 740°C over periods required for the δ measurements does not alter its physical properties. It thus seems safe to assume that the values of δ given by Johnson and McKay for a given crystal actually pertain to a crystal with a fixed set of physical properties. From the measured points published in Fig. 5 of the paper referred to above, the following table may be set up for one of their samples in the high primary energy region:

TABLE I. Values of the secondary emission coefficient of MgO as function of the primary energy E_{p0} at room temperature and at 740°C. The data are taken from Fig. 5 in the paper by Johnson and McKay and only refer to the region beyond the maximum of the yield curve.

E_{p0} (kev)	$\delta_{740°\mathrm{C}}$	$\delta_{20°\mathrm{C}}$	δ_{740}/δ_{20}	$(E_{p0}\delta)_{20°\mathrm{C}}$ (kev)
5.0	2.18	2.74	0.79	13.7
4.5	2.18	2.86	0.76	12.9
4.0	2.74	3.64	0.75	14.6
3.5	2.89	3.69	0.78	13.0
3.0	3.19	4.69	0.68	14.1
2.5	3.81	4.92	0.77	12.4
2.0	4.61	5.55	0.83	11.1
1.75	5.15	6.15	0.84	10.8

E_{p0} represents the energy of the bombarding primaries. We note that for the sample under consideration the maximum yield at room temperature is 7.0, occurring for $E_{p0}\simeq 1100$ ev; at 740°C the maximum yield of 6.2 is obtained for $E_{p0}\simeq 900$ ev. From the table the following conclusions may be drawn:

(a) For primary energies $\gtrsim 2$ kev the ratio δ_{740}/δ_{20} is practically constant; the average value is 0.79 ± 0.04 if we exclude the perhaps doubtful value at $E_{p0}=3.0$ kev.

(b) For primary energies $\gtrsim 3$ kev the secondary yield varies approximately inversely proportional to E_{p0}.

Moreover, their measurements show that for primary energies below 500 ev the temperature effect is negligible, whereas the value of E_{p0} at which the yield is a maximum shifts to lower values at higher temperatures. For a qualitative explanation of the temperature effect we refer to the paper of Johnson and McKay.

2. THE LOW PRIMARY ENERGY REGION

Before discussing the conclusions drawn from the table a few remarks about the low primary energy region may be in order. Denoting the number of secondary electrons produced by a primary at a depth between x and $x+dx$ below the surface by $n(x)dx$ and the probability of escape by $f(x)$, the secondary yield may be written as

$$\delta=\int_0^\infty n(x)f(x)dx. \tag{1}$$

If the penetration depth x_p of the primaries is very small compared with the range x_s of the secondaries, the function $f(x)$ is approximately a constant and (1) reduces to

$$\delta\simeq nf(0), \quad x_p \ll x_s, \tag{2}$$

where n is the total number of secondaries produced per incident primary electron. Now, for primary energies larger than several times the energy required to produce a secondary, it seems reasonable to assume that the

---

† Work supported by the U. S. Army Signal Corps.

[1] J. B. Johnson and K. G. McKay, Phys. Rev. 91, 582 (1953).

[2] K. C. Nomura, Phys. Rev. 89, 894 (1953).

total number of secondaries produced is proportional to E_{p0}. For such primary energies then (as long as $x_p \ll x_s$), the yield should according to (2) be proportional to E_{p0}. For MgO this seems to be approximately the case up to $E_{p0} \simeq 400$ ev, according to the results of Johnson and McKay. The importance of yield measurements in the low primary energy region is primarily that they may be used to study possible changes in the electron affinity of a crystal, because the latter determines in part the escape probability. For example, at present it is not known whether a stoichiometric excess of oxygen in the MgO crystals as a whole is correlated with a change in electron affinity.

A tentative indication about the electron affinity of MgO may be inferred from the first crossover point ($\delta = 1$) which for the sample under consideration occurs at a primary energy of 32.8 ev. Primaries of this energy may be expected to produce about two secondaries, indicating that $f(0) \simeq 0.5$. Now, the probability of escape of an electron at the surface with an energy ϵ in the conduction band is $0.5[1-(X/\epsilon)^{\frac{1}{2}}]$, where X is the electron affinity. Although this formula cannot be applied immediately to the case under consideration, the implication seems to be that $X \ll \epsilon$, i.e., X is probably only a fraction of an ev.

3. THE HIGH PRIMARY ENERGY RANGE

Beyond the maximum in the yield curve, the range of the secondaries x_s is smaller than the penetration depth of the primaries. It will be assumed that, at any rate over the secondary range, the primaries move essentially along straight lines. Also, it will be assumed that the number of secondaries produced per unit depth is proportional to the energy loss per unit length of the primaries. As is well known, the primaries lose energy in accordance with the equation[3]

$$-\frac{dE_p}{dx} = \text{const}\frac{1}{E_p}\log\frac{2E_p}{\epsilon}, \qquad (2)$$

where ϵ represents an excitation energy. Over a limited range of E_p this expression may be approximated fairly well by Whiddington's law:

$$-\frac{dE_p}{dx} = \frac{A}{E_p}, \quad \text{or} \quad E_p(x) = (E_{p0}^2 - 2Ax)^{\frac{1}{2}}. \qquad (3)$$

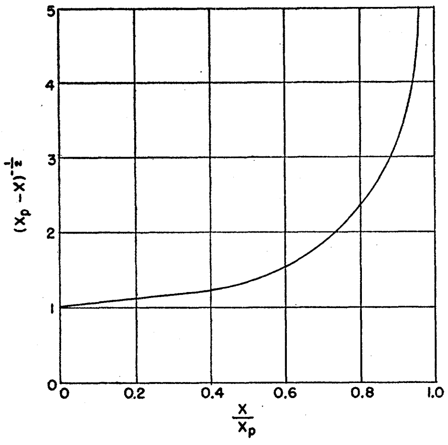

FIG. 1. The function $(x_p - x)^{-\frac{1}{2}}$ *versus* x/x_p which determines the production of secondaries as function of depth for a Whiddington law.

For the yield at large values of E_{p0} one is in fact only interested in the production of secondaries over a depth range over which the primaries have lost only a fraction of their total energy, because the function $f(x)$ decreases rapidly beyond a certain value. From (3) it follows that

$$n(x) = -\frac{1}{\epsilon_e}\frac{dE_p(x)}{dx} = \left(\frac{A}{2}\right)^{\frac{1}{2}} \frac{1}{\epsilon_e (x_p - x)^{\frac{1}{2}}}, \qquad (4)$$

where ϵ_e is the average energy required to produce a secondary and $x_p = E_{p0}^2/2A$. Thus, as long as the range of the secondaries is small compared with x_p, the production of secondaries is practically constant over this range and equal to

$$n(x) \simeq A/(\epsilon_e E_{p0}), \qquad (5)$$

as may be seen from Fig. 1. For a primary energy of 3 kev, for example, $x_s/x_p \simeq 0.1$ in MgO; and for 5 kev, $x_s/x_p \simeq 0.04$. From (1) and (5) we thus obtain for the yield at high primary energies,

$$\delta = \frac{A}{\epsilon_e E_{p0}} f, \quad \text{with} \quad f = \int_0^{x_p} f(x)dx \simeq \int_0^{\infty} f(x)dx. \qquad (6)$$

The integral f is determined by the crystal under consideration and depends on its temperature, electron affinity, density of electron traps, etc. However, if these quantities are fixed, the product $E_{p0}\delta$ should be constant according to (6), in agreement with the findings of Johnson and McKay. It also shows the importance of yield measurements for primary energies far beyond the maximum, because such measurements provide direct information about the integrated probability of escape. One example is the influence of temperature on the secondary yield for high primary energies, discussed below. In a similar fashion, the influence of electron traps may be studied and models for the escape mechanism may be checked. Unfortunately, most yield measurements have been carried out too close to the maximum yield to allow a quantitative interpretation because in such cases one is not permitted to employ the simplifying assumption made here.

4. THE TEMPERATURE DEPENDENCE OF THE YIELD AT HIGH PRIMARY ENERGIES

As a first step it seems reasonable to attempt an explanation of the temperature effect on the basis of electron-lattice scattering alone, neglecting possible

---

[3] See A. J. Dekker and A. van der Ziel, Phys. Rev. **86**, 755 (1952).

trapping, etc. It seems that indeed this provides already a quantitative interpretation of the results obtained by Johnson and McKay, as shown below. From the theory of electron-lattice scattering (see appendix) in ionic crystals it follows that per collision a secondary loses on the average an amount of energy

$$-(d\epsilon/dx)\lambda = h\nu/(2n_\nu+1)=\alpha(T), \tag{7}$$

where λ represents the mean free path and ν the frequency of the longitudinal optical vibrational modes. The quantum numbers n_ν are given by

$$n_\nu = 1/[e^{h\nu/kT}-1]. \tag{8}$$

From the optical data[4] on MgO one finds that $h\nu \simeq 1300k$, where k is Boltzmann's constant. It should be noted that $\alpha(T)$ in (7) is a function of temperature alone and independent of the energy of the electron. If for simplicity it is assumed that all secondaries are produced with the same energy ϵ_0, it follows from (7) that the energy of a secondary after N collisions is equal to

$$\epsilon(N)=\epsilon_0 - N\alpha(T). \tag{9}$$

Moreover, the mean free path of the secondaries may be written as

$$\lambda = \lambda_0\epsilon/(2n_\nu+1), \tag{10}$$

where for the energy range of interest λ_0 is approximately constant as indicated in Fig. 2. According to the last two equations, the mean free path as function of the number of collisions suffered by the secondary after its birth is

$$\lambda = [\lambda_0/(2n_\nu+1)](\epsilon_0 - N\alpha). \tag{11}$$

For the escape mechanism the following, admittedly simplified, model will be adopted: on their way to the surface the secondaries carry out a one-dimensional random walk perpendicular to the surface, with steps decreasing in accordance with (11). Furthermore, it will be assumed that secondaries that have lost an energy ϵ_0' do not take part in the secondary emission process any longer. In other words, the life of the secondaries is limited to a maximum number of collisions N_m given by

$$N_m\alpha = \epsilon_0'. \tag{12}$$

Two remarks may be injected at this point. From (7) it follows that the energy lost per collision at room temperature and 740°C is respectively

$$\alpha(298°\mathrm{K})=0.108 \text{ ev}, \quad \alpha(1013°\mathrm{K})=0.063 \text{ ev}.$$

In other words, the energy loss is smaller at the higher temperature. This by itself would lead to an increase in yield with increasing temperature and the observed decrease must thus result from a decrease in λ which swamps this effect. In this connection it is interesting to consider the requirement (12) for the hypothetical case that the secondaries move in straight lines towards the surface, rather than in a random walk fashion. Under these circumstances the secondary range would be equal to $N_m\langle\lambda\rangle_{\mathrm{Av}}$, where $\langle\lambda\rangle_{\mathrm{Av}}$ represents the average mean free path during the life of the electron measured in numbers of collisions. It follows immediately from (7), (11), and (12) that $N_m\langle\lambda\rangle_{\mathrm{Av}}$ is independent of temperature. One thus concludes that the observed temperature effect is essentially determined by the zig-zag nature of the paths of the secondaries.

Continuing now with the random walk model, one can readily show that the mean square displacement corresponding to N steps, the length of which decreases with a fixed amount for each collision in accordance with (11), is given by

$$\langle x^2\rangle_{\mathrm{Av}} = N\langle\lambda^2\rangle_{\mathrm{Av}}, \tag{13}$$

where the average of λ^2 is taken over N collisions. If $N=N_m$ the corresponding quantity $\langle x^2\rangle_{\mathrm{Av}}$ will be considered as the square of the secondary range x_s in the model adopted here. According to (11) and (13) the range of the secondaries for a given temperature is then equal to

$$x_s = \left[N_m\frac{\lambda_0^2}{(2n_\nu+1)^2}(\epsilon_0^2+\alpha^2\langle N^2\rangle_{\mathrm{Av}} - 2\epsilon_0\alpha\langle N\rangle_{\mathrm{Av}}\right]^{\frac{1}{2}}, \tag{14}$$

where the average value of N is $N_m/2$ and where for $N_m \gg 1$ one may write

$$\langle N^2\rangle_{\mathrm{Av}} = \int_0^{N_m}\frac{N^2dN}{N_m}=\frac{1}{3}N_m^2. \tag{15}$$

By employing (7) and (12), expression (14) may then be written as

$$x_s = \mathrm{const}(2n_\nu+1)^{-\frac{1}{2}}. \tag{16}$$

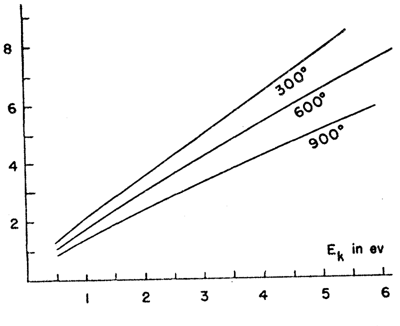

FIG. 2. The mean free path λ as function of energy in magnesium oxide as calculated from (26a) and (26b) for $T=300°$, $T=600°$, and $T=900°$ absolute.

---

[4] See H. Frohlich, *Theory of Dielectrics* (Oxford University Press, Oxford, 1949), pp. 155 and 158. [The maximum optical absorption (transverse waves) has been observed at 17.3×10^4A. For longitudinal waves the frequency is larger by a factor $(K/K_0)^{\frac{1}{2}}$ where K and K_0 are, respectively, the static and high-frequency dielectric constant. For MgO these constant are $K\simeq10$ and $K_0\simeq3$, leading to $h\nu\simeq1300k$.]

Hence, as T increases, n_ν increases and the range of the secondaries decreases. Now, for high primary energies the number of secondaries produced at a depth x is given by (5) so that the yield is proportional to

$$(A/\epsilon_e E_{p0})x_s. \tag{17}$$

For a given value of E_{p0} the ratio of the yields at two temperatures T_1 and T_2 will thus be given by

$$\delta_1/\delta_2 = [(2n_{\nu 2}+1)/(2n_{\nu 1}+1)]^{\frac{1}{2}}. \tag{18}$$

The fact that δ_1/δ_2 is independent of the primary energy far enough beyond the maximum in the yield curve is in agreement with the observations of Johnson and McKay. Also, for the ratio corresponding to $T_1 = 1013$°K and $T_2 = 298$°K (18) yields $\delta_1/\delta_2 = 0.76$, in good agreement with the experiments. It may be noted that (18) is also in agreement with the observations of Johnson and McKay in so far as δ is approximately a linear function of T over the range between room temperature and 740°C.

One may raise objections to the one-dimensional model employed here. In view of the fact that only the ratio of the ranges of the secondaries at the two temperatures is involved, it would seem, however, that a three-dimensional treatment would lead to the same result because both (11) and (12) retain the same form so that the temperature dependence of $\langle\lambda^2\rangle_{\mathrm{Av}}$ is the same as in the case treated above. The simple model employed here thus gives a quantitative interpretation of the observations.

APPENDIX

A summary of the theory of interaction between slow electrons and lattice vibrations in ionic crystals, initiated by Frohlich,[5] may be found in a paper by Seitz.[6] Let Ze be the charge per ion, M the effective mass per ion pair, a the shortest interionic distance, n the number of ion pairs per unit volume, ν the frequency of the optical vibrations, q the wave vector of a particular vibrational mode, and E the energy of the electron. The mean free path between collisions may then be shown to be given by

$$\frac{1}{\lambda} = \frac{\pi m Z^2 e^4}{2nMa^6 h\nu}\,\frac{2n_\nu+1}{E}\int\frac{dq}{q}. \tag{19}$$

The lower limit of the integration over q is equal to $2\pi mv/\hbar k$, where k is the wave vector of the electron. The maximum wave vector of the phonons is about equal to π/a and this value may be employed as the upper limit as long as $2k \geqslant \pi/a$. If, however, $2k < \pi/a$, the upper limit must be taken as $2k$. For MgO the mean free path has been plotted in Fig. 2 as function of the energy of the electron for three different temperatures, assuming (19) to be valid. This shows that λ is approximately a linear function of E over the energy range of interest and thus justifies Eq. (10). It must be admitted, however, that because the mean free path turns out to be small, it is doubtful whether the perturbation method employed by Frohlich is valid.[7] On the other hand, the treatment by Seeger and Teller[8] leads to a similar energy dependence of λ and it would seem therefore that (10) is formally correct.

Although the loss of energy by an electron per unit time is an essential quantity in calculating mobilities, the energy loss per unit path length is essential in the escape mechanism. One can show on the basis of the ideas developed by Frohlich that

$$-\frac{dE}{dx} = \frac{\pi m Z^2 e^4}{2nMa^6}\cdot\frac{1}{E}\int\frac{dq}{q}. \tag{20}$$

Equation (7) then follows immediately from (19) and (20).

The author wishes to acknowledge the interest of Drs. W. G. Shepherd and A. van der Ziel in this problem.

---

[5] H. Frohlich, Proc. Roy. Soc. (London) **160**, 230 (1937); **172**, 94 (1939).

[6] F. Seitz, Phys. Rev. **76**, 1376 (1949).

[7] See F. Seitz, *The Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), p. 534.

[8] R. Seeger and E. Teller, Phys. Rev. **54**, 515 (1938); **56**, 352 (1939).