

New Galvanomagnetic Effect

COLMAN GOLDBERG AND R. E. DAVIS

Westinghouse Research Laboratories, East Pittsburgh, Pennsylvania

(Received February 5, 1954)

A galvanomagnetic effect is described which is observed by measuring the induced voltage normal to the direction of current flow as in the Hall effect but with the magnetic field in the current-voltage plane. The measurements enable us to test the assumptions of spherical energy surfaces and spherical relaxation time surfaces for semiconductors. If these assumptions are correct, the effect should be isotropic. Preliminary room temperature measurements show this effect to be anisotropic in both *p*- and *n*-type germanium, in agreement with magnetoresistance measurements.

I. INTRODUCTION

THE galvanomagnetic effect to be discussed here is not new in the sense that it is based on new principles, but is new because it apparently has not been previously discussed and investigated. As will be shown in the following sections, if a crystal carrying an electric current is placed in a magnetic field, an electric field will be induced perpendicular to the current in the current-magnetic field plane. We call this field the "planar Hall field" since the experimental conditions are similar to those used in Hall experiments, except that in observing the Hall effect the magnetic field, electric field, and current are mutually perpendicular.

The electric field induced by the planar Hall effect analytically involves the same coefficients (but in a different arrangement) as does magnetoresistance. The magnetoresistance effect, of course, is concerned with a change in electric field parallel to the current direction.

II. GENERAL

In the absence of temperature gradients the electric current carried by a crystal with cubic symmetry is¹

$$\mathbf{i} = \sigma_0 \mathbf{E} + \alpha \mathbf{E} \times \mathbf{H} + \beta \mathbf{E} H^2 + \gamma \mathbf{H} (\mathbf{E} \cdot \mathbf{H}) + \delta M \mathbf{E}, \quad (1)$$

$$M = \begin{bmatrix} H_1^2 & 0 & 0 \\ 0 & H_2^2 & 0 \\ 0 & 0 & H_3^2 \end{bmatrix}, \quad (1a)$$

where E and H are the electric and magnetic field, respectively, and the subscripts 1, 2, and 3 refer to the axes of cubic symmetry. Equation (1) is actually an approximation obtained by ignoring terms containing E to a power higher than the first or H to a power higher than the second. The coefficients in (1), σ_0 , α , β , γ , and δ are expressed in terms of integrals which can be evaluated, in principle, if assumptions are made concerning the statistics to be used and the dependence of relaxation time and energy upon the wave vector \mathbf{k} . If δ is nonzero, the presence of the tensor M complicates this equation since the current passing through the crystal in the presence of electric and magnetic fields will be anisotropic, being dependent upon the orientation of the crystal.

¹ F. Seitz, Phys. Rev. **79**, 372 (1950).

Equation (1) gives the current density in terms of the electric and magnetic fields. Since in an experiment it is customary to fix the current and magnetic field and to measure the resulting electric field, it is more useful to have an equation of the following form²:

$$\mathbf{E} = \rho_0 [\mathbf{i} + A \mathbf{i} \times \mathbf{H} + B \mathbf{i} H^2 + C \mathbf{H} (\mathbf{i} \cdot \mathbf{H}) + D M \mathbf{i}]. \quad (2)$$

Here we have the electric field as a function of the current and magnetic field. If we substitute this equation for the electric field into Eq. (1), we can determine the coefficients in Eq. (2) in terms of the coefficients in the current Eq. (1).

$$\rho_0 = 1/\sigma_0, \quad (3a)$$

$$A = -\alpha \rho_0, \quad (3b)$$

$$B = -(\beta + \rho_0 \alpha^2) \rho_0, \quad (3c)$$

$$C = -(\gamma - \rho_0 \alpha^2) \rho_0, \quad (3d)$$

$$D = -\delta \rho_0. \quad (3e)$$

In obtaining Eqs. (3) we ignore terms containing the magnetic field to a power higher than the second.

If for a semiconductor we assume Maxwell-Boltzmann statistics for the carriers and a mean free path independent of energy, and if we assume that the energy and relaxation time surfaces in \mathbf{k} space are spherical, then we can integrate the coefficients of (1). If the resulting values for σ_0 , α , β , γ , and δ are substituted into Eqs. (3), we have the following as the coefficients of (2) for these assumptions:

$$\rho_0 = 1/n e_0 \mu, \quad (4a)$$

$$A = -(3\pi/8c)(q/e_0)\mu, \quad (4b)$$

$$B = -C = (9\pi/16c^2)(1 - \pi/4)\mu^2, \quad (4c)$$

$$D = 0, \quad (4d)$$

where

$$\mu = 4e_0 \lambda / 3(2\pi m k T)^{\frac{1}{2}}$$

is the mobility, n is the carrier density, c is the velocity of light, λ is the mean free path, and q is the charge of the current carrier, being $+e_0$ for holes and $-e_0$ for electrons. Since for these assumptions, D is zero, the field as given by Eq. (2) will be isotropic. The only

² G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).

crystal parameter involved in the coefficients A , B , or C is the mobility so that these parameters should not vary too much from one crystal to another of the same type but of different resistivity, as long as we are concerned only with relatively pure crystals. Thus, measurement of A , B , or C should enable us to calculate the mobility. Essentially (4b) is used to calculate mobility from Hall and conductivity measurements and (4c) is used to calculate mobility from magnetoresistance measurements. From (4c) we see that

$$\mu = 1.623B^{\frac{1}{2}}, \quad (5a)$$

$$\mu = 1.623(-C)^{\frac{1}{2}}, \quad (5b)$$

and, anticipating Eq. (13), it is found from (4b) that

$$(3\pi/8)\mu = R\sigma_0. \quad (5c)$$

The mobilities calculated from (5a), (5b), and (5c) should be equal to the mobility determined from drift measurements.³ Since mobilities determined by (5c) for germanium do not agree with the mobilities determined by drift measurements, the isotropic assumptions must be incorrect for germanium. The correct set of assumptions must lead to the observed experimental relations between $R\sigma_0$, B , and C and must give the observed value for D . If the correct assumptions enable us to calculate the mobility from B , C , and $R\sigma_0$, the mobilities obtained should equal the drift mobility. Experimental values for B , C , and D will thus be useful in checking assumptions made about energy and relaxation time surfaces.

Experimentally it is convenient to set up a coordinate system in a crystal which may or may not coincide with the axes of cubic symmetry. If the axes we define experimentally are x , y , and z , then the components of the electric field in these directions are represented by

$$E_r = \rho_0[\dot{i}_r + A(\mathbf{i} \times \mathbf{H})_r + B\dot{i}_r H^2 + CH_r(\mathbf{i} \cdot \mathbf{H}) + D \sum_{s=1}^3 \sum_{r'=x}^z \sum_{r''=x}^z \sum_{r'''=x}^z \dot{i}_{r'} a_{sr'} a_{sr''} a_{sr'''} H_{r'} H_{r'''}], \quad (6)$$

where r may equal x , y , or z and where the a_{sr} are the direction cosines of the x , y , z axes with respect to the 1, 2, 3 axes of cubic symmetry.

The coefficients B , C , and D can be calculated from magnetoresistance measurements.² These measurements give B directly, but C and D always appear in additive combinations with B or with each other and B . Hence, the accuracy of the values of C and D obtained from magnetoresistance measurements will depend upon the relative magnitudes of the three coefficients.

III. THE NEW EFFECT

If we use the experimental conditions that $i = i_x$, $H_z = i_y = i_z = 0$, i.e., the current is along the x axis and the magnetic field is in the xy plane, we find from

³ J. R. Haynes and W. Shockley, Phys. Rev. **81**, 835 (1951).

Eq. (6) that the field in the y direction is

$$E_y = \rho_0 C \dot{i}_x H_x H_y + \rho_0 D \dot{i}_x \left[\left(\sum_s a_{sx}^3 a_{sy} \right) H_x^2 + \left(\sum_s a_{sx} a_{sy}^3 \right) H_y^2 + 2H_x H_y \left(\sum_s a_{sx}^2 a_{sy}^2 \right) \right]. \quad (7a)$$

If $H_z = 0$ and we define ψ as $\arctan(H_y/H_x)$, then (7a) may be written as

$$E_y = \dot{i}_x H^2 [G \sin(2\psi + \Delta) + g], \quad (7b)$$

where

$$G = \rho_0 \left\{ \left[\frac{1}{2}C + D \sum_s a_{sx}^2 a_{sy}^2 \right]^2 + \frac{1}{4}D^2 \left[\sum_s (a_{sx}^3 a_{sy} - a_{sx} a_{sy}^3) \right]^2 \right\}^{\frac{1}{2}}, \quad (7c)$$

$$g = \frac{1}{2}D\rho_0 \left[\sum_s (a_{sx}^3 a_{sy} + a_{sx} a_{sy}^3) \right], \quad (7d)$$

$$\Delta = \arctan \frac{\left[\sum_s (a_{sx}^3 a_{sy} - a_{sx} a_{sy}^3) \right]}{\left[\frac{1}{2}C + D \sum_s a_{sx}^2 a_{sy}^2 \right]}. \quad (7e)$$

It is the field given by Eq. (7b) that we wish to investigate. This field which has apparently not been previously discussed or investigated, will be anisotropic unless $D=0$, and measurements of it for crystals with known orientations should enable us to calculate the coefficients C and D .

If we use a crystal in which x , y , z coincide with the (100) axes of cubic symmetry, (7b) becomes simply

$$E_y = G \dot{i}_x H^2 \sin(2\psi), \quad (8)$$

with

$$G = C\rho_0/2. \quad (9)$$

If x coincides with the (110) axis, y with the ($\bar{1}$ 10) axis, and z with the (001) axis, (7) again reduces to (8) only for this crystal orientation:

$$G = (C+D)\rho_0/2. \quad (10)$$

Equation (8) shows that the field should have a maximum at $\psi = 45^\circ$ for crystals with the two orientations mentioned above. G can be calculated by making measurements at this angle and by using the relation,

$$G = (V_y t / H^2 I) \times 10^{16}, \quad (11)$$

where t is the sample thickness. Equation (11) follows directly from (8). The factor 10^{16} is used so that G and hence C and $(C+D)$ calculated by (9) and (10) will be in laboratory units ($\text{cm}^5/\text{joule sec}$) if the units of V_y , t , H , and I are volts, centimeters, gauss, and amperes, respectively.

Thus, this new effect enables us to calculate C directly and D indirectly and enables us to check our assumptions of isotropy discussed in Sec. I. It should be pointed out that crystal orientations other than those discussed above could be used in studying this effect. The orientations discussed in this section, however, are readily obtainable, at least for germanium, and also lead to simple expressions for G , g , and Δ of Eq. (7b).

IV. PRELIMINARY EXPERIMENTS

In order to investigate the effect described in Sec. III several properly oriented samples of both *n*- and *p*-type germanium were prepared. The samples had their *x*, *y*, *z* axes parallel to either the (100), (010), (001) axes or the (110), (110), (001) axes. Several samples of both orientations were cut from a single slice of a single-crystal germanium ingot grown by the method of Teal and Little.⁴ By preparing the samples in this way it was hoped to obtain a group of samples that had the same characteristics except for orientation. Three groups of samples were prepared from three ingot slices: 9.5 and 20.5 ohm cm *p* type and 13 ohm cm *n* type. Leads were soldered to the samples as in a Hall measurement. For the effect we wish to investigate (which we shall refer to as the "planar Hall effect") the magnetic field lies in the *xy* plane, i.e., the plane of the sample. The voltage developed in the *y* direction may contain a contribution from the Hall effect if the magnetic field does not lie precisely in the *xy* plane. By proper reversals of the magnetic field and sample current we can obtain that component of the voltage in the *y* direction which is a function of H^2 and I , thus eliminating any Hall voltage.

All measurements were made at room temperature. The planar Hall voltage was found to vary linearly with the sample current and sinusoidally with 2ψ as predicted by Eq. (7b). The dependence upon the angle ψ is shown in Fig. 1. The coefficient G was calculated by means of Eq. (13) and Fig. 2 shows the dependence of G upon H^2 . The horizontal lines obtained from *n*-type samples indicate that E_y is proportional to H^2 . The finite slope obtained for the *p*-type samples indicates the presence of an H^4 contribution to E_y which is not un-

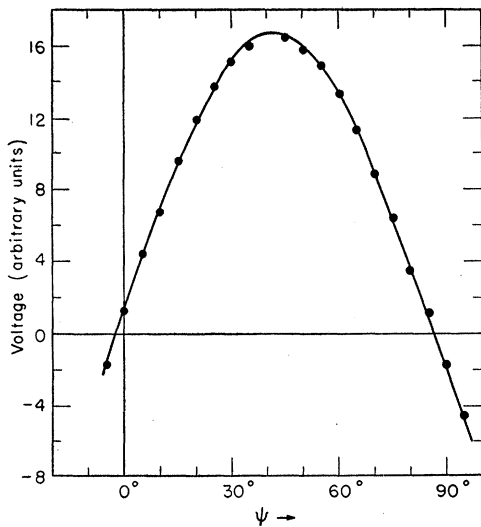


FIG. 1. Dependence of the planar Hall voltage upon the angle between current and magnetic field. The solid line fits the equation $V = 16.47 \sin 2(\psi + 3^\circ) - 0.50$. [Compare with Eq. (7b).]

⁴ G. K. Teal and J. B. Little, Phys. Rev. 78, 647 (1950).

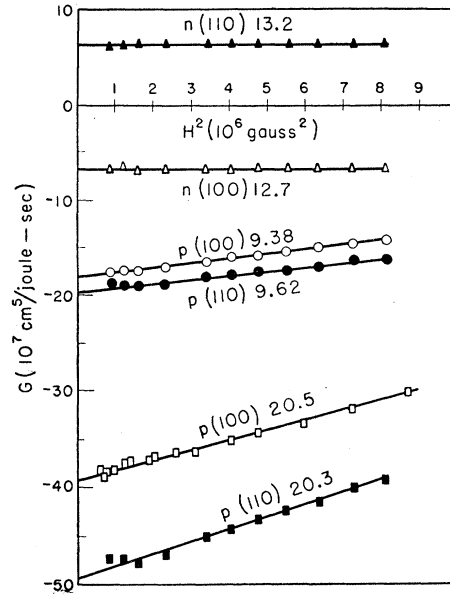


FIG. 2. Dependence of coefficient G on magnetic field for germanium at room temperature. The numbers in parentheses refer to the crystal orientation (see footnotes to Table I). The letter before the parentheses indicates whether the particular curve is for an *n*- or *p*-type sample and the number after the parentheses is the resistivity in ohm-cm.

expected since our derivation has ignored higher-order terms. (The values of G for *n* type indicate a very small H^4 contribution which is not detectable in Fig. 2.) If we consider the (100) *p*-type samples we see that for a ratio of resistivity of 2.19 the G values have a ratio of 2.17 which is in good agreement with Eq. (9).

The G values for the two *n*-type crystals in Fig. 2 show a decided anisotropy. Not only the magnitude of G but also its sign is dependent upon crystal orientation. The anisotropy is also evident in the *p*-type samples.

The values of C and D obtained by means of (9) and (10) from Fig. 2 are shown in Columns I of Table I. In order to compute D it is, of course, necessary to use data from two samples with different orientations. Since the values obtained for C and D differed from those obtained by Pearson and Suhl² by magnetoresistance measurements (Table II), magnetoresistance measurements were made on the same samples for which the coefficient G had been measured. With these data it was possible to obtain B and C from measurements on single samples while D could be obtained by combining measurements on the two samples with different orientation. The results of these magnetoresistance measurements are shown in columns II of Table I. By combining planar Hall measurements with magnetoresistance measurements it is possible to determine all three coefficients by measurements on a single sample as shown in columns III. The values of B and C for *n*- and *p*-type crystals and of D for *n*-type material are fairly consistent for the different crystals and the different techniques of measurement. The values of D

TABLE I. Measured values of the coefficients for germanium (room temperature).
 (The units of the values given for B , C , and D are 10^7 cm⁴/volt² sec².)

Sample	Type	Orienta- tion	ρ_0 (ohm cm)	$R \times 10^{-4}$ (cm ² /coul)	(I) Planar Hall effect		(II) Magnetoresistance			(III) Magnetoresistance plus planar Hall effect			(IV)
					C	D	B	C	D	B	C	D	μ_z (cm ² / volt sec)
33-2 Both	n	(110) ^a	13.2	-4.44									
33-5	n	(100) ^b	12.7	-4.36	-1.09	+2.04	+1.14		+1.82	+1.08	-0.98	+1.93	
9-7 Both	p	(110)	9.62										
9-9	p	(100)	9.38	2.94	-3.86	-0.24	+4.60		-0.84	+4.62	-3.22	-0.88	
9-2 Both	p	(110)	20.3	5.56									
9-6	p	(100)	20.5	5.60	-3.83	-1.03	+4.97		-0.70	+5.23	-3.64	-1.22	10 000

^a The symbol (110) refers to a crystal whose x , y , z axes are parallel to the (110), ($\bar{1}$ 10), and (001) crystallographic axes.

^b The symbol (100) refers to a crystal whose x , y , z axes are parallel to the (100), (010), (001) crystallographic axes, respectively.

for p -type material are not consistent. This may be due to the fact that in computing D it is necessary to subtract two large numbers in order to obtain a much smaller number. For p -type germanium it would apparently be better to measure the coefficient D by the method proposed in Sec. V.

The mobilities as calculated by the isotropic formula from the value of C obtained by the planar Hall effect (columns I of Table I) are given in column IV of Table I. These values are appreciably different from those obtained from drift experiments or the Hall coefficient. As pointed out in Sec. I, this indicates that the assumptions used in evaluating the coefficients of Eq. (4) are incorrect. Furthermore, the nonzero values of D obtained for both n - and p -type germanium also indicate that either the energy surfaces or the relaxation time surfaces in \mathbf{k} space or both are not spherical. Seitz¹ has derived expressions for B , C , and D by assuming that the energy surfaces are spherical and that the expression for relaxation time as a function of the wave vector \mathbf{k} has a spherically symmetrical term plus a term containing the simplest cubically symmetric anisotropy. He finds that with these assumptions D and C should be of the same sign. Our experimental results show this to be true for p -type germanium but not n -type so that Seitz's theory does not describe the proper situation for electrons in germanium. It can also be shown² from Seitz's derivation that for his assumptions the ratio $(B+C+\frac{1}{2}D)/(B+C+D)$ should equal 2.5. For our data of Table I we find this ratio varies from 1.19 to 2.65 in p -type germanium. The uncertainty in the ratio is due to the uncertainty in the value of the coefficient D . Seitz's theory, however, does not (but should) explain the serious discrepancy in

 TABLE II. Values of the coefficients for germanium (room temperature) obtained by Pearson and Suhl.^a (Units are 10^7 cm⁴/volt² sec².)

Type	B	C	D
p	1.3	-1.09	0
n	1.03	-0.69	1.3

^a See reference 2.

mobility between Hall and drift measurements for p -type germanium.

V. A METHOD FOR DIRECT MEASUREMENT OF COEFFICIENT D

As indicated in Sec. IV, it would be useful to have a method for measuring the coefficient D directly. In this section it will be shown that this can be done by observation of the dependence of the Hall field upon H^2 .

The experimental conditions when measuring the Hall coefficient are $i=i_x$, $H=H_z$, i.e., $i_y=i_z=H_x=H_y=0$. If our assumptions of Sec. I are correct, then (6) shows that for these experimental conditions

$$E_y = -A\rho_0 i_x H_z, \quad (12)$$

and R , the Hall coefficient, is

$$R = -A\rho_0 = \alpha\rho_0^2. \quad (13)$$

There will be no terms quadratic in the magnetic field in the expression for the Hall field. However, if the assumptions are incorrect and D is nonzero then

$$E_y = -A\rho_0 i_x H_z + D\rho_0 i_x H_z^2 \left(\sum_s a_{sx} a_{sy} a_{sz}^2 \right). \quad (14)$$

Thus for $D \neq 0$ the Hall field will be anisotropic.⁵ For the x , y , z axes respectively parallel to the (211), ($\bar{1}$ 11), (011) crystallographic axes the Hall field is

$$E_y = -A\rho_0 i_x H_z + 0.236 D \rho_0 i_x H_z^2. \quad (15)$$

Equation (15) shows that the coefficient D could be obtained directly by a direct measurement of the H^2 dependence of the Hall field. It should be possible to measure this field dependence directly by using an ac magnetic field and a dc sample current.

VI. SUMMARY

The coefficients C and D have been measured by means of the effect described in Sec. III and the values obtained agree with values calculated from magneto-

⁵ This anisotropy has been pointed out by Mason, Hewitt, and Wick, J. Appl. Phys. 24, 166 (1953).

resistance measurements except for the coefficient D in p -type germanium. The coefficient D could probably be evaluated most accurately by the method proposed in Sec. V.

The nonzero values obtained for D indicate that the assumptions of Sec. I are incorrect, i.e., the energy surfaces and/or the relaxation time surfaces are not spherical in \mathbf{k} space.

PHYSICAL REVIEW

VOLUME 94, NUMBER 5

JUNE 1, 1954

Threshold Field and Free Energy for the Antiferroelectric-Ferroelectric Phase Transition in Lead Zirconate

A. P. DEBRETTEVILLE, JR.

Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey

(Received May 20, 1953; revised manuscript received February 9, 1954)

Sawaguchi and Kittaka obtained the threshold field of PbZrO_3 —that field necessary to change the antiferroelectric to the ferroelectric phase—below the Curie temperature. The free-energy difference is the difference between the area under the ferroelectric portion of the curve and that under the antiferroelectric portion. An expression for free-energy difference has been obtained as a function of voltage, by eliminating the temperature parameter. This experimental relationship is compared with the author's theoretical expression—one that involves the product of the constant spontaneous polarization (16×10^{-6} coulomb/cm² in the ferroelectric phase), the threshold voltage, and the molar volume (43 cm³); the free energy so calculated is small—of the order of 4 calories/mole at 25 kv/cm. Equations are given for calculating the threshold field-strength and free-energy difference as functions of temperature. The equation for threshold field-strength resembles the Clapeyron-Clausius equation, with field strength analogous to pressure and spontaneous polarization analogous to volume. The threshold voltage *versus* the downward shift of the Curie temperature is in good agreement with the experimental results of Sawaguchi and Kittaka.

I. INTRODUCTION

IT is known that pure lead zirconate has an antiferroelectric phase below and a paraelectric phase above 230°C, the Curie point. Sawaguchi and Kittaka have shown that one can apply a field of the order of 10 kv/cm to pure lead zirconate ceramic and cause the ferroelectric phase to appear below the Curie point.¹

Below the Curie point the sample is normally antiferroelectric; the dielectric constant ϵ_a is approximately 200, and has a positive temperature coefficient. At the Curie point, the dielectric constant increases sharply about twenty-fold.²

Figure 1 shows the experimental data obtained by Sawaguchi for the threshold field of the antiferroelectric-to-ferroelectric phase transition, as a function of temperature.¹

Figure 2(b) shows the point O , from which the threshold field E_t was calculated; the cross-hatched area is proportional, essentially, to the free energy.¹

The author intends to calculate the value of the threshold field, and the free-energy difference ΔF , between the ferroelectric phase and the more stable or lower free-energy antiferroelectric phase, as a function of temperature and voltage.

CALCULATION³

Sawaguchi and Kittaka have derived an expression for the free-energy difference $\Delta F = (F_f - F_a)$ between

¹ E. Sawaguchi and T. Kittaka, *J. Phys. Soc. Japan* **7**, 336-337 (1952); E. Sawaguchi, *J. Phys. Soc. Japan* **8**, 615-629 (1953).

² Shirane, Sawaguchi, and Takagi, *Phys. Rev.* **84**, 476-481 (1951).

³ Background for this type of calculation can be found in the

the ferroelectric state F_f and the antiferroelectric state F_a , in terms of the internal energy of the antiferroelectric state U_a and the internal energy of the ferroelectric state U_f ¹

$$F_f(T_m) - F_a(T_m) = U_f(T_f) - U_a(T_a). \quad (1)$$

The temperatures of the antiferroelectric and ferro-

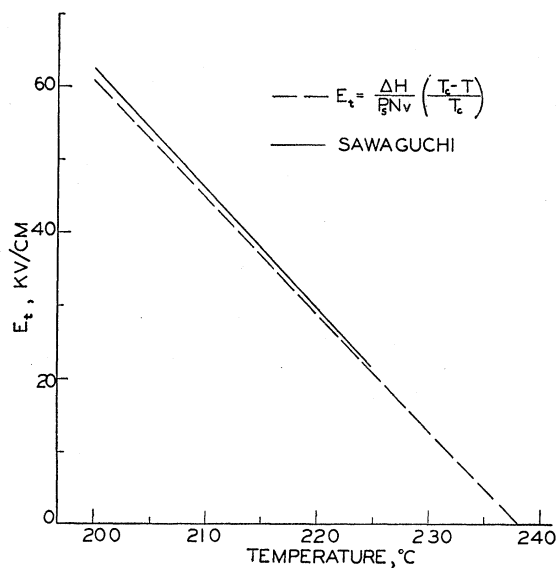


FIG. 1. Relation between threshold field and Curie temperature. field of low-temperature physics. See J. C. Slater, *Quantum Theory of Matter* (McGraw-Hill Book Company, Inc., New York, 1951), pp. 504-508; E. T. Jaynes, *Ferroelectricity* (Princeton University Press, Princeton, 1953), pp. 70-80.