

where  $\alpha$  is the molecular polarizability. The quadrupole energy is  $V_q = -\frac{1}{2}eQ_{\text{mole}}r^{-3}(1-3\cos^2\theta)$ , where  $Q_{\text{mole}}$  is the quadrupole moment of the molecule relative to the molecular axis and  $\theta$  is the angle between the molecular axis and the line joining the center of the molecule with the ion. When the measured<sup>3,4</sup> values of  $\alpha$  and  $Q_{\text{mole}}$  for  $N_2$  are inserted into the above expressions, it is seen that  $|V_p| > |V_q|$  for the whole range of separations at which  $|V_p|$  is of the order of  $kT$ . For pure polarization scattering the momentum transfer cross section averaged over all thermal energies is approximately equal to the cross section at the kinetic energy  $(64/9\pi)kT$ . Comparing the two interactions at the separation  $r=3.8\text{A}$ ,  $|V_p| = (64/9\pi)kT = 0.059\text{ eV}$  ( $T=300^\circ\text{K}$ ) while  $|V_q(\theta=\pi/2)| = 0.018\text{ eV}$  and  $|V_q(\theta=0)| = 0.035\text{ eV}$ .

An exact calculation of the momentum transfer cross section resulting from the interaction  $V_p+V_q$  is intractable because the potential is not spherically symmetric and translational kinetic energy is not conserved. However, in the case of a pure polarization interaction the Langevin formula<sup>5</sup> gives the mobility of ions (single positive charge, atomic mass  $M$ ) in nitrogen as  $2.11(1+28/M)^{1/2}\text{ cm}^2/\text{volt sec}$ . The inclusion in the Langevin potential model of a repulsive force at small separations would cause an increase in this calculated mobility by at most 15 percent, depending on the effective radius of the repulsion. This agrees reasonably well with the empirical result<sup>2</sup> of  $2.04(1+28/M)^{1/2}\text{ cm}^2/\text{volt sec}$  and indicates that the quadrupole interaction is not of primary importance in determining the ionic mobility at room temperature.

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## Electrical Conduction in Halide-Contaminated Ice

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THE anomalous electrical properties of halide-contaminated ice have been the subject of several papers by the authors.<sup>1-4</sup> Such ice, frozen according to the methods prescribed, exhibits unusual features: (1) it acts as a rectifier; (2) it possesses an extremely high dielectric constant for fields applied in the non-conductivity direction; and (3) the conductive and dielectric properties depend in a complicated way upon temperature, bias, and frequency. The general behavior with respect to the properties enumerated is such as to suggest electronic conduction. This interpretation was given further support by the fact that the passage of an electric current for long time intervals produced no marked change in the conductive behavior of the crystal. It appeared reasonable, therefore, to formulate an explanation based on the premise that the partially shielded proton is displaced under the stress of an applied electric field in such a manner as to increase considerably the ionic contribution to the hydrogen bonding between neighboring molecules. In spite of the relative ease of proton displacement considered essential, the proton appeared to be bound because of the high-energy barrier postulated for proton transfer (26 kcal/mole or even more<sup>5</sup>). Moreover, there is persistence of a sense of direction to the  $c$  axis under all conditions of applied field—a condition considered incompatible with the process of molecular rotation inherent in the transfer mechanism.

An experimental re-examination of the conduction process has revealed that the conduction of electricity in the forward direction through the contaminated ice results in the liberation of oxygen

and hydrogen at the positive and negative electrodes respectively. The amount of gas liberated is in quantitative agreement with that associated with the total charge transfer. Contrary to our earlier interpretations it now appears certain that the conduction process is ionic—by a proton transfer mechanism.

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## Polarization of Nucleons Elastically Scattered from Nuclei\*

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SEVERAL recent experiments<sup>1-3</sup> have shown that protons of energy 200-350 Mev scattered from nuclei are polarized. The elastically scattered protons have a polarization ( $\sim 60$  percent) that is somewhat larger than the inelastically scattered ones. Fermi<sup>4</sup> has proposed an explanation of the polarization for elastic scattering in terms of a nuclear spin-orbit interaction potential similar to that assumed in the nuclear shell model. He used the Born approximation in his estimates. The purpose of this note is to investigate the polarization effects of a nuclear spin-orbit potential using the transparent nuclear model of Serber.<sup>5</sup> That is, we add to the nuclear complex potential which is constant over a sphere of radius  $R$ , a term

$$-\hbar^{-1}U(r)\mathbf{L}\cdot\boldsymbol{\sigma}, \quad (1)$$

where  $\mathbf{L}$  and  $\hbar\boldsymbol{\sigma}/2$  are the orbital and spin angular momenta of the proton.  $U$  is taken to be real since the absorption cross section of protons in nuclear matter is independent of its spin. The radial and energy dependence of  $U$  are unknown, as well as its variation with atomic number. We have assumed two specific forms for  $U$ :

$$U_1(r) = u_1 R \delta(r-R), \quad U_2(r) \cong u_2 (R/r)^2. \quad (2)$$

$U_1(r)$  is the one considered by Fermi and is suggested by the Thomas precession of a particle with spin under acceleration, which, in the transparent nuclear model, is concentrated at the boundary of the potential hole. By contrast,  $U_2(r)$  is concentrated near the origin. [The exact form of  $U_2$ , which was chosen for convenience in numerical computation, is implied in Eq. (8).]

The calculations are performed by a partial-wave analysis. The degree of polarization (as defined by Oxley *et al.*<sup>1</sup>) is

$$P(\theta) = (|f-ig|^2 - |f+ig|^2) / (|f-ig|^2 + |f+ig|^2), \quad (3)$$

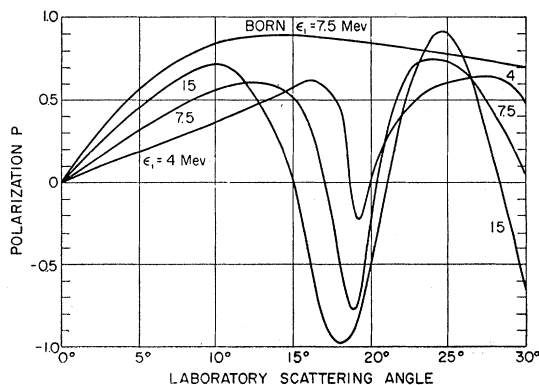


FIG. 1. Polarization  $P$  as a function of scattering angle  $\theta$  for various  $\epsilon_1$  calculated from potential  $U_1$  [Eq. (7)] for 316-Mev protons scattered by Be. The top curve gives the Born approximation values for  $\epsilon_1 = 7.5$  Mev.

where

$$f(\theta) = \sum_{l=0}^L [(l+1)A_l^+ + lA_l^-] P_l^0(\cos\theta), \quad (4)$$

$$g(\theta) = - \sum_{l=0}^L [A_l^+ - A_l^-] P_l^1(\cos\theta), \quad (5)$$

with

$$A_l^\pm = [\exp(2i\delta_l^\pm) - 1]/2i. \quad (6)$$

$\delta_l^\pm$  are the phase shifts for the partial waves with  $J=l\pm\frac{1}{2}$ ,  $P_l^m$  = associated Legendre polynomial, and  $L$  = largest integer  $< kR$ . Using the WKB approximation to evaluate  $\delta_l^\pm$ , we obtain

$$(\delta_l^\pm)_1 = [(iK/2) + k_1][R^2 - (l+\frac{1}{2})^2/k^2]^{\frac{1}{2}} \pm \frac{1}{2}\epsilon_1 l T^{-1} [1 - (l+\frac{1}{2})^2/(L+\frac{1}{2})^2]^{-\frac{1}{2}}, \quad (7)$$

and

$$(\delta_l^\pm)_2 = [(iK/2) + k_1 \pm (k\epsilon_2/2T)][R^2 - (l+\frac{1}{2})^2/k^2]^{\frac{1}{2}}, \quad (8)$$

for the two forms of spin-orbit interaction respectively. Here  $T$  = kinetic energy,  $k_1$  and  $K$  are the usual optical parameters.  $\epsilon_{1,2}$  are energies that characterize the depth of the spin-orbit coupling.  $\epsilon_1 = Lu_1$  and  $\epsilon_2 = Lu_2$ . The two potentials differ in that  $U_1$  emphasizes the high,  $U_2$  the low angular momenta phase shifts.

For definiteness, we have calculated  $P$ , using Eqs. (3)–(8), for 316-Mev nucleons scattered by Be. This energy corresponds to the experiments of Marshall *et al.*<sup>2</sup> We take the optical parameters from a recent paper by Taylor,<sup>6</sup> i.e.,  $R = 3.2 \times 10^{-13}$  cm,  $k_1 = 0.86 \times 10^{12}$  cm<sup>-1</sup>, and  $K = 1.7 \times 10^{12}$  cm<sup>-1</sup>, corresponding to a complex nuclear potential  $V = (-13 + 25.6i)$  Mev, and  $L = 13$ . Figure 1 shows  $P(\theta)$  for the spin-orbit potential  $U_1(r)$  with  $\epsilon_1 = 4, 7.5$ , and 15 Mev. For comparison, the top curve in this figure shows the Born approximation result with  $\epsilon_1 = 7.5$  Mev and the same nuclear parameters. Figure 2 shows  $P(\theta)$  for  $U_2(r)$  with  $\epsilon_2 = 4, 7.5$ , and 15 Mev and Fig. 3 shows the differential cross section averaged over spin directions,

$$\langle d\sigma/d\Omega \rangle = \frac{1}{2} k^{-2} [ |f - ig|^2 + |f + ig|^2 ], \quad (9)$$

for  $\epsilon_2 = 0-15$  Mev.

It is clear from the curves that one can fit the result of Marshall *et al.*<sup>2</sup> that  $P(14^\circ) \cong 0.6$  with  $\epsilon_1 = 7.5$  Mev (i.e.,  $u_1 = 0.58$  Mev) or  $\epsilon_2 = 12$  Mev (i.e.,  $u_2 = 0.92$  Mev). These represent spin-orbit interactions that agree roughly in order of magnitude with that in the shell model theory provided one assumes a linear dependence of the spin-orbit interaction with the momentum of the nucleon.

The curves show that for small  $\theta$ ,  $P$  is proportional to  $\epsilon$ . For a given  $\epsilon$ ,  $P$  is larger for interaction  $U_1$  than  $U_2$ . This results from the fact already mentioned that  $U_1$  emphasizes the larger and more heavily weighted  $l$  values more than  $U_2$ .

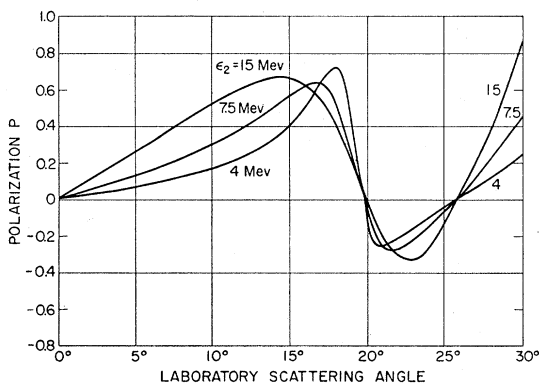


FIG. 2. Polarization  $P$  as a function of scattering angle  $\theta$  for various  $\epsilon_2$  calculated from potential  $U_2$  [Eq. (8)] for 316-Mev protons scattered by Be.

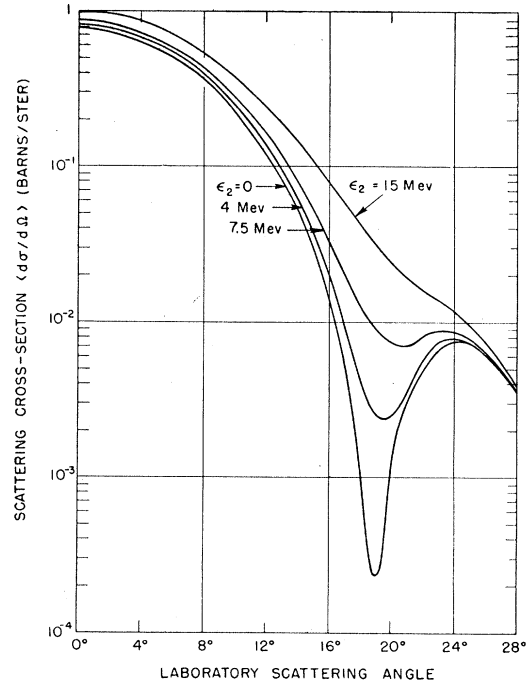


FIG. 3. Differential scattering cross section  $\langle d\sigma/d\Omega \rangle$  averaged over spin directions for 316-Mev protons scattered by Be. The curves were calculated from potential  $U_2$  for various  $\epsilon_2$ .

Perhaps the most interesting result of these calculations is the oscillatory nature of  $P(\theta)$ .  $P(\theta)$  first becomes negative in the region of the first diffraction minimum ( $\sim 20^\circ$ ) due to a change of sign of  $f(\theta)$ . It becomes positive again when  $g(\theta)$  also changes sign. The fact that the Born approximation applied to  $U_1$  does not show this, is fortuitously due to  $f$  and  $g$  having identical angular dependences. The search for negative  $P(\theta)$  may be hampered of course by the low intensity of the diffracted beam beyond the first minimum.

Finally it is interesting to note in Fig. 3 how the spin-orbit potential has the effect of washing out the deep minima in  $\langle d\sigma(\theta)/d\Omega \rangle$ . There are some experimental indications of such an effect.<sup>7</sup> It is also seen that the total diffraction cross section  $\sigma_d$  increases with  $\epsilon$ . The values of  $\sigma_d$  as obtained from integration of  $\langle d\sigma(\theta)/d\Omega \rangle$  are 61, 67, 75, and 105 mb for  $\epsilon_2 = 0, 4, 7.5$ , and 15 Mev, respectively.

Perhaps detailed investigations of  $P(\theta)$  for various elements and at lower energies will be able to determine the radial and velocity dependence of  $U(r)$ , although it must be borne in mind that  $P(\theta)$  is a sensitive function of all the optical parameters  $R$ ,  $k_1$ , and  $K$  as well as  $U(r)$ . One can also conjecture that the polarization of elastically scattered nucleons will still be present in the Bev range. For example, a calculation was carried out for 1.4-Bev neutrons on Be with  $\epsilon_2 = 15$  Mev, and one obtains  $P(\theta = 5^\circ) = 0.46$  and  $P(\theta = 7^\circ) = 0.81$ .

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