

The second integrals in the expressions (13) and (14) can be shown to be equal; thus they can introduce no asymmetry between the scattered intensities for the two positions of the foil. Further, if G is assumed to be a delta function about the direction of scattering, then

$$2 \int_{-2\theta_2}^{2\theta_2} \int_{\frac{1}{2}(\pi-\alpha)-2\theta_2}^{\frac{1}{2}(\pi-\alpha)+2\theta_2} S(\theta)G(\psi) \sin\gamma d\gamma d\phi = S\left(\frac{\pi}{2}, \frac{h}{2 \cos\delta}\right). \quad (16)$$

In view of the results of Eq. (15) it seems reasonable to assume that this approximation introduces an error of at most five percent in the absolute scattering intensity. Although this is sufficiently large to mask any double-scattering contribution to the number of electrons scattered through a given angle, it introduces a comparatively small error in the asymmetry ratio R/T , the contribution of the two integrals being entirely symmetric.

Adding these contributions, one obtains the ratio

$$R/T = 1.05 \pm 0.01. \quad (17)$$

Comparing Eqs. (12) and (15) it is obvious that for electrons at oblique incidence the only serious deviation from single scattering is that due to double scattering. This is essentially the condition for the validity of the approximation made by Goertzel and Cox. Using Rutherford's formula with relativistic electron mass for the single-scattering distribution, $S(\xi)$, they obtain an asymmetry ratio slightly less than 1.02 for the parameters considered above; the discrepancy is due almost entirely to the choice of different single-scattering distributions. The accuracy, apart from any assumptions as to scattering distributions, of the Goertzel-Cox approximation permits an essential simplification in calculations of this type.

Note added in proof.—A calculation similar to that of Goertzel and Cox has been published by Norio Ryu, *J. Phys. Soc. Japan* **5**, 423 (1950).

The Excitation of Plasma Oscillations*

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A beam of high-energy electrons, injected into the plasma of a dc discharge from an auxiliary electron gun, excited oscillations in the plasma at the plasma electron oscillation frequency given by the Tonks-Langmuir equation. A movable probe showed the existence of standing-wave patterns of the oscillatory energy in the region of the plasma in and around the electron beam. Nodes of the patterns coincided with the electrodes which limited the region of the plasma traversed by the beam. The standing-wave patterns were independent of the frequency of the oscillation. At any particular frequency, the standing wave was determined by the thickness of the ion sheaths on the bounding electrodes. The mechanism of the energy transfer from the electron beam to the oscillation of the plasma electrons was established as a velocity-modulation process by the detailed behavior of the frequency of oscillation and the transitions in the standing-wave patterns as the sheath thickness was varied. Experimental attempts to produce plasma oscillations as predicted by Bohm and Gross proved to be fruitless.

PLASMA electron oscillations have been observed in a number of experiments¹⁻⁴ in which a beam of high-energy electrons penetrated a neutral plasma of a low-pressure gas discharge. In all these experiments the beam of high-energy electrons was formed by electrons which emerged from the cathode fall region of a low-pressure arc without colliding with neutral gas atoms. The oscillations were usually accompanied by an abrupt increase in the energy spread of the beam

particles. The frequency of oscillation was related to the electron density by the Tonks-Langmuir relation,

$$\omega_p^2 = ne^2/m\epsilon_0, \quad (1)$$

where ω_p is the radian frequency of the plasma electron oscillation, n the electron density, m the mass of an electron, and ϵ_0 the permittivity of free space. The experimental evidence indicated that energy was transferred from the electron beam to an organized oscillation of the plasma electron.

EXPERIMENTAL APPARATUS

The use of fast electrons coming from the cathode region of an arc as a beam to excite plasma oscillations has several disadvantages. One of the most obvious drawbacks is that the plasma itself is controlled by the

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¹ L. Tonks and I. Langmuir, *Phys. Rev.* **33**, 195 (1929).

² H. J. Merrill and W. Webb, *Phys. Rev.* **55**, 1191 (1939).

³ T. R. Neill and K. G. Emeleus, *Proc. Roy. Irish Acad.* **A53**, 197 (1951).

⁴ G. Wehner, *J. Appl. Phys.* **22**, 761 (1951).

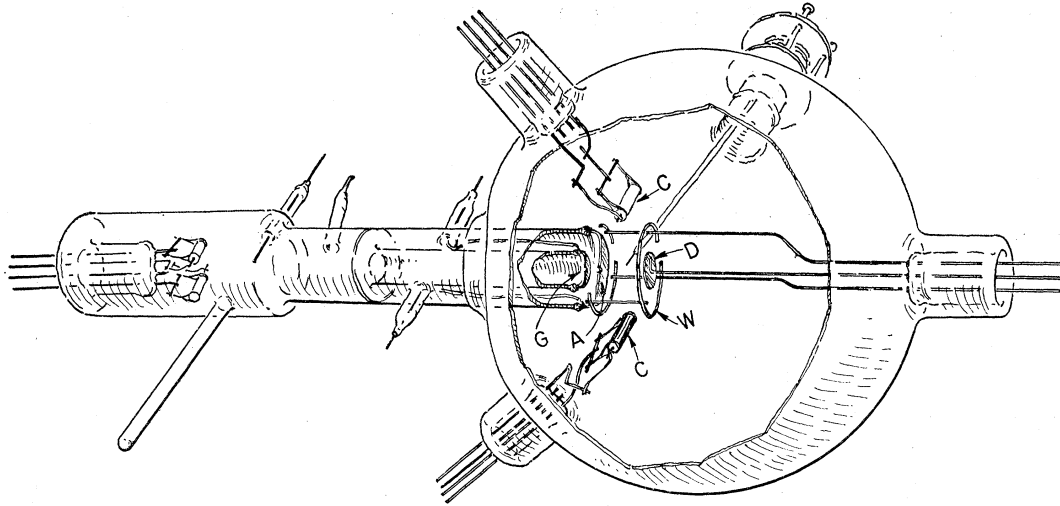


FIG. 1. Sketch of the experimental tube.

density and energy of the beam electrons since these electrons maintain the plasma. In addition, neither the density of electrons in the beam nor their energy is easily varied. The use of a beam of high-energy electrons from an auxiliary source rather than from the cathode of the arc eliminates these difficulties, and the present apparatus was designed with this object in view.

A glass tube was constructed in which a controlled electron beam was injected into the plasma of a low-pressure (about three microns) mercury arc. The main plasma was drawn from two large oxide-coated cathodes, marked *C-C* in Fig. 1. A second discharge in the side arm acted as a source of electrons for an electron gun. A beam of electrons was drawn from a 1-mm hole in the anode of the gun discharge, *G* of Fig. 1, accelerated through several hundred volts, and injected into the plasma through a 1-mm hole in the accelerating anode marked *A* in Fig. 1. The electron beam finally impinged on an anode disk *D*, which was electrically connected to the accelerating anode and 1.5 cm away from it. The geometry of the tube and the electrical circuits controlling the arc were arranged to give large ion sheaths on the accelerating electrode and the anode disk. These two electrodes could be biased negatively with respect to the wire ring anode, *W*, of the main discharge through a range of 200 volts. This allowed the thickness of the ion sheaths to be changed. A probe, attached to the tube by a stainless steel bellows, could be moved through the interaction region of the plasma and the electron beam. The probe was a wire 3 thousandths of an inch in diameter and 5 mm long; its position was measured by a cathetometer to an accuracy of ± 0.1 mm.

Oscillations were detected by a superheterodyne receiver capacitatively coupled to the movable probe. The probe picked up the oscillation whether it was in or near the electron beam. The selectivity of the receiver determined the frequency of the oscillation to ± 5

Mc/sec whereas the band width of the oscillation was usually less than 10 Mc/sec.

EXPERIMENTAL RESULTS

The behavior of the plasma oscillation was studied by varying the electron density of the beam (controlled by the electron gun discharge current) and the thickness of the ion space-charge sheath (controlled by the sheath bias voltage). The electron density of the beam was so large that the density of the plasma region around the beam was determined by the density of the beam particles. Probe measurements showed that the plasma electron density varied as a linear function of the gun discharge current if the energy of the beam particles was kept constant. The variation of the thickness of the sheath on the beam electrodes had no effect on the plasma electron density until the sheath thickness was comparable to the plasma length—*AD*.

The frequency of oscillation was observed to vary in a discontinuous manner with the gun discharge current. The frequency would remain nearly constant over small variations of the gun discharge current and then jump abruptly to a new frequency. Along one of these frequency plateaus, the intensity of the oscillation would rise to a maximum and then fall to a much lower value before a jump to a new frequency occurred. It was observed that the frequency corresponding to the strongest signal for each frequency range was consistent with the predictions of Eq. (1). Since the plasma electron density was a linear function of the gun discharge current, the square of the frequency of the oscillation at each intensity maximum was a linear function of the gun discharge current. Equation (1) was used to determine a conversion factor, relating the electron density and the gun discharge current, from two observed points of the plot of gun discharge current *versus* the square of the frequency. The electron density was then calculated from the observed gun

discharge current. The measured frequency of oscillation as a function of this normalized electron density is shown in Fig. 2.

With the electron density held constant, motion of the probe (coupled to the high-frequency receiver) through the plasma, showed that the entire plasma between the accelerating anode and the anode disk was oscillating in a standing wave. As the electron density was varied so that the tube oscillation jumped from one frequency plateau to another, the standing-wave pattern changed, each frequency plateau corresponding to a given wave pattern. The standing-wave pattern associated with each range of frequency is also shown in Fig. 2. In general, the standing-wave pattern had an integral number of half-waves between the two beam electrodes with nodes at these electrodes.

The standing-wave patterns were measured as the thickness of the plasma sheaths on the accelerating anode and anode disk was varied, keeping the electron density constant. The results of these measurements are illustrated in Fig. 3. As the two beam electrodes were made more negative with respect to the wire ring anode of the main discharge, the plasma sheaths became thicker. Sudden transitions in the frequency of oscillation occurred as the sheath thickness was varied. These transitions were accompanied by abrupt changes in the wavelength of the plasma oscillations. As in the case of the sudden frequency transitions with changes in density, small changes in frequency occurred over a certain change in sheath thickness, and then a major jump in oscillation frequency occurred as the wavelength of the standing wave changed. The positions of the visible sheath edge show that both the frequency of oscillation and the transitions in the standing wave patterns were associated with this edge. The abrupt changes in the wavelength adjusted the standing-wave pattern to keep a node of the pattern near the visible sheath edge.

The thickness of the sheath, X_s , may be calculated

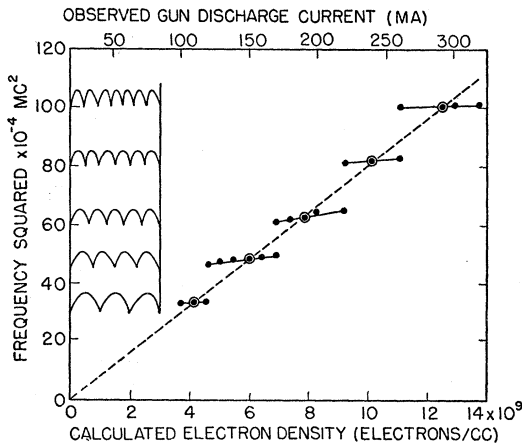


Fig. 2. Oscillation frequency as a function of the gun discharge current and electron density.

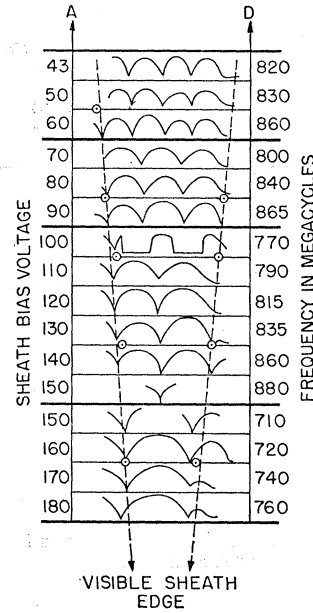


Fig. 3. Standing-wave pattern as a function of the thickness of the plasma sheath keeping the electron density constant. *A* and *D* represent the positions of the accelerating anode and anode disk as shown in Fig. 1.

from the space charge law for an infinite plane to give

$$X_s = 375V_s^{1/2}/n^{1/2},$$

where V_s is the sheath bias voltage and a directed ion velocity of 1.7×10^6 cm/sec was assumed. The electron density may be eliminated by using Eq. (1) to give

$$X_s = 3.4V_s^{1/2}/f_0, \tag{2}$$

where f_0 is the average frequency in Mc/sec of a given frequency range. The variation of the measured frequency of oscillation as a function of the sheath thickness calculated from Eq. (2) is shown in Fig. 4. In each frequency range, the frequency is a linear function of the sheath thickness. The standing-wave pattern of each frequency range is also shown in the figure.

MECHANISM OF EXCITATION

An explanation of an experimental plasma oscillator has been given by Wehner.⁴ The mechanism of energy transfer from the electron beam to the oscillation of the plasma electrons is described by him as a transit time process similar to that controlling a klystron. The frequency of the oscillation is determined fundamentally by the resonant properties of the plasma given by Eq. (1). The electron beam enters the plasma through an ion sheath and is velocity-modulated at a point near the sheath edge of the plasma. As the beam traverses the plasma, the velocity modulation of the beam causes the electrons to bunch. An electric field of the proper phase for retarding the bunched electrons, and thus extract energy from the beam, must be assumed.

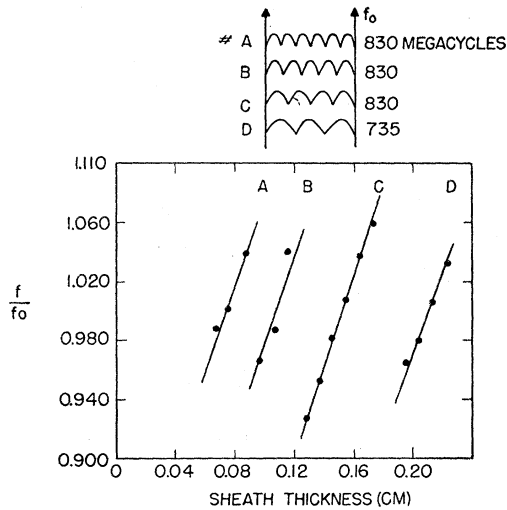


FIG. 4. The relation between oscillation frequency and sheath thickness.

Wehner pointed out that the proper phase relation between the electric field where the beam is modulated and where the energy is extracted from the beam requires that the average transit time t of a beam electron across the distance X_t between the modulation and extraction points be

$$t = X_t/v_0 = [(4k+1)/4]/f, \quad (3)$$

where $k=0, 1, 2, \dots$; v_0 is the average velocity of the beam electrons calculated from the accelerating voltage of 500 volts and the sheath bias voltage; and f is the frequency of oscillation. If the distance X_t is assumed constant, Eq. (3) may be written as $\lambda\sqrt{V} = \text{constant}$, where λ is the free-space wavelength of the oscillation and V is the voltage accelerating the beam. This relation has been observed by Neill and Emeleus³ and by Wehner.⁴

In the present experiment, the actual point in the plasma where the beam was modulated was not detected since the probe did not have sufficient motion to pass through the sheath. If the modulation point is assumed to be at the sheath edge and the extraction point is assumed to be fixed by the standing-wave pattern, the deviations of the measured frequency from the frequency predicted by Eq. (1) may be understood by considering Eq. (3). If X_s is the sheath thickness on the accelerating anode and X_t is the distance from the modulation point at the sheath edge to the extraction point, then the extraction point is fixed at a distance $X_s + X_t$ from the accelerating anode, or $X_s + X_t = \text{constant}$. If the sheath thickness X_s increases, and the plasma density remains constant, X_t must decrease and the frequency of oscillation increase to satisfy the transit time relation. This is shown in Fig. 4. If the sheath voltage is held constant and the plasma density is increased, the frequency of the oscillation should increase according to Eq. (1), as was discussed above.

However, the increase in the plasma density decreases the sheath thickness and consequently increases X_t . To satisfy Eq. (3), the frequency should decrease. As a result of these two opposing effects the increase in the measured frequency is less than the increase predicted by Eq. (1). This effect accounts for the frequency plateau curves of Fig. 2.

If the transit-time mechanism is correct, the sheath thickness X_s must be linearly related to the transit distance X_t . By use of the measured frequency of oscillation and the velocity of the beam determined by the applied potentials, the distance X_t can be calculated from Eq. (3). For every case where sufficient data were taken and the plasma density remained constant, X_s varied linearly with X_t . An example of these data is illustrated in Fig. 5. The slope of the X_t versus X_s line is equal to minus one, indicating that the extraction point was fixed and the sheath thickness varied as assumed above.

The modulation of the electron beam occurs in the large fields at the sheath edge, whereas the beam may be demodulated in any region where the field has the opposite sign. This means that the modulation and the extraction points must always be separated by a node in the standing wave pattern. Both the sheath thickness and the transit distance can be calculated from the observed data. The distance X_s determines the location of the modulation point with respect to the accelerating anode, and the sum of X_s and X_t determines the location of the extraction point with respect to the accelerating anode. Superimposing the calculated sheath and extraction points on a diagram indicating the positions of the nodes of various standing waves observed, we find this phase relation is indeed met, as illustrated in Fig. 6. The transitions in the standing-wave patterns occur as the plasma oscillation adjusts itself to satisfy both the condition expressed by Eq. (1) and that expressed by Eq. (3) by keeping the extraction point and the modulation point separated by a node in the standing-

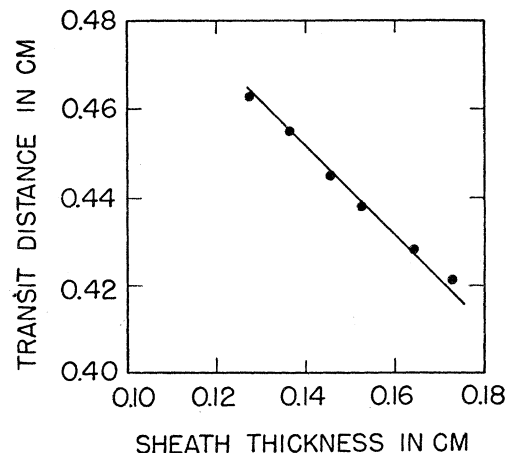


FIG. 5. Relation between the sheath thickness and the transit distance keeping the electron density constant.

wave pattern while oscillating in the frequency range required by the Tonks-Langmuir equation. The amplitude of the observed oscillation increased as the extraction region approached the maximum of the standing wave and decreased on approaching a node.

DISCUSSION

In 1949 Bohm and Gross⁵ published a theory of plasma electron oscillations based on a one-dimensional analysis of a uniform infinite plasma. The theory postulated that a traveling longitudinal potential field could be excited by a beam of high-velocity electrons in the plasma and that this could be used to explain the energy transfer from a beam of high-energy particles to the oscillation existing in the plasma. A large portion of our experimental effort was devoted to injecting an electron beam into a discharge plasma so as to generate a plasma oscillation by the mechanism proposed by Bohm and Gross. The electron beam injected into a uniform plasma from an electron gun did not excite observable plasma oscillations in any of the experimental tubes constructed to satisfy the conditions of the Bohm and Gross theory. However, as soon as the discharge in the tube shown in Fig. 1 was modified by introducing large sheaths on the beam electrodes, oscillations were immediately found at the same plasma density where they were not observed in the "infinite" plasma.

All previous experimental investigations of plasma oscillations are consistent with the interpretation of the excitation process suggested by these present experiments. However, none have established the existence of an organized oscillation throughout the plasma region

⁵ David Bohm and E. P. Gross, Phys. Rev. 75, 1851 and 1865 (1949); Phys. Rev. 79, 992 (1950).

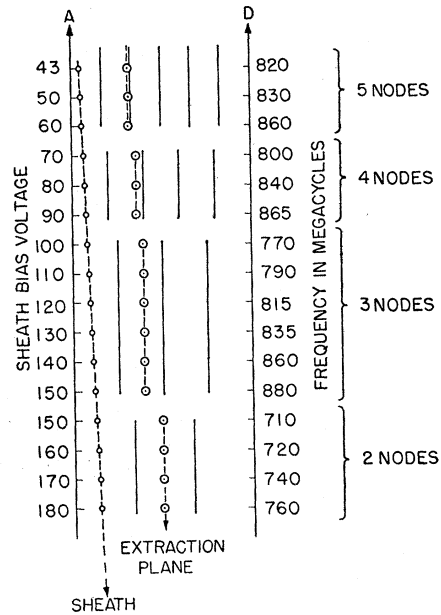


Fig. 6. Calculated positions of the sheath and extraction planes superimposed on a diagram of the positions of the nodes of various standing waves observed.

which modulated and demodulated the electron beam and provided a feedback mechanism to sustain the oscillation. Our experiments indicate the existence of a standing wave, set up in the plasma, which provides the necessary mechanism for maintaining the oscillations. It is apparent from calculation of the phase velocity of the components of the standing wave that it is not electromagnetic in origin. The experimental evidence indicates the oscillation is a longitudinal pressure wave set up in the plasma electrons.