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The Measurement and the Calculation of the Liquid Helium Vapor Pressure-Temperature Scale from 1° to 4.2°K†

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Measurements between 1.2 and 4.2°K on a paramagnetic salt using a mutual inductance bridge apparatus indicate errors in the presently accepted liquid helium temperature scale amounting to 12 millidegrees at the λ point. These data, together with those of Kistemaker, are found to be in good agreement with a temperature scale calculated from the other thermodynamic properties of helium. The calculated scale is further confirmed by a comparison between the calculated and observed latent heat of vaporization. Accepting as correct the boiling point determination of Keesom and Clusius, the estimated accuracy of the proposed temperature scale is 1.5 millidegrees below the λ point and 3 millidegrees above the λ point.

I. INTRODUCTION

The presently accepted temperature scale in the liquid helium region was adopted[1] in 1948. Between 1.6 and 4.2°K, this scale is based upon the vapor pressure measurements of Schmidt and Keesom[2] (1937 scale) using a helium gas thermometer calibrated at liquid hydrogen temperatures. Below 1.6°K the scale follows the formula derived by Bleaney and Simon[3] which was experimentally verified by Bleaney and Hull[4] using a magnetic thermometer.

Measurements by Kistemaker and Keesom[5] of the isotherms of helium gas indicated the possibility of errors in the 1937 scale of Schmidt and Keesom. Subsequently Kistemaker[6] extended the range of these measurements and at the same time made measurements of the vapor pressure using a helium gas thermometer. These results were in reasonable agreement with the formula of Bleaney and Simon below 1.6°K but disagreed with the 1937 scale above 1.6°K. A maximum deviation of -0.013°K was found at the λ point. Unfortunately, these measurements were not sufficiently extensive above the λ point and could not, therefore, be conveniently incorporated in the 1948 scale.

In connection with an experiment on the polarization of nuclei in manganese ammonium sulfate hexahydrate (MAS), it was desirable to evaluate the specific heat and Weiss constant of this material. From measurements to determine the latter constant, it became apparent that rather large irregularities in the susceptibility temperature data were present. These could logically arise from a pecularity in the magnetic properties of MAS, instrumental errors, or from errors in the temperature scale. After a series of experiments designed to distinguish these possibilities, we attribute the irregularities to the latter effect and interpret our results as showing essential agreement with Kistemaker's scale below T_λ and that there are in addition small errors in the 1948 scale above the λ point.

These results are shown to be in good agreement with the vapor pressure calculated from the known thermodynamic properties of helium. Finally, the latent heat of vaporization is calculated and found to agree satisfactorily with the experimental measurements.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The vapor pressure of liquid helium was measured with either a mercury or an oil manometer. The manometers were of sufficiently large bore (25 mm) to make the meniscus correction negligible. The column heights were measured with an accuracy of 0.05 mm using a sliding vernier cursor on a steel scale. In the usual practice, the pressure was measured at the top of the cryostat over the helium bath and a small correction was applied for the gas flow pressure gradient in the Dewar flask. In all cases the dimensions of the pressure measuring apparatus were large enough to make thermomolecular pressure corrections negligible. In the helium I region a correction for the hydrostatic pressure gradient was made using the liquid densities measured by Kamerlingh Onnes and Boks.[7] Below the λ point the temperature gradient in the liquid is negligible because of the large thermal transport in helium II, so that the latter correction does not apply. For convenient reference to the 1948 scale, the observed pressures were corrected to a standard gravity (980.7 cm sec^{-2}) and 20°C.

The temperature of the liquid helium bath was varied by controlled pumping of the helium vapor. After the pressure was reduced to the desired value, the pumping speed was adjusted until the temperature was essentially stationary. The usual thermal drift rate while making observations was about 0.5 millidegree

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[2] G. Schmidt and W. H. Keesom, Physica **4**, 963, 971 (1937).

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[6] J. Kistemaker, Physica **12**, 272, 281 (1946).

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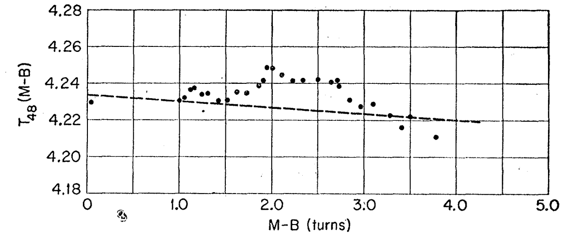

FIG. 1. The relative susceptibility of manganese ammonium sulfate hexahydrate. The straight line corresponds to a Weiss constant of 3 millidegrees.

per minute. To avoid nonequilibrium thermal distributions while in the helium I region, the sequence of observations was always toward lower temperatures. Measurements below the λ point were made from either direction according to convenience.

The temperature of the bath was obtained through the measurement of the magnetic susceptibility of a sample of paramagnetic salt in contact with the bath. This salt in the shape of a 12-mm diam sphere of compressed powder was supported in a plastic holder and located in the central region of a set of mutual inductance coils mounted in the liquid nitrogen bath. In all cases the paramagnetic sample was shielded from high-temperature radiation through the glass apparatus or down the sample tube. The Dewar coils were connected with a variable mutual inductance and a null detector into an alternating-current bridge arrangement in which the dial reading of the variable mutual inductance is proportional to the susceptibility of the sample. The measurements were made at a frequency of either 165 cps or 500 cps with an oscillating magnetic field at the sample of 3-gauss amplitude. The details of the mutual inductance bridge apparatus are reported elsewhere.[8]

With a paramagnetic sample whose susceptibility follows a Curie-Weiss law, the mutual inductance readings M at the bridge box will have the temperature dependence, $M-B=A/(T+\Delta)$, where B is the residual unbalance of the Dewar coils in the absence of a sample, A is proportional to the Curie constant, and Δ is the Weiss constant. Since B changes somewhat with each cooling of the apparatus, it is necessary to infer its value in each experiment. This is most satisfactorily done by measuring M with the sample in equilibrium with the nitrogen bath before helium is transferred. Since $M-B$ is relatively small at 77°K, rather approximate values of A and Δ are then sufficient to accurately determine B.

The results of the first experiment (I) with MAS are shown in Fig. 1. If the salt obeys the Curie-Weiss law and the 1948 vapor pressure-temperature scale T_{48} is correct, then $T_{48}(M-B)$ should be a linear function of $M-B$, with slope Δ and intercept A. This is not the case, but rather there are considerable deviations from a linear behavior.

Since the estimated uncertainties in our measurement of $M-B$ and T_{48} are, respectively, 0.05 and 0.005 percent at 4.2°K and 0.02 and 0.2 percent at 1.2°K, most of the deviation is well outside the limit of normal experimental error. The irregularities might, however, result from the anomalous behavior or errors in the measurement of any of the following: (a) the mutual inductance bridge apparatus; (b) the paramagnetism of MAS; (c) the vapor pressure; (d) the helium temperature scale. To resolve these possibilities, 3 additional experiments were carried out.

The first possibility was ruled out by simply repeating experiment I with the same sample of MAS. Although the values of A in the two runs agreed within 0.15 percent, the values of B differed by about $A/4$. For the same values of $M-B$ and T_{48}, therefore, the bridge reading M was vastly different in the two experiments. The results of experiment II, as shown in Table I, have the same behavior as those in experiment I; so it is concluded that the anomaly does not originate in the electrical measurements. The observed pressures in Table I have been corrected for gravity, temperature, manometer back pressure, and the gas-flow pressure drop in the cryostat. The correction in the second column is for the hydrostatic head of liquid nitrogen or helium over the sample. Additional liquid helium was added after the 2.47°K point, which accounts for the larger correction at the next lower points. The depth corrections in this second experiment were somewhat smaller than in the first, so the agreement of the results has some bearing on the validity of this correction.

Although possibility (b) was deemed quite unlikely in view of the many studies of MAS, it nevertheless seemed desirable to conduct an experiment that would conclusively eliminate this possibility. To this purpose experiment III was performed. The paramagnetic salt was iron alum, $FeNH_4(SO_4)_2\cdot 12H_2O$; the other procedures were identical with those in I and II. The results are shown in Fig. 2. In spite of the relatively large Weiss constant (~0.030°K), the results are seen to be entirely consistent with those of the previous experiments.

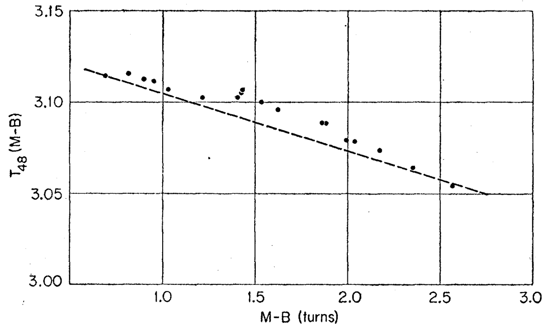

FIG. 2. The relative susceptibility of iron ammonium alum. The indicated Weiss constant is 32 millidegrees.

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[8] Erickson, Roberts, and Dabbs, Rev. Sci. Instr. (to be published).

Table I. Susceptibility—vapor pressure data for MAS in experiment II. The correction in the second column is for the hydrostatic pressure head.

P_{obs} mm Hg, 20°C	Correction mm Hg	P mm Hg, 20°C	T_{48} °K	M Dial turns	$M-B$	$T_{48}(M-B)$	No. of obs
737.5 N_2	30.0	767.5	77.44	−1.3656	0.0546	4.228	
737.5	2.3	739.8	4.184	−0.4109	1.0093	4.2229	3
542.7	2.1	544.8	3.883	−0.3316	1.0886	4.2270	6
259.5	2.1	261.6	3.263	−0.1232	1.2970	4.2321	5
128.7	2.05	130.75	2.791	0.0930	1.5132	4.2233	3
70.6	2.05	72.65	2.471	0.2917	1.7119	4.2301	4
61.05	2.5	63.55	2.407	0.3373	1.7575	4.2303	2
39.07	2.45	41.52	2.218	0.4888	1.9090	4.2342	1
37.99		37.99	2.182	0.5252	1.9454	4.2449	1
37.70		37.70	2.179	0.5275	1.9477	4.2440	2
10.61		10.61	1.758	0.9900	2.4102	4.2371	2
4.56		4.56	1.552	1.3051	2.7253	4.2297	2
7.50		7.50	1.668	1.1205	2.5457	4.2462	2
5.90		5.90	1.611	1.2120	2.6320	4.2402	3
3.455		3.455	1.492	1.4034	2.8236	4.2128	3
1.936		1.936	1.381	1.6298	3.0500	4.2120	2
1.142		1.142	1.290	1.8361	3.2563	4.2006	2
0.590		0.590	1.192	2.1071	3.5273	4.2045	2
0.410		0.410	1.142	2.2532	3.6734	4.1950	2

There remained the more difficult problem of showing that the pressure measurements and corrections properly described the conditions at the sample. In the customary procedure, as employed in the previous experiments, the salt was enclosed in a glass envelope tube and held in thermal contact with the bath by means of helium exchange gas. As the sample temperature was independent of the amount of exchange gas for pressures greater than 15μ, it seems highly improbable that the sample temperature could have been different from that of the bath. However, the hydrostatic depth corrections were quite large (~0.025°K) in the region immediately above the λ point. Since the simply calculated correction assumes no heat conduction or convection in the helium bath, this was considered a possible source of large error. It must be noted, however, that no change in the depth correction procedure would affect the results below the λ point.

To clarify this situation a fourth experiment was performed using a fresh sample of MAS. Here the capsule tube was again in thermal contact with the bath, but the sample was immersed barely below the surface in helium liquid condensed within the sample tube and the vapor pressure was measured in the sample tube as well as over the bath. The sample tube was large enough (18 mm) so that the pressure gradients due to refluxing condensation in the upper tube should have been negligible. Thus, the capsule tube pressure together with a quite small depth correction should fairly represent the pressure at the sample. The observations in experiment IV demonstrated that the previous technique of pressure measurement and correction is quite appropriate. Thus the observed anomalies must arise from errors in the temperature scale.

Because of the parameters A and Δ, it is not possible with the data available to establish uniquely a temperature scale. To do this it would be necessary to determine these parameters from observations at higher temperatures, e.g., with liquid hydrogen. However, the data do permit of an accurate interpolation (or extrapolation) based on any 2 points in the observed interval of temperature. The validity of the temperature scale resulting will then be determined primarily by the correctness of these adopted points. Following this procedure we have adopted as correct the 1948 scale temperatures at 4.2 and 1.3°K. For each experiment the parameters A and Δ can then be calculated, and with these and $M-B$ a magnetic temperature T_m is obtained. The results are given in Fig. 3 where the difference between the accepted temperature and the magnetic temperature is shown as a function of T_{48}. Kistemaker's suggested correction below the λ point is shown as a dashed curve, and it is seen to be quite consistent with these results. The deviation around 3.5°K indicates that the 1937 correction to the 1932 scale was too large in this region. It is of interest that the values of the Weiss constant for the two different samples of MAS, although very small, were of opposite sign. We conclude that the Weiss constant of a perfectly spherical specimen of powdered MAS is probably negative but very small in this temperature region, and that the measured Weiss constant in any actual sample is determined more by the aggregate state and surface condition than by the intrinsic properties of MAS.

It is noted that in each of these experiments the deviation just above the λ point is greater than that just below the λ point. This apparent discontinuity was consistently 3–5 millidegrees. Since a vapor in thermodynamic equilibrium with its liquid cannot possess such a discontinuity, this behavior must originate in some dynamic mechanism, e.g., a slight superheating of the boiling liquid in the He I region.

According to the usual criterion for the determination of the λ point, the vanishing of thermal gradients in the liquid, we observe the λ pressure to be 38.05

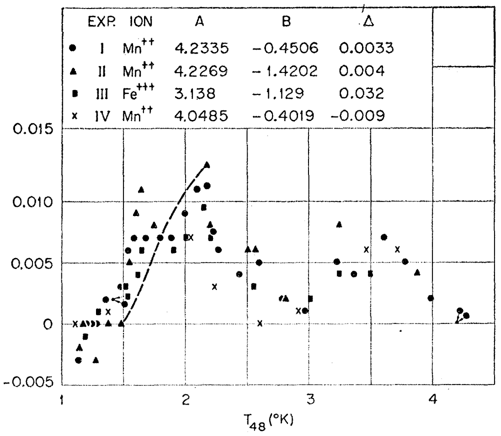

FIG. 3. The deviation of the magnetic temperature data from the 1948 temperature scale. Kistemaker's proposed correction is given by the dashed curve.

±0.05 mm Hg (20°C), in agreement with the result reported by Long and Meyer.[9]

III. CALCULATION OF THE VAPOR PRESSURE AND LATENT HEAT

From the thermodynamic properties of liquid helium and a knowledge of the virial coefficients of the vapor, it is possible to calculate the vapor pressure and latent heat as a function of the temperature. Calculations of this type have been made by Bleaney and Simon[3] and by Kistemaker.[6] We repeat these calculations using more recent data on the liquid specific heat and the fundamental atomic constants, and employing a more exact evaluation of the virial correction terms.

If a liquid and its vapor are in equilibrium, then their Gibbs free energies are equal. Thus[10]

$$RT\left\{\ln\frac{P}{T^{5/2}}-i_0-\phi(v)\right\}-L_0=\int_0^T V_l\left(\frac{dP}{dT}\right)dT-\int_0^T S_l dT, \quad (1)$$

where $i_0=\ln(2\pi m)^{3/2}k^{5/2}h^{-3}$ is the chemical constant; $\phi(v)=2B/v+3C/2v^2-\ln(Pv/RT)$ is the virial correction in which $Pv/RT=1+B/v+C/v^2$; L_0 is the latent heat of vaporization at $T=0$; V_l and S_l are, respectively, the volume and entropy of the liquid.

For the values of atomic constants given by Bearden *et al.*,[11] $m=6.6456\times10^{-24}$ gram, $k=1.38039\times10^{-16}$ erg/deg, $h=6.62509\times10^{-27}$ erg sec, whence $i_0=12.244$ cgs, or with P in mm Hg at 20°C, $i_0'=2.1940$. This is somewhat smaller than the previous value of the chemical constant for helium.

Kistemaker and Keesom[5] have measured the virial coefficients of helium below 2°K and have found them to be consistent with the adopted values[12] for higher temperatures. The second virial term is conveniently described by $B=15.31-386.5/T$ cm^3/mole; the third virial coefficient has been taken from a graphical plot of the measured values. Having values for the virial coefficients, it is then a simple matter to solve the cubic equation for the mole volume of the vapor at a given pressure and temperature and then to evaluate the correction term $\phi(v)$.

The liquid volume is obtained from the data of Kamerlingh Onnes and Boks,[8] and dP/dT from the 1948 temperature scale.[1] The liquid entropy is evaluated from an integration of a smooth curve through the specific heat data of Dana and Kamerlingh Onnes,[13] Keesom and Clusius,[14] Keesom and Miss Keesom,[15] Hull, Wilkinson, and Wilks,[16] and Kramers, Wasscher, and Gorter.[17]

There is insufficient knowledge of the interatomic forces in liquid helium to make an accurate calculation of L_0 from first principles. L_0 must therefore be derived either from Eq. (1), using observed $T-P$ data, or from an expression equivalent to (1) involving the measured latent heat of vaporization. Since the latent heat measurements are not sufficiently extensive nor accurate for this determination, we use the smoothed T_m-P data and calculate L_0 values from Eq. (1). The validity of the T_m scale is then demonstrated by the consistency of the L_0 values.

The results from this calculation are summarized in Table II. Values of L_0 derived from the 1948 scale are also given, and it is seen that they are much less consistent than the values obtained with T_m. Considering the T_m data around 2°K and those above 3°K as most valid, the suggested value of L_0 is 59.50 joules/

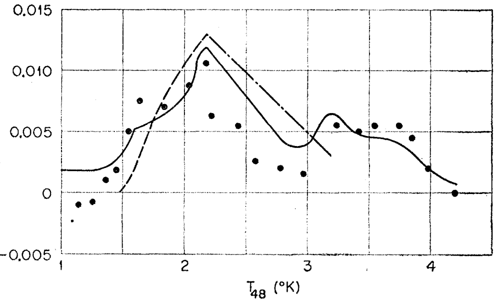

FIG. 4. Deviations from the 1948 temperature scale. The solid line is for the calculation according to Eq. (1). The points are the averaged data shown in Fig. 3. The broken line represents the correction consistent with the measurements of Kistemaker and of Abraham, Osborne, and Weinstock.

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[10] See J. E. Mayer and M. G. Mayer, *Statistical Mechanics* (John Wiley and Sons, Inc., New York, 1940), p. 292.

[11] Bearden, Earle, Minkowski, and Thompsen, "Values of the Atomic Constants as of April 1953" (unpublished).

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[14] W. H. Keesom and K. Clusius, Communs. Kamerlingh Onnes Lab. Univ. Leiden 219e (1932).

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[16] Hull, Wilkinson, and Wilks, Proc. Phys. Soc. (London) **A64**, 379 (1951).

[17] Kramers, Wasscher, and Gorter, Physica **18**, 329 (1952).

TABLE II. Calculated values of the latent heat of varpoization at $T=0$.

P mm Hg, 20°C	T_m °K	T_{48} °K	v cc/mole	$-RT\phi(v)$ joules/mole	$\int_0^T S_l dT$ joules/mole	$\int_0^P V_l dp$ joules/mole	L_0 joules/mole T_m	L_0 joules/mole T_{48}
751.7	4.2	4.200	234.2	9.02	22.91	2.95	59.52	59.52
617.0	4.0	4.002	291.1	7.70	20.20	2.38	59.50	59.57
501.0	3.8	3.805	358.2	6.55	17.65	1.90	59.47	59.63
400.0	3.6	3.605	443.5	5.50	15.25	1.50	59.50	59.65
314.3	3.4	3.405	552.3	4.57	12.99	1.16	59.50	59.67
242.3	3.2	3.206	694.3	3.77	10.86	0.90	59.53	59.73
181.5	3.0	3.001	895	3.00	8.86	0.67	59.61	59.65
133.2	2.8	2.802	1162	2.42	6.99	0.49	59.59	59.67
93.6	2.6	2.602	1573	1.85	5.25	0.34	59.68	59.77
63.4	2.4	2.406	2190	1.36	3.65	0.23	59.64	59.88
40.4	2.2	2.207	3214	0.95	2.22	0.14	59.69	60.01
38.00	2.17	2.182	3382	0.88	2.02	0.13	59.47	60.00
31.65	2.1	2.110	3955	0.76	1.63	0.11	59.53	59.99
24.00	2.0	2.008	5007	0.61	1.18	0.08	59.50	59.89
12.65	1.8	1.807	8684	0.35	0.58	0.05	59.46	59.83
5.80	1.6	1.607	1.70×10^4	0.19	0.26	0.02	59.42	59.79
2.175	1.4	1.402	4.03×10^4	0.08	0.10	0.01	59.55	59.68
0.626	1.2	1.200	12.0×10^4	0.03	0.03		59.64	59.64
0.120	1.0	1.000	51.7×10^4	0.01	0.01		59.58	59.58

mole. This is a somewhat lower value than that given by Bleaney and Simon[3] or by Kistemaker,[6] the difference resulting primarily from the smaller chemical constant employed here. Adopting $L_0=59.50$ joules/mole, one calculates a $P-T$ relation from Eq. (1). The deviation of this calculated temperature from the 1948 scale is shown as the solid line in Fig. 4; also shown are the average data points from Fig. 3, and Kistemaker's correction is shown by the broken line. The dashed curve above the λ point is that which Abraham, Osborne, and Weinstock[18] have taken from Kistemaker's data, and which seems reasonably confirmed by their measurements of the vapor pressure of He^3.

The latent heat of vaporization can now be evaluated on a basis consistent with the calculated $P-T$ function according to the expression,

$$L/T=-R\{\ln(P/T^{5/2})-i_0-5/2\} -R\phi(v)-RT(\partial\phi/\partial T)_P-S_l, \quad (2)$$

where the second and third terms are the viral corrections to the ideal gas entropy. The results for this calculation are given in Table III, and compared with the experimental observations of Dana and Kamerlingh Onnes[19] in Fig. 5. Kistemaker's calculation of the latent heat function is given by the broken line in Fig. 5.

IV. DISCUSSION

Below the λ Point

In this region the calculated $P-T$ and latent heat functions agree very well with the observations and with Kistemaker's results. The disagreement at the lowest temperatures between the observed and calculated temperatures, as shown in Fig. 4, results of course from the adoption of the 1.3°K point on the 1948 scale. According to the calculated scale, this point is in error by about -1.5 millidegrees. Renormalizing the T_m data to this new low-temperature point would give a correction to the T_m points amounting to 0.0005° at 3°K, 0.001° at 2.2°K, and 0.0017° at 1.0°K. This would in general give an even better agreement between the T_m points and the calculated curve.

Up to the λ point the gas and liquid corrections to the ideal vapor pressure are sufficiently small to be represented by simple analytic expressions. The vapor pressure is then described by the formula,

$$\log P_{mm}(20°\mathrm{C})=-\frac{3.1071}{T}+2.5\log T+2.1940 -5.00\times10^{-4}T^6+\frac{0.0522}{T}\int_0^T \exp[-20(2.20-T)^2] \times dT+0.448T^{\frac{1}{2}}e^{-7.15/T}. \quad (3)$$

TABLE III. Calculated values of the latent heat of vaporization of liquid helium as a function of temperature.[a]

T °K	S_g joules/mole deg S_{ideal}	Virial correction	S_g	S_l joules/mole deg	L joules/mole
4.2	37.56	4.07	33.49	13.97	81.98
4.0	38.19	3.41	34.78	13.13	86.60
3.8	38.94	2.95	35.99	12.38	89.72
3.6	39.61	2.56	37.05	11.65	91.44
3.4	40.43	2.24	38.19	10.98	92.51
3.2	41.24	1.97	39.27	10.31	92.67
3.0	42.35	1.70	40.65	9.67	92.94
2.8	43.50	1.46	42.04	9.04	92.40
2.6	44.85	1.18	43.67	8.37	91.78
2.4	46.44	0.92	45.52	7.58	91.06
2.2	48.35	0.66	47.69	6.56	90.49
2.17	48.68	0.64	48.04	6.15	90.90
2.1	49.49	0.55	48.94	5.06	92.15
2.0	50.78	0.45	50.34	3.90	92.88
1.8	53.94	0.25	53.69	2.23	92.63
1.6	58.01	0.14	57.87	1.13	90.78
1.4	63.30	0.06	63.24	0.52	87.81
1.2	70.38	0.02	70.36	0.18	84.22
1.0	80.29	0.01	80.28	0.06	80.22

[a] S_g is the gas entropy, given by the first three terms on the right-hand side of Eq. (2).

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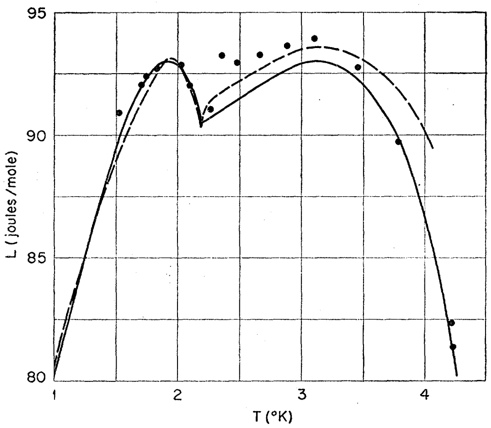

FIG. 5. The latent heat of varpoization of liquid helium. The data points are the measurements of Dana and Kamerlingh Onnes. The solid curve is calculated from Eq. (2), and the dashed curves follows the calculation of Kistemaker.

The fourth and fifth terms in this formula result from the liquid entropy integral, and the last term represents the virial correction and liquid volume integral. In this formula the λ pressure of 38.05 mm corresponds to the temperature 2.170°K.

We estimate the uncertainty in the last 3 terms of Eq. (3) to be one percent, and in L_0 to be ± 0.05 joule/mole. This gives rise to an uncertainty in the calculated temperature scale amounting to 1.5 millidegrees at T_λ and 0.7 millidegrees at 1°K.

Above the λ Point

Although the calculated and observed functions agree quite satisfactorily in this region also, it is interesting to note the seemingly consistent departures in both the $P-T$ and latent heat function in the interval 2.2–3.2°K. The largest potential source of error in these calculations results from the scatter of the specific heat measurements above the λ point. However, it is not possible to modify the liquid entropy in such a manner as to improve the agreement of the calculation with both the $P-T$ and latent heat data, and yet be consistent with the measured specific heat. Since these departures barely exceed the experimental uncertainty of the data, and there seems to be no methodical way of improving the agreement, it is difficult to assess the possible origin or significance of the disagreement.

If the error in the liquid and gas correction terms is again taken to be one percent then the resulting uncertainty in the calculated *absolute* temperature amounts to 11 millidegrees at 4.2°K and 5 millidegrees at 3°K. The T_m data have, therefore, a somewhat greater significance than the calculated scale in this region, for, if the susceptibility data are corrected according to the calculated scale below the λ point, then an extrapolation of T_m according to a Curie-Weiss law for the paramagnetic salt determines the 4.2°K point within 5 millidegrees of the accepted value. Since this is not so precise as the determination of the normal boiling point by Schmidt and Keesom we prefer to continue the adoption of their point on the absolute scale. The calculated scale is then of the nature of an interpolation between the λ temperature of 2.170°K and the normal boiling temperature of 4.216°K. The adoption of the latter point considerably reduces the uncertainty in the liquid entropy determination, and as a result the *relative* temperature is given by the calculated scale with an estimated accuracy of 3 millidegrees. As shown in Fig. 4, this is in agreement with the magnetic temperature data as well as the correction curve adopted by Abraham, Osborne, and Weinstock.[18] It seems reasonable, therefore, to propose that the temperature interpolation given by the calculated scale is correct to 3 millidegrees in the region above the λ point.

The validity of the calculated scale is further demonstrated by the agreement between the calculated and observed latent heat of vaporization at the higher temperatures, as shown in Fig. 5. Whereas Kistemaker's calculation of the latent heat suggested rather large errors in the experimental observations around 4°K, the results here are in good agreement with the data. The discrepancy in the two calculations originates in the manner of evaluating the virial corrections in Eq. (2). We have calculated $\phi(v)$ and $d\phi/dT$ in a manner exact through the third virial coefficient, whereas the more usual procedure is to expand the virial correction as a power series of the pressure and use only the first few terms.

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