the Dirac Hamiltonian must be enlarged by the addition of a term which does not commute with m_0.

We introduce an operator O such that

$$[m_0,O]=i\hbar/c^2. \tag{5}$$

With the aid of this operator we can now generalize Dirac's Hamiltonian to give (4). We write

$$H=c\boldsymbol{\alpha}\cdot\mathbf{p}+\beta(m_0+QO/c^2)c^2. \tag{6}$$

We must now investigate the operator O. We see first of all from (5) that it has the dimensions of a time. Secondly, we note that it plays the same conjugate role to m_0c^2 that $\mathbf{p}$ does to $\mathbf{r}$ and H to the time. It appears reasonable, therefore, to identify O with the proper time τ of the moving particle. This means that the rest mass and proper time of a particle are canonically conjugate variables and therefore do not commute.

Since the proper time of a moving particle is a measure of the line element, the introduction of an uncertainty relationship of the form (5) is equivalent to saying that it is impossible to introduce an exact metric geometry for a system whose rest mass is accurately known. This does not seem unreasonable since an exact geometry presupposes the possibility of localizing a physical particle (e.g., an electron) at any desired point of our frame of reference, and any attempt to do this would bring with it the creation of pairs. This would result in an indeterminancy in the rest mass of our original particle.

Let us now consider a particle which is under observation between the two space-time points (x_k') and (x_k''). The proper time associated with this observation is given by

$$\tau^2=-\frac{1}{c^2}\sum_{k=1}^{4}(x_k'-x_k'')^2. \tag{7}$$

From this we obtain

$$\tau=\pm\frac{i}{c}\sum_{k=1}^{4}\gamma_k(x_k'-x_k''), \tag{8}$$

where the γ_k are the 4×4 Dirac matrices. We thus obtain from (6):

$$H=c\boldsymbol{\alpha}\cdot\mathbf{p}+\beta\left[m_0\pm i\frac{Q}{c^3}\sum_k\gamma_k(x_k'-x_k'')\right]c^2. \tag{9}$$

It is interesting to note that this form of the Hamiltonian has a formal similarity to that suggested by Pais.[4]

Since under ordinary conditions the rest mass of a particle such as an electron is constant, it is clear that the time-average value of Q must vanish and (1) then represents the instantaneous fluctuations of rest mass of the particle. This is obviously connected with the virtual creation and destruction of pairs in the neighborhood of the particle produced by the virtual emission of photons.

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Artificially Produced Negative Heavy Meson*

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Negative heavy mesons, of mass about $1000m_e$, are known in cosmic rays,[1,2] but the energies of the causative particles are unknown. An experiment for finding out if such mesons are produced in bombardment of a beryllium target by the Cosmotron's 2.2-Bev protons was undertaken, and one such particle has now been found.

The experimental arrangement is sketched in Fig. 1. Negative particles ejected from the target A are analyzed by the magnetic field of the accelerator, leave the vacuum chamber E through the $\frac{1}{16}$-in. aluminum window F, pass through a collimator C in a lead housing B, and enter a stack D containing 2-inch$\times$3-inch 400μ G5 Ilford emulsions with their planes horizontal. The particles arriving at the stack were ejected between 2° and 17° to the direction of the proton beam P, traverse a distance A to D of 235 cm, enter the stack nearly parallel to the emulsion plane with a horizontal angular spread of 6°, and cover the momentum interval of 310–380 Mev/c. Particles heavier than $700m_e$ come to rest in the stack; endings were looked for in a region in which particles of about half the proton mass should stop. Loss of heavy particles from an emulsion layer by multiple scattering was compensated by embedding pairs of plates in material with the same range-energy relationship as emulsion.

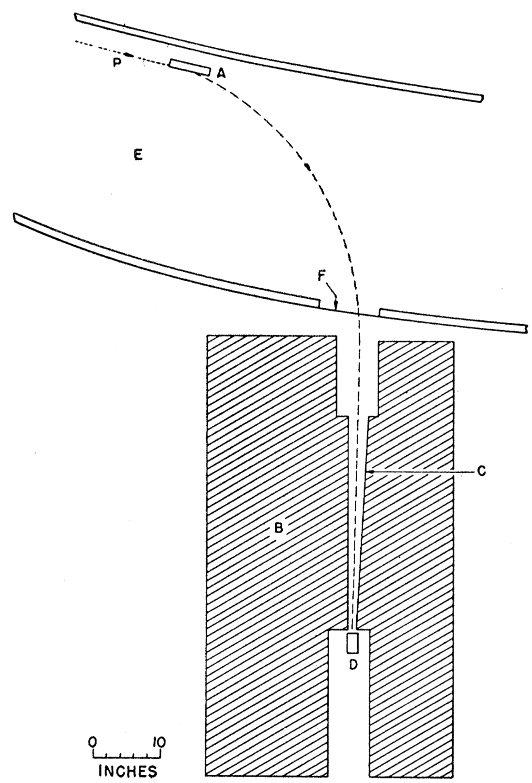

Fig. 1. Schematic diagram of experimental arrangement. Top view.

In a scanned area of 28.2 cm² there were observed (besides neutron and conventional σ stars) 2000 stars with 2 or more prongs caused by the fast pions of the incoming beam and also the star shown in Fig. 2. The particle making track 1 (3.6 mm long in the emulsion) moves in the direction of the incoming beam and must have traversed a total of 43 mm in the stack to the end of its range. Track 2 has a grain count less than 1.5 minimum ($E_{\text{kin}}>0.3$ mc²); it is too short (110μ), steep, and distorted to identify the particle. Track 3 is 840μ long, stops in the emulsion, and, according to multiple scattering, is probably due either to a triton or an alpha particle. The spatial angle between tracks 2 and 3 is 112°.

Measurements of gap length *vs* range[3] and of multiple scattering *vs* range both verify that the particle of track 1 is incoming to the star. The gap measurements give the mass of the particle as $1050\pm150m_e$ and the scattering measurements as $1200\pm300m_e$; both values were obtained by comparison with proton and pion tracks, and the quoted errors include the errors of the proton and pion measurements. From the range within the stack and the momentum, the mass of the particle is $1080\pm220m_e$. These results are consistent with the masses reported by Peters[2] for negative

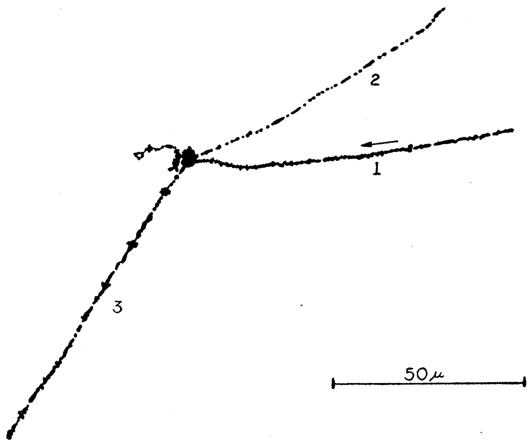

FIG. 2. Trace of projection of heavy meson star. Short curly track is typical of an Auger electron.

heavy mesons and overlap the values given for τ and K mesons[2–4] and also the larger masses found by Daniel *et al.*[5]

The target-to-star path corresponds to 1.2×10^{-8} sec for the heavy meson in its rest system. If the mean life of such mesons were as low as 1 millimicrosecond, it can easily be shown (using cosmic ray evidence that the ratio R of heavy mesons to pions created is less than 0.1) that the probability of observing 1 heavy meson in our experimental arrangement would be less than 0.01. If, on the other hand, the mean life of 8 millimicroseconds indicated for cosmic ray heavy mesons[6] is adopted, the observed proportion of 1 heavy meson star to 2000 pion stars (a proportion that may, of course, change markedly with more observations) would correspond to a value of about 10^{-4} for R.

The creation process of this meson cannot be analyzed unambiguously because of the thickness of the target; the heavy meson could have lost as much as 100 Mev in the beryllium and, besides, could have been made by a fast secondary (pion or proton).

We are particularly indebted to Dr. William H. Moore for his essential share in setting up the experiment, and to Dr. Morton F. Kaplon for initiating us in the gap-length determinations. We take great pleasure in acknowledging the invaluable aid given by our scanning and processing staff, R. Mildred Bracker, Margaret D. Carter, Barbara M. Cozine, Jeanne C. Dennison, Mary C. Hall, Alice C. Lea, and Jacqueline D. Leek.

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Radiative Corrections in Positronium

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ALL corrections of order α^5 to the $n=2$ energy levels of positronium have been evaluated using the relativistic two-body equation[1,2] with techniques recently proposed by several authors.[3,4] The corrections arise from three sources:

(1) The self-energy and vacuum polarization effects analogous to those which yield the main part of the Lamb shift in hydrogen.

(2) Recoil and retardation corrections to the Breit interaction. Similar contributions have been calculated by Salpeter[5] for hydrogen. In hydrogen such terms account for less than 0.05 percent of the Lamb shift. The effect is much larger in positronium since the mass dependence is of the form $m_1^2m_2^2/(m_1+m_2)^3$.

(3) Corrections to the α^4 virtual annihilation interaction[6–8] and α^5 annihilation processes.[9] These are of course peculiar to a particle-antiparticle system.

Items (1) and (2) above have been calculated with arbitrary masses m_1 and m_2 so as to determine the mass dependence in general. In the limit m_1 small and m_2 large, we confirm Salpeter's numerical result[5] and Arnowitt's hyperfine structure calculation.[10] In the limit $m_1=m_2$ we confirm the results of Karplus and Klein.[9] We also agree with them on item (3). The energy shift due to items (1) and (2) is given by the expression below:

For the $2S$ state:

$$\Delta E_1=\frac{\alpha^5\mu^2}{m_1+m_2}\frac{1}{8\pi}\left\{-\frac{4}{3}+\frac{8}{3}\left[\frac{25}{12}+\ln\frac{2Ry_\infty}{\alpha k_0(2,0)}\right]\right.$$
$$+2\left[\ln\alpha+\frac{7}{4}+\frac{4}{3}(1-\ln2)\right.$$
$$\left.-(m_1^2-m_2^2)^{-1}(m_2^2\ln\eta_2-m_1^2\ln\eta_1)\right]$$
$$\left.-2\langle 2S^2-3\rangle m_1m_2(m_1^2-m_2^2)^{-1}\ln\frac{m_1}{m_2}\right\},\quad(1)$$

$$\Delta E_2=\frac{\alpha^5\mu^2}{m_1+m_2}\frac{1}{6\pi}\left\{\frac{m_1^2+m_2^2}{m_1m_2}\left(\frac{5}{6}-\frac{1}{5}^{\dagger}-\ln\frac{\alpha^2k_0(2,0)}{Ry_\infty}\right)\right.$$
$$\left.-\frac{m_2}{m_1}\ln\eta_2-\frac{m_1}{m_2}\ln\eta_1+\frac{1}{2}\langle 2S^2-3\rangle\right\}.\quad(2)$$

For the $2P$ state:

$$\Delta E_1=-\frac{\alpha^5\mu^2}{m_1+m_2}\frac{1}{8\pi}\left(\frac{7}{18}-\frac{8}{3}\ln\frac{Ry_\infty}{k_0(2,1)}\right),\quad(3)$$

$$\Delta E_2=\frac{\alpha^5\mu^2}{m_1+m_2}\left\{\frac{-1}{6\pi}\left(\frac{m_1^2+m_2^2}{m_1m_2}\right)\ln\frac{k_0(2,1)}{Ry_\infty}\right\}$$
$$+\frac{\alpha^2}{m_1m_2}\left\{\frac{1}{4\pi}\left\langle\left(\frac{m_2}{m_1}\boldsymbol{\sigma}_1+\frac{m_1}{m_2}\boldsymbol{\sigma}_2\right)\cdot\frac{\mathbf{L}}{r^3}\right\rangle\right.$$
$$\left.+\frac{1}{2\pi}\left\langle\frac{\mathbf{S}\cdot\mathbf{L}}{r^3}-\frac{1}{2}\left[\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{r^3}-\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{r}\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^5}\right]\right\rangle\right\},\quad(4)$$

where

$$\mu=\frac{m_1m_2}{m_1+m_2},\quad \eta_1=\frac{m_1}{m_1+m_2},\quad \eta_2=\frac{m_2}{m_1+m_2},\quad \mathbf{S}=\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2),$$

and $k_0(2,0)$ and $k_0(2,1)$ are given by Bethe, Brown, and Stehn.[11] $\langle A\rangle$ denotes the expectation value of A. The dagger in Eq. (2) signifies that in positronium, this vacuum polarization term is 1/10.

In positronium, there is no accidental α^4 degeneracy. Hence, the α^5 shifts must be added to the much larger α^4 fs and hfs in order to obtain the corrected level spacing. Table I gives the α^4 corrections determined by Ferrell[8] and the α^5 contributions we have found.

TABLE I. Additions to the nonrelativistic $n=2$ level of positronium, Mc/sec.

Order	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_2$	$^3P_1$	$^3P_0$
α^4	−18135	7413	−3536	−981	−5360	−10835
α^5	357	232	−3	1	−5	−16
Total	−17778	7645	−3539	−980	−5365	−10851