low energies the mean free path becomes longer than the average nuclear radius, and thus below about 40 Mev even the largest nucleus should be fairly transparent. Consequently, for low pion energies, the dependence of production cross sections upon atomic mass number, A, may be expected to deviate from the usual $A^{\frac{2}{3}}$ law and, except for Coulomb effects, approach a linear dependence. It should also be noted that Heckman and Bailey[20] find an A dependence for the backscattering of 30-Mev pions.

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Elementary Particles and the Lamb-Retherford Line Shift*

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According to the principles of contemporary quantum electrodynamics, the existence of a charged particle field of any kind results in a vacuum polarization contribution to the Lamb-Retherford line shift of an amount proportional to the square of the charge and inversely proportional to the square of the mass of the field particle. On the basis of the present agreement between theory and experiment with respect to the line shift in hydrogen, one may conclude that no singly charged particles of spin-$\frac{1}{2}$ with mass less than four electron masses, other than the electron and positron, can exist without spoiling this agreement. Similar reasoning argues against the existence of singly charged particles of spin-0 with mass less than twice the electron mass. For doubly charged particles these limits are quadrupled. The assumptions involved in these conclusions as well as some experimental evidence are briefly discussed.

THE polarization of the vacuum[1] as a consequence of the existence of the electron-positron field seems now to be a well established phenomenon since it appears to be essential to obtain agreement between experiment[2] and theory[3] for the Lamb-Retherford line shift in the hydrogen atom. To this line shift the vacuum polarization term contributes -27.13 Mc/sec which is some fifty times the discrepancy at present between theory (including fourth order corrections) and experiment.[4] It is characteristic of the theory of the line shift that the existence of any charged particle field which is invariant under charge conjugation will also make a corresponding contribution to the line shift from its vacuum polarization. The contribution of such a particle field is always of the same sign[5] (negative) and, for given spin of the particle, is proportional to the square of the charge and inversely proportional to the square of the mass of the particle. It follows that if one assumes that the present structure of quantum electrodynamics with its renormalization procedure is substantially correct, then one may draw conclusions as to the very existence of other elementary charged particles than those already known, based simply on the present accuracy of agreement of the theory (which neglects the existence of any particles other than electrons and positrons) and experiment for the line shift.

One can see immediately that all well-established charged particles known at present, other than electrons and positrons, will, because of their large mass, make a contribution to the line shift many orders of magnitude below the present difference between theory and experiment[4] which is of the order of 0.5 Mc/sec (= theoretical − observed line shift). However, the existence of elementary singly-charged spin-$\frac{1}{2}$ particles (other than electrons and positrons) with mass less than four electron masses would more th n double the present discordance between theory and experiment. One may thus conclude that the present agreement between theory and experiment argues against the existence of such particles. For doubly-charged particles of spin-$\frac{1}{2}$ this limit of four electron masses would be quadrupled to sixteen electron masses.

For a spin-0 charged particle field[6] theory predicts a vacuum polarization effect which is only one-eighth that

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for a spin-$\frac{1}{2}$ particle of the same charge and mass. Thus we also have an argument against the existence of singly charged spin-0 particles of mass less than about two electron masses and doubly-charged particles of mass less than about eight electron masses. For spin-1 particles one encounters a divergence in the vacuum polarization calculation[6] which precludes any trustworthy conclusion from being drawn.

It is appropriate to examine briefly the assumptions underlying the above conclusions. These are (1) the validity of the principle of charge conjugation invariance, (2) the assumption that all spin-$\frac{1}{2}$ particles are properly described by the Dirac equation and that all spin-0 particles are properly described by the Klein-Gordon equation (or its equivalent), and (3) the assumption that the current scheme of quantum electrodynamics is valid down to distances of the order of the Compton wavelength of the particles whose contribution to the vacuum polarization is under consideration. Unfortunately very little may be positively asserted with respect to these assumptions. Our principal evidence for charge-conjugation invariance stems from the existence of positive-negative partners for the electron, mu meson, and pi meson. However, the antiparticle to the proton is still unknown, but there is also no evidence against its existence. For the various recently discovered particles the evidence is still unclear.

With respect to the second assumption little more can be said but that no other satisfactory representations than the Dirac and Klein-Gordon equations have been proposed. It is also not unlikely that should it be possible to construct other representations for spin-$\frac{1}{2}$ and spin-0 particles, the vacuum polarization effects associated with such particles would still be of the same order of magnitude as that predicted in the familiar cases; hence only small quantitative changes in our conclusions might be expected. It should be mentioned that the calculations of vacuum polarization which have been employed in the above are actually made under the assumption that the spin-$\frac{1}{2}$ particle has a normal Dirac magnetic moment. The inclusion of a Pauli moment term in the equations leads to a divergence in the vacuum polarization similar to that encountered in the case of spin-1 particles.

Relative to the third assumption above we should mention that the charge distribution associated with the vacuum polarization resulting from particles of mass M extends over a region of dimensions $\hbar/Mc$ about an external point charge. Now we have every reason to believe that with the present renormalization procedure, the quantum electrodynamics of the electron is valid at least to the point where the electron explores regions of the order of 10^{-13} cm in extent. Thus we should expect the theory to be valid in its applications to the cases in which we obtained our conclusions. Some question might however be raised concerning the applicability of the theory to the case of vacuum polarization associated with particles of mesonic mass for which the polarization charge would be distributed over regions of the order of 10^{-13} cm in linear dimensions.

It should finally be remarked that there exists some meagre and controversial experimental evidence relative to the conclusions we have drawn above. There have repeatedly been reported observations of positive particles, with masses of several times the electron mass, in the neighborhood of certain negative beta-emitting nuclei (particularly P^{32}) which are energetically incapable of emitting positrons.[7] On the other hand, some observers have not been able to confirm their existence. Without taking any position in this controversy we can only suggest that our conclusions would favor these particles, *if* they do indeed exist, being bosons unless their masses have been underestimated. It is probably only a fortuitous circumstance that the real existence of charged spin-0 particles of mass 2.5 times the electron mass would make up the present difference between theory and experiment on the hydrogen line shift—a difference which has been suggested to be a little large to be accounted for by (at present, unknown) sixth order terms in the theoretical calculation.[4] If only because of its interest relative to the theory of the line shift, it would seem to be of importance to clarify the experimental situation relative to the existence of these particles.

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