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Production of Positive Mesons in Heavy Nuclei

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The data on the A dependence of low-energy meson production by protons on heavy nuclei are explained by taking into account the following effects: (1) the energy degeneration of the incident proton in nuclear matter, which tends to make meson production possible only in the "front" of the struck nucleus; (2) the subsequent re-absorption of the meson by the nuclear matter it traverses. This last effect is further strengthened by the reflectivity of the Coulomb barrier.

INTRODUCTION

RECENT experiments[1] on the production of positive mesons in collisions of protons with heavy nuclei indicate that for fixed proton and meson energies, $\sigma(A)/A^{\frac{2}{3}}$, and, hence, $\sigma(A)/A$, decrease with A, the number of nucleons in the target nucleus. This suggests that neither a direct "independent-particle" nor a "surface" production mechanism is in itself sufficient to explain the data. It is the purpose of this note to point out that the decrease in the "meson-production efficiency" with A is a consequence of the rapid energy degeneration of the incident proton in nuclear matter, which makes meson production possible only in the "front" of the struck nucleus, together with a meson-attenuation effect which becomes more important with increasing A.

Let us first describe our model of the nucleus. We consider the nucleus as composed of nucleons moving in a self-consistent nuclear potential. This model has been explored by Johnson and Teller[2]; it leads to roughly constant neutron and proton densities, and the consequence of interest to us is that the presence of the Coulomb potential tends to push the protons away from the center as well as from the boundary thus confining them to a region of somewhat smaller radius compared to the nucleus as a whole[3] (see Fig. 1).

PROTON ENERGY DEGENERATION

Let us consider a proton of 240–340 Mev entering a heavy nucleus. The proton has a mean free path for elastic scattering[4] which can be computed by using the free-particle cross sections $\sigma_{pp}\simeq 25$ mb and $\sigma_{np}\simeq 40$ mb in this energy range. The approximate constancy of these values over a fairly wide energy range allows us to ignore the dependence of the scattering mean free path on the momentum distribution of the nucleons forming the target nucleus, provided that the "width" of such a distribution $\lesssim 200$ Mev/c.[5] We thus obtain

$$\lambda_s=(4\pi/3)R^3[Z\sigma_{pp}+N\sigma_{np}]^{-1}$$
$$=RA^{\frac{2}{3}}[0.26Z+0.42N]^{-1}. \quad (1)$$

Now, on the average, after traveling a distance λ_s, the proton will undergo an elastic collision and (again on the average) lose 50 percent of its energy in the process. Hence for a proton of 240 Mev only one zone of depth λ_s is available for meson production, whereas for protons of 340 Mev all particles in the first zone, and roughly half the particles in the second zone (those whose momentum vector have a positive component pointing towards the incoming proton), are available for meson production.

Experiments[6] indicate that the process

$$p+p\rightarrow\pi^{+}+d$$

is a great deal more probable than the process

$$p+n\rightarrow\pi^{+}+2n.$$

The large suppression of the latter process is partly due

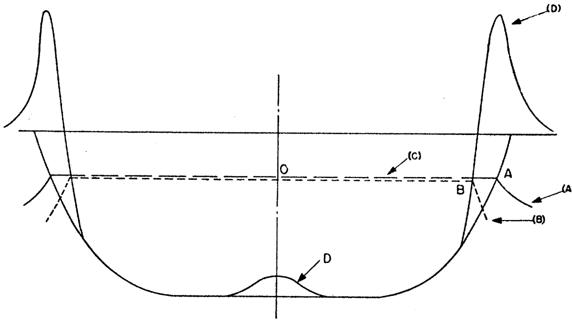

FIG. 1. A schematic representation of potentials acting on the nucleons in a nucleus. $OA=R$, the nuclear radius; $OB=P$, the charge distribution radius; (A) is the neutron tail; (B), the proton tail; (C), the highest proton and neutron levels; and (D), the Coulomb potential.

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[1] R. Sagane and W. Dudziak, Phys. Rev. **92**, 212 (1953); D. Clark, Phys. Rev. **87**, 157 (1952).

[2] M. H. Johnson and E. Teller (to be published).

[3] The fact that the radius of the nuclear-charge distribution appears to be smaller than the nuclear radius is indicated by recent measurements of x-rays from mesonic atoms [Proceedings of the Third Annual Conference on High-Energy Physics at Rochester, 1952 (Interscience Publishers, New York, 1953)] and by the scattering of 100-Mev electrons from heavy nuclei.

[4] The concept of a mean free path is meaningful here since, at the energies being considered, the de Broglie wavelength of the proton is much smaller than the inter-nucleonic distance so that diffraction effects are expected to be negligible.

[5] Assuming that the "width" of the momentum distribution is insensitive to changes in A, this condition is met for all A, since it is satisfied for deuterium and carbon. See, Cladis, Hess, and Moyer, Phys. Rev. **87**, 425 (1952).

[6] For example, J. Carothers and C. G. André, Phys. Rev. **88**, 1426 (1952). These experiments have been extended to lower energies by W. Dudziak (private communication).

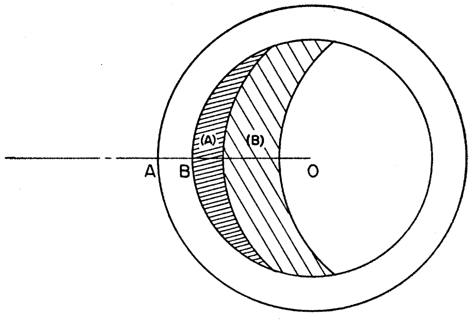

FIG. 2. The meson-production zones in the target nucleus. $OA=R$, the nuclear radius; $OB=P$, the charge distribution radius; (A) first zone; and (B) second zone.

to the exclusion principle: in the latter case there are three particles in the final state and since the two identical neutrons tend to keep away from each other the phase space available to each particle is appreciably reduced. A much larger suppression of the process can be expected if conservation of isotopic spin is assumed together with a strong interaction in the $T=\frac{3}{2}$ state.[7]

We thus assume that only the protons in the target nucleus are effective in producing positive mesons, and so we write

$$\begin{aligned} &\text{for 240-Mev protons:} \quad \sigma=\sigma_0 Z_1, \\ &\text{for 340-Mev protons:} \quad \sigma=\sigma_0'(Z_1+\tfrac{1}{2}Z_2), \end{aligned} \tag{2}$$

where Z_i is the number of protons in the ith zone. We remark here that as pointed out above the protons are confined to a somewhat smaller volume by the Coulomb barrier, and this serves to reduce the number of protons in the first zone (Fig. 2). We find that

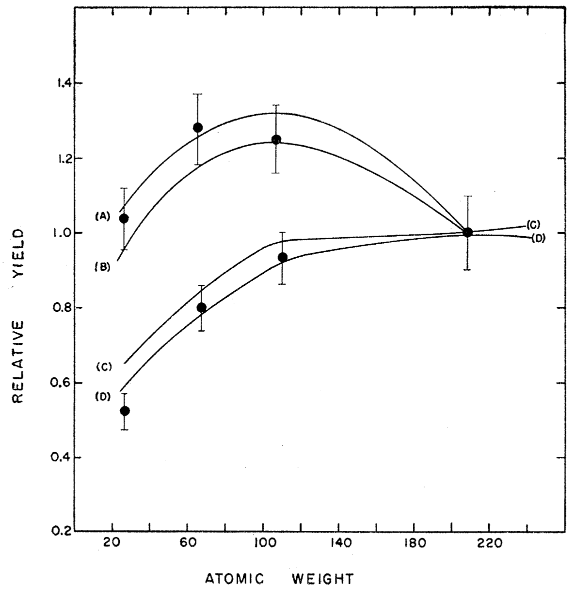

FIG. 3. Comparison with the data of Dudziak and Sagane. The solid lines are the theoretical curves. (A) $E_\pi=13$ Mev, $f=\frac{1}{2}$; (B) $E_\pi=13$ Mev, $f=\frac{1}{3}$; (C) $E_\pi=27$ Mev, $f=\frac{1}{2}$; (D) $E_\pi=27$ Mev, $f=\frac{1}{3}$. All curves and the experimental points are normalized to unity at Pb.

$$Z_1(\lambda_s)/Z=(R/P)^3\{\tfrac{1}{2}(P/R)^2[x/R+\lambda_s/R-1] \\ +\tfrac{1}{2}(x/R)[(\lambda_s/R)-1]+\tfrac{1}{2}(P/R)^3-\tfrac{1}{4}(x/R)^2(\lambda_s/R)\}, \tag{3}$$

and

$$(Z_1+Z_2)/Z=Z_1(2\lambda_s)/Z,$$

where

$$(x/R)=[(P/R)^2-(1-\lambda_s/R)^2](R/2\lambda_s),$$

R is the nuclear radius (taken to be $1.5\times 10^{-13}A^{\frac{1}{3}}$ cm), and P is the radius of the "charge distribution." We assume that $P=0.90R$.[8]

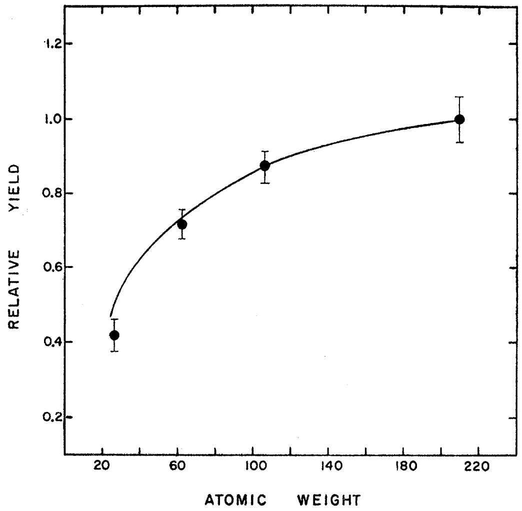

FIG. 4. Comparison with the data of Clark. The solid line is the theoretical curve for $E_\pi=40$ Mev, $f=\frac{1}{3}$. The curve and the experimental points are normalized to unity at Pb.

MESON ABSORPTION

Once a meson is produced, it travels through nuclear matter for a distance d. The chance that the meson will undergo absorption may be described by a mean free path for absorption λ_a of the order of $6-7\times 10^{-13}$ cm,[9] and the probability of the meson's reaching the boundary is $\exp(-d/\lambda_a)$. The presence of the Coulomb barrier gives rise to a reflection probability $(1-T)$. Consequently the probability that a meson finally gets out of the nucleus without being absorbed is

$$\frac{T\exp(-d/\lambda_a)}{1-(1-T)\exp(-2R/\lambda_a)}\simeq T\exp(-d/\lambda_a). \tag{4}$$

We have calculated T for $l=0$ waves only, since it turns out that the centrifugal barrier contributes less

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[7] M. A. Ruderman, Phys. Rev. **88**, 1427 (1952). Note that the conclusions regarding the process $p+n\rightarrow\pi^-$ must apply identically to the process $p+n\rightarrow\pi^+$ by charge symmetry.

[8] A charge distribution radius 10 percent smaller than the nuclear radius is suggested by an interpretation of the experiments of Fitch and Rainwater on x-rays from μ-mesonic atoms. See L. N. Cooper and E. M. Henley, Phys. Rev. **91**, 480 (1953).

[9] Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).

than 10 percent to the transmission coefficient. The factor $\exp(-d/\lambda_a)$ was also computed with λ_a chosen equal to 6.5×10^{-13} cm and an average $d=fR$. The calculation was carried out with $f=\frac{1}{2}$ and $f=\frac{1}{3}$,[10] for Al, Cu, Ag, and Pb. A comparison with the data of Dudziak and Sagane and of Clark is made in Figs. 3 and 4, respectively.

CONCLUSION

As Figs. 3 and 4 show, the agreement of theory with experiment is quite satisfactory. In view of uncertainties in the value of λ_a and f, the experiments cannot be said to confirm or deny the conclusions of other experiments which suggest a radius of the charge distribution smaller than the nucleus as a whole. With a choice $P=R$, the theoretical curves would be shifted upwards by 5–10 percent.

In view of difficulties in obtaining Coulomb wave functions for attracting particles, no detailed comparison with the π^- data is here made. The author is grateful to Professor Sagane and Dr. Dudziak for communicating to him the results of their experiments prior to publication. Discussions with Dr. J. Lepore, Dr. W. Heckrotte, Dr. M. H. Johnson, and Dr. T. Kinoshita are gratefully acknowledged.

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[10] Small values of f were chosen for the experiments under consideration since the mesons were observed at 90° and ∼140°. For mesons at 0° one would have to take $f\simeq1.0-1.5$.

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Inelastic Scattering of 220-Mev π^- Mesons in Emulsions*

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The nuclear interaction of 220-Mev negative π mesons has been studied in photographic emulsions. A particular type of inelastic scattering events has been analyzed. Among 1960π^- meson interactions, 17 events are observed where the π meson transferred a large fraction of its initial energy to a single proton. The incident π meson, the scattered π meson, and the proton are nearly coplanar in most of these cases. However the three tracks are not exactly coplanar indicating that the meson collision was not with a hydrogen nucleus. The 17 events are interpreted as the interaction of the incident π meson with a single proton of the target nucleus.

I. INTRODUCTION

SEVERAL studies have been made in photographic emulsions[1–3] of the nuclear interaction of π mesons of energy less than 250 Mev. The phenomena most frequently observed are (a) "stars" produced by the absorption of the incident π meson, (b) "stars" produced by inelastic scatterings, (c) elastic scatterings, and (d) "stops" due in part to charge exchange. The nucleus, in part or as a whole, is involved in the collision in events of types (a), (b), and (c). In most cases, the energy lost by the meson is shared among several nucleons. For this reason it is difficult to obtain information on the initial interaction of the π meson in the nucleus.

Several events have been found where nearly all of the energy lost by the π meson was transferred to a single proton. The analysis of these events is described.

II. PROCEDURE

Ilford G-5, 600- and 1000-micron thick plates were exposed in the 220-Mev negative π-meson beam of the University of Chicago cyclotron. The general features of the meson beam have been described elsewhere.[4] The plates were area-scanned for π-meson interactions with a total magnification of 250×. An appreciable fraction of the elastic scatterings and inelastic scatterings where no charged nuclear particles were ejected was undoubtedly missed. Undoubtedly, a large fraction of the "stops" were also not seen. Although it is somewhat difficult to estimate the scanning efficiency for various types of events, it is believed that a very high percentage of the events were found where one or more charged nuclear particles are associated with the event. The scanning time that would be required to find the particular events of interest would be excessively long if scanning "along the track" or under a higher magnification were employed. It is estimated, by using the data of Morrish[3] on the number of interactions of various types, that about 70 percent of the total number of interactions were observed.

All events were studied where a lightly ionizing track

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[1] Bernardini, Booth, and Lederman, Phys. Rev. **83**, 1075 (1951); Bernardini, Booth, and Lederman, Phys. Rev. **83**, 1277 (1951); G. Bernardini and F. Levy, Phys. Rev. **84**, 610 (1951).

[2] H. Bradner and B. Rankin, Phys. Rev. **87**, 547 (1952); B. Rankin and H. Bradner, Phys. Rev. **87**, 553 (1952).

[3] A. H. Moorish, Phys. Rev. **90**, 674 (1953).

[4] R. L. Martin, Phys. Rev. **87**, 1052 (1952).