between the first excited level to the ground state, while the 0.73- and 0.94-Mev quanta come from higher levels. If it is assumed that these higher levels have high spin values, it would not be expected that they would be excited strongly by neutrons of the energy used.

The same argument is applicable in the case of nickel. The radioactive decay of Co<sup>60</sup> gives two gamma of energy 1.33 and 1.17 Mev, but the neutron bombardment of Ni<sup>60</sup> produces only one of these, corresponding to the excitation of the first excited state.

It is seen that the pulse-height distributions obtained by neutron scattering in lead, iron, chromium, and nickel are successfully interpreted in terms of gamma rays arising from nuclear energy levels that are known to be excited by other means. This serves as a partial corroboration of the validity of the measurements, and enhances the credence of the results for bismuth, where there is no previous information concerning the energy level structure. The four gamma rays from bismuth give evidence of the presence of a number of excited states which should be observable by other methods of excitation.

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# Elastic Scattering of Alpha Particles by Neon\*

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Differential cross sections for the elastic scattering of alpha particles by neon have been measured at four angles for alpha energies from 2 to 4 Mev. Absolute values of the cross section are good to  $\pm 4$  percent. The angles chosen correspond to center-of-mass angles such as to simplify assignments of angular momentum to the scattered resonance wave. Because no nuclear spins are involved, the angular momentum of the scattered wave also fixes the parity and angular momentum of the compound state.

Thirteen resonances were observed. Eleven are attributed to virtual states of Mg<sup>24</sup> and two are assigned to virtual states of Mg<sup>20</sup>. The experimental data were analyzed in terms of the Wigner-Eisenbud formalism to determine the additional level parameters:  $E_r$ ,  $\gamma \lambda^2$  and  $\Delta_{\lambda}$ . The laboratory energies of the (Ne<sup>20</sup>+ $\alpha$ ) resonances in Mev, and the total angular momentum and parity assignments of the virtual states are: 2.488(1-), 2.573 (0<sup>+</sup>), 2.652 (2<sup>+</sup>), 2.903 (0<sup>+</sup>), 3.062 (1<sup>-</sup>), 3.184 (2<sup>+</sup>), 3.548 (3<sup>-</sup>), 3.780 (1<sup>-</sup>), 3.801 (2<sup>+</sup>), 3.839 (4<sup>+</sup>), and 3.923 (2<sup>+</sup>). Similarly, the assignments for (Ne<sup>22</sup>+ $\alpha$ ) resonances are 3.245 (3<sup>-</sup>) and 3.418 (3<sup>-</sup>).

The resonances above 3.0-Mev bombarding energy were investigated for competing reactions. No competing reaction with cross sections greater than about one percent of the elastic value were observed.

## I. INTRODUCTION

'HE elastic scattering of alpha particles by nuclei of zero spin provides a very simple method for classifying the resonant states of the compound nucleus. The method has been discussed in some detail by Cameron<sup>1</sup> and Hill<sup>2</sup> who have studied virtual states of Ne<sup>20</sup> and O<sup>16</sup> by alpha scattering on O<sup>16</sup> and C<sup>12</sup>. Very briefly stated, the method consists of measuring the elastic scattering cross section as a function of alpha energy at those angles for which the various low-order Legendre polynomials have zeros. The angles at which the resonance scattering vanishes serve to identify clearly the partial waves involved in the resonance scattering. Since no nuclear spins are involved, the

-mangular momentum of the resonance partial wave fixes uniquely the J and parity of the compound nuclear state. The Wigner-Eisenbud dispersion formalism<sup>3,4</sup> may be used to extract the following additional level parameters,  $E_{\lambda}$ ,  $\Delta_{\lambda}$ ,  $(E_r = E_{\lambda} + \Delta_{\lambda})$ , and  $\gamma_{\lambda}^2$ , where  $E_{\lambda}$  is the "characteristic" energy of the level,  $\Delta_{\lambda}$  is the level shift,  $E_r$  is the resonance energy, and  $\gamma_{\lambda}^2$  is the "reduced" width of the level.

The present experiment has been undertaken to investigate the nuclear energy levels of Mg<sup>24</sup> by observing resonances in the elastic scattering of alpha particles by Ne<sup>20</sup>. Earlier work on the scattering of alpha particles by neon<sup>5,6</sup> was done with natural radioactive sources. The energy and angular resolution was too poor to permit observation of resonances below 5-Mev alpha energy. Brubaker studied the reaction from 4 to 7 Mev and observed broad anomalies in the scattering cross-

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<sup>&</sup>lt;sup>1</sup> National Science Foundation Fellow. <sup>1</sup> J. R. Cameron, Phys. Rev. **90**, 839 (1953). <sup>2</sup> R. W. Hill, Phys. Rev. **90**, 845 (1953).

<sup>&</sup>lt;sup>8</sup> E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29 (1947).
<sup>4</sup> T. Teichmann and E. P. Wigner, Phys. Rev. 87, 123 (1952).
<sup>5</sup> G. Brubaker, Phys. Rev. 54, 1011 (1938).
<sup>6</sup> W. Riezler, Ann. Physik 23, 198 (1935).



FIG. 1. Schematic diagram of the gas scattering chamber and the gas circulating system. A—differential pumping column, B—roughing pumps (250 liters/sec), C—oil trap, D—liquid air trap (with magnesium turnings for better heat contact), E—gas purifier (magnesium turnings in inconel tube heated to 600°C), -heat exchanger with copper turnings, G-collector cup, H-proportional counters with slit systems.

section curves above 5.0 Mev but made no attempt at analysis. Riezler's work was too scanty to permit any conclusions regarding resonances to be drawn.

## **II. EXPERIMENTAL PROCEDURE**

The experimental method has been described previously.<sup>1,2,7</sup> Monoergic ( $\pm 0.05$  percent) alpha particles from an electrostatic generator were scattered by neon in the differentially pumped gas scattering chamber (Fig. 1). The elastically scattered alphas were detected by two proportional counters mounted on a turntable and positioned  $114.0^{\circ}$  apart. The energy of the alpha particles was varied from 2.0 to 4.0 Mev and yields were recorded for laboratory angles of 78.7°, 114.2°, 131.8°, and 167.3° with respect to the incident beam. In a small region above 3.5 Mev, additional data were taken at 98.5° and 147.5°. The angles chosen correspond roughly to center-of-mass angles for which the Legendre polynomials of order l=1 to 4 have zeros.

In order to determine the absolute cross section, neon pressure readings were taken. A beam-current integrator was used<sup>8</sup> to shut off the counting circuits as soon as a predetermined charge (20 microcoulombs) had been collected in the collector cup. Data above 3 Mev were taken with 0.0005-mm nickel foils as counter windows. Below 3.0 Mev, alpha particles scattered through 131.8° and 167.3° had such low energies that a 0.00025-mm window had to be used. Propane was employed as a counter gas. The pulseheight spectrum of the counters showed a well-defined group of pulse heights due to elastically scattered alpha particles and some considerably smaller pulses probably due to  $\gamma$  rays. In order to monitor continuously the number of pulses outside the main group, two discriminator-scalers were used for each counter, the

discriminators being adjusted to 30 and 60 percent, respectively, of the pulse height of the main group. The counting rates of the two scalers so set seldom differed by more than one percent. A precision pulser<sup>9</sup> was incorporated in the electronic arrangement to set and check the amplifiers and discriminators.

The neon gas used in the experiment was recirculated by transferring the gas from the exhaust stacks of the forepumps through a purifying stage back to the chamber (Fig. 1). In the purifying stage the gas had to pass over magnesium turnings that were maintained at a near-melting temperature. Tests showed that air leaking into the system was removed efficiently by the purifier. To guarantee neon gas of high purity, frequent mass spectroscopic analyses were made with a Nier-type Consolidated Mass Spectrometer. Nitrogen was the major contaminant but never exceeded 0.2 atom percent. The oxygen content was lower by at least a factor of five. A few other contaminants such as argon were detected but none were greater than 0.1 atom percent. Continuous monitoring of the gas purity was possible by watching the color of the alpha beam in the neon gas. A contamination of several tenths of a percent of air was easily seen as a bluish tinge to the orange color of neon. The neon gas was replaced with fresh gas occasionally and the magnesium was changed once.

The data at  $\theta_{lab} = 78.7^{\circ}$  and  $167.3^{\circ}$  were taken at 3-kev intervals. At  $\theta_{lab} = 114.2^{\circ}$  and 131.8° 3-kev steps were taken at the resonances and 15-kev steps off resonances. High-resolution work was done on all of the resonances by changing the resolution of the cylindrical analyzer from its normal setting of  $\pm 0.05$  percent to  $\pm 0.02$  percent and reducing the chamber pressure so that the target thickness at one of the angles was approximately equal to the energy spread of the beam. Because of the  $\sin\theta$  factor in the thickness of the gas target which the counter sees, the target thickness at  $167.3^{\circ}$  was almost five times as great as that at  $78.7^{\circ}$ for the same gas pressure. Consequently the target thickness at 167.3° was normally larger than the cylindrical analyzer resolution.

#### **III. EXPERIMENTAL RESULTS**

The cross-section values computed from the data are given in Figs. 2 and 3. The drawings include the results of both the survey work and high-resolution work on the resonances. Not all of the off-resonance data for  $\theta_{lab} = 78.7^{\circ}$  and  $167.3^{\circ}$  have been illustrated. Approximately every other point was plotted to avoid overlapping of the circles. All of the pertinent data for  $\theta_{lab} = 114.2^{\circ}$  and  $131.8^{\circ}$  have been illustrated.

The measured yield of elastically scattered particles Y is related to the cross section by

# $\sigma(\theta_{\rm lab}) = Y \sin\theta / NnG.$

 $\theta$  is the scattering angle, N and n are the number of

9 W. C. Elmore and M. Sands, *Electronics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 323.

<sup>&</sup>lt;sup>7</sup> Jackson, Galonsky, Eppling, Hill, Goldberg, and Cameron, Phys. Rev. **89**, 365 (1953). <sup>8</sup> Worthington, McGruer, and Findley, Phys. Rev. **90**, 899

<sup>(1953).</sup> 



FIG. 2. Experimental cross sections for alpha particles on neon from 2.4 Mev to 3.2 Mev.

incident particles and the number of target nuclei per unit volume, respectively, and G is the geometry factor of the counter slit system.<sup>8</sup> The major error in the cross section was due to the uncertainty in Y. The statistical error was usually about  $\pm 3$  percent. The error introduced by the slight dependence of Y upon the discriminator setting of the scalers is estimated to be appreciably less than 1 percent. The uncertainty in  $\theta$  ( $\pm 0.1$  degrees) introduces an error of  $\pm 1$  percent in the  $\sin\theta$  factor at 167.3°. The gas density at the target is known to about  $\pm 1$  percent, the G-factor of the slit system to  $\pm 0.2$  percent.

The major error in the number of incident particles was not given by the accuracy of the current integrator (better than 0.1 percent) but by the uncertainty in the equilibrium ratio of  $\alpha^+/\alpha^{++}$  collected at the Faraday cup. The only experimental data on the equilibrium fraction of singly charged alphas in this energy range is that of Rutherford<sup>10</sup> and is for air. In the present experiment the nickel foil preceding the collector cup determines the  $\alpha^+/\alpha^{++}$  ratio. However, at lower energies it has been shown that the equilibrium charge ratio is not strongly dependent on the nature of the medium<sup>11</sup> (with the exception of the very light elements<sup>12</sup>).

Therefore Rutherford's data for air was used to correct the measured incident charge. The correction varied from seven percent at  $E_{\alpha} = 2.0$  MeV to one percent at 4.0 Mev. Hence even if the equilibrium ratios for nickel and air are slightly different no appreciable error is introduced into the final cross-section data. It is reasonable to assume that the error will be less than  $\pm 1.0$  percent. The arithmetical sum of all uncertainties is 7.0 percent while the rms value of the errors is 3.6 percent. Errors introduced by impurities of the neon gas are not included. Assuming that the impurities scatter classically this error would be of the order of 0.2 percent.

The energy of the alpha particles before entering the chamber was known through the setting of the cylindrical analyzer which was calibrated with the  $Li^{7}(p,n)Be^{7}$  threshold.<sup>13</sup> The calibration was thought to be reliable to  $\pm 0.1$  percent. The major error in the determination of the alpha-particle energy was, however, the uncertainty in the energy loss in the gas. This loss was determined by noting the energy shifts of the resonances as a function of chamber pressure. The correct energy was then found by linear extrapolation to zero pressure. Since the extrapolation method was dependent on energy differences, small fluctuations in energy could affect the final value significantly. The

 <sup>&</sup>lt;sup>10</sup> E. Rutherford, Phil. Mag. 47, 277 (1924).
 <sup>11</sup> M. C. Henderson, Proc. Roy. Soc. (London) A109, 157 (1925).
 <sup>12</sup> E. Snitzer, Phys. Rev. 89, 1237 (1953).

<sup>&</sup>lt;sup>13</sup> Herb, Snowden, and Sala, Phys. Rev. 75, 246 (1949).



FIG. 3. Experimental cross sections for alpha particles on neon from 3.2 Mev to 4.0 Mev.

data from the thirteen resonances showed that the quantity  $\Delta E/\Delta P$  (Mev/cm of oil pressure) fluctuated appreciably over the energy range studied. As a consequence all alpha-particle energies are assigned a  $\pm$ 5-kev uncertainty.

Ne<sup>20</sup> is known to have an excited state at 1.634 Mev and Ne<sup>22</sup> has one at 1.277 Mev.<sup>14</sup> Also the Ne<sup>20</sup>( $\alpha, p$ )Na<sup>23</sup> and  $Ne^{22}(\alpha, p)Na^{25}$  reactions are energetically possible above 2.4 and 3.4 Mev, respectively.<sup>15,16</sup> To see the degree to which these reactions and inelastic scattering compete with the elastic scattering, the counter angle and counter-window foil were selected to allow possible inelastic groups to appear separated from the main group of pulse heights. An oscilloscope camera of the Land type was employed for quick semiquantitative pulse-height analyses. The response of the photographic emulsion to a given number of alpha pulses seen on the scope was noted in order to place an upper limit on unobserved groups. Photographs of the pulse-height distribution were taken for all resonances above 3.0 Mev. There was no evidence of inelastic scattering at any of the resonances checked. An upper limit of one percent of the elastic cross section may be given. At

3.839 and 3.923 Mev, small pulses thought to be caused by gamma-ray electrons were seen. At the 3.923-Mev resonance, a group about one-fourth the height of the main group was seen. The photograph is shown in Fig. 4. This pulse height is expected for protons from the Ne<sup>20</sup>( $\alpha, \phi$ )Na<sup>23</sup> reaction.

# IV. RESULTS OF THE ANALYSIS

Immediate assignments of total angular momenta and parities were possible in most cases by a qualitative examination of the data. Since the angles were chosen to correspond to the zeros of the Legendre polynomials,  $P_1(\cos\theta)$  to  $P_4(\cos\theta)$ , the partial wave responsible for the formation of the compound nucleus was identified by noting the angle or angles at which the anomalies did not appear. The reason for neglecting higher-order partial waves will be discussed later.

Since the main part of the off-resonance cross section is Rutherford scattering, resonance dips in excess of 10 percent could only be associated with anomalies produced by an isotope whose abundance was greater than 10 percent (i.e., Ne<sup>20</sup>). The experimental results were fitted with the theoretical curves from the Wigner-Eisenbud formalism<sup>3,4,17</sup> to give the parameters  $E_r$ ,  $\gamma_{\lambda^2}$ , and  $\Delta_{\lambda}$ , and as a check on level assignments. The interaction parameter, a, was chosen as  $a = 1.4(A^{\frac{1}{3}} + 4^{\frac{1}{3}})$ 

<sup>14</sup> F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 24, 321 (1952).

 <sup>&</sup>lt;sup>(1)</sup> 5C. W. Li, Phys. Rev. 88, 1038 (1952) and Li, Whaling, Fowler, and Lauritsen, Phys. Rev. 83, 512 (1951).
 <sup>16</sup> E. Bleuler and W. Zünti, Helv. Phys. Acta 20, 95 (1947).

<sup>&</sup>lt;sup>17</sup> R. K. Adair, Phys. Rev. 86, 155 (1952).

 $\times 10^{-13}$  cm. The results are given in Table I and illustrated in Figs. 5 and 6 as a comparison of experimental results to theory.

The formalism also provides a sum rule that gives an upper bound to  $\gamma_{\lambda}^2$  of  $3\hbar^2/2\mu a$ , which is often called the "Wigner limit". Since  $\Gamma_{\lambda}/\gamma_{\lambda}^2$  is proportional to the barrier penetration of the l'th partial wave,  ${}^{4}\Gamma_{\lambda}$  is also limited. Since resonances with  $\Gamma_{\lambda} < 1$  kev could not be experimentally detected with the apparatus, the Wigner limit excludes the possibility of high-order partial waves being involved in the observed scattering anomalies. For example, at  $E_r(lab) < 4.0$  Mev, J must be <5 if a Ne<sup>20</sup>+ $\alpha$  resonance is to be seen. No resonances should be resolvable much below 2.0 MeV and J must be  $\leq 2$  to give an experimentally detectable anomaly below  $\sim 2.4$  Mev.

The Ne<sup>20</sup> resonances at 2.488, 2.573, 2.652, 3.184, 3.548, and 3.839 Mev which were assigned J=1, 0, 2,2, 3, and 4, respectively, are not resolved, but evidence available at all four angles of observation supports the quoted assignments. The Wigner limit restricts these resonances to J < 5. Their values for  $\Gamma_{\lambda}$  in Table I are very approximate values calculated on the basis

Oscilloscope Fig. 4. picture of the proportional counter pulses at a bombarding energy of 3.923 Mev and  $\theta_{lab} = 90^{\circ}$ . The group of large pulses results from elastically scattered alpha particles while the smaller pulses are believed to be caused by protons from the  $Ne^{20}(\alpha, p)Na^{23}$  reaction.



of the known target thickness and beam resolution and hence are enclosed in parentheses.

The resonances at 3.245 and 3.418 Mev are both thought to be J=3 levels in Mg<sup>26</sup>. The experimental values for cross sections at the peaks are slightly in excess of the calculated values for both resonances. If Nier's values for the isotopic ratios are used,<sup>18</sup> the discrepancy is about 8 percent but if Hibbs' are used,<sup>19</sup> the discrepancy is only 4 percent. However, the resonances appear to be resolved and the magnitudes and shapes of the experimental cross-section curves are in closest agreement to those predicted from the dispersion theory with the J=3,  $Mg^{26}$  assignment. The resonances at E=3.780 and 3.801 Mev involve scattered p and dwaves, respectively. The experimental widths  $(7\pm 2)$ and  $5\pm 1$  kev) were assigned from observation of the data at  $\theta_{lab} = 78.7^{\circ}$  and  $167.3^{\circ}$  where high beam resolution and a thin target were used. The data and theoretical curves are shown in Fig. 7. The level density in this region is sufficiently high that interference from the tails of neighboring levels has a marked effect on

<sup>18</sup> A. O. Nier, Phys. Rev. 77, 789 (1950).
<sup>19</sup> R. F. Hibbs, U. S. Atomic Energy Commission Report AECU-556, August, 1949 (unpublished).



FIG. 5. Comparison of experimental and calculated cross sections from 2.4 Mev to 3.2 Mev. The calculated cross sections neglect the finite experimental energy resolution.

the scattering cross section. For example the predicted single level scattering maximum at 3.783 Mev and  $\theta_{\rm lab} = 167.3^{\circ}$  is reduced by  $\sim 25$  percent by this interference.

In general the off-resonance cross sections measured in the experiment differ by less than 5 percent from the Rutherford plus hard-sphere cross sections with a the radius of the hard sphere taken as  $a=1.40(A^{\frac{1}{3}}+4^{\frac{1}{3}})$  $\times 10^{-13}$  cm.

A marked deviation of experimental points from calculated values above the 3.923-Mev resonance is suggestive of a resonance slightly above 4.0 Mev. The signs of the deviations are those that would be caused by a resonance with  $J = 1^{-}$ . However, a decrease of the interaction parameter, a, would have the same effect. Considering the expected high density of resonances, it is not unlikely that a neighboring resonance is the cause.

#### V. DISCUSSION OF RESULTS

The energy level diagram (Fig. 8) and Table I summarize the experimental results. While the alpha



FIG. 6. Comparison of experimental and calculated cross sections from 3.2 Mev to 4.0 Mev. The calculated cross sections neglect the finite experimental energy resolution.

Er(lab) (Mev)	Level assignment	Excitation energy, $E_x$ (Mev)	Γ <sub>λ</sub> (lab) (kev)	$\begin{array}{c} \gamma \lambda^2 (\mathrm{c.m.}) \\ (\mathrm{Mev}\text{-cm}) \\ \times 10^{15} \end{array}$	$\frac{\gamma \lambda^2(\text{c.m.})}{3\hbar^2/2\mu a}$	Δ <sub>λ</sub> (c.m.) (Mev)	$\gamma_{\lambda^2(c.m.)/D}$ ×10 <sup>13</sup> , cm
	$Mg^{24}$						·····
3.923 3.839 3.801 3.780 3.548 3.184 3.062 2.903 2.652 2.573 2.488	$2^+$ $4^+$ $2^+$ $1^-$ $3^-$ $2^+$ $1^-$ $0^+$ $2^+$ $0^+$ $1^-$	$12.601 \\ 12.531 \\ 12.499 \\ 12.481 \\ 12.288 \\ 11.985 \\ 11.883 \\ 11.751 \\ 11.542 \\ 11.476 \\ 11.476 \\ 11.405 \\ 1$	$\begin{array}{c} 6\pm 1 \\ (1) \\ 5\pm 1 \\ 7\pm 2 \\ (1) \\ (0.5) \\ 8\pm 2 \\ 10\pm 2 \\ (0.5) \\ (1) \\ (0.5) \end{array}$	$\begin{array}{c} 8.3 \\ (25) \\ 8.3 \\ 7.0 \\ (10) \\ (4.2) \\ 42 \\ 51 \\ (25) \\ (17) \\ (18) \end{array}$	$\begin{array}{c} 0.03 \\ (0.08) \\ 0.03 \\ 0.02 \\ (0.03) \\ (0.01) \\ 0.14 \\ 0.16 \\ (0.08) \\ (0.06) \\ (0.06) \end{array}$	+0.013 +0.026 +0.018 +0.15 +0.29	$\begin{array}{c} 0.22\\ (0.28)\\ 0.22\\ 0.14\\ (0.08)\\ (0.11)\\ 0.84\\ 0.60\\ (0.67)\\ (0.20)\\ (0.20)\end{array}$
	$Mo^{26}$		()	(20)	(0100)		(0.00)
3.418 3.245	3- 3-	13.534 13.388	$3.2 \pm 0.5$ $2.5 \pm 0.5$	34 45	0.11 0.15	+0.070 +0.11	0.57 0.75

TABLE I. Summary of resonances seen by alpha scattering in neon gas.

resonance energy is uncertain by only  $\pm 5$  kev, the corresponding excitation energy in Mg<sup>24</sup> has an additional 12-kev systematic uncertainty arising from the uncertainty of the Ne<sup>20</sup>+ $\alpha$ -Mg<sup>24</sup> energy difference. This energy difference is best calculated by the following cycle: Na<sup>23</sup>(d,p)Na<sup>24</sup>, Na<sup>24</sup> $\rightarrow$ Mg<sup>24</sup>+ $e^-$ , Na<sup>23</sup>(p, $\alpha$ )Ne<sup>20</sup>, H<sup>2</sup>-(H+n), n-H. In order that our Ne<sup>20</sup>+ $\alpha$ -Mg<sup>24</sup> difference corresponds to Li's current mass scale,<sup>15</sup> the adjusted Q's of Table I of Li's paper were used.

The only other experimental information concerning the levels of  $Mg^{24}$  in this same excitation range comes from the  $Na^{23} + p$  reaction. Burling<sup>20</sup> observed  $Na^{23}(p,\gamma)Mg^{24}$  and  $Na^{23}(p,p')Na^{23*}$  resonances for



FIG. 7. Detailed comparison between experimental points and calculated cross-section curve between 3.75 Mev and 3.88 Mev. The calculated cross sections neglect the finite experimental energy resolution.

<sup>20</sup> R. L. Burling, Phys. Rev. 60, 340 (1941).

proton energies from 0.3 to 1.9 Mev. Burling's data below  $E_p \sim 1.1$  Mev overlaps the upper portion of the Mg<sup>24</sup> excitation region studied in the present experiment. For comparison purposes his levels in Mg<sup>24</sup> are included at the right of the energy-level diagram (Fig. 8). It will be noted that more Mg<sup>24</sup> levels are seen by the



FIG. 8. Energy level diagram of  $Mg^{24}$  between 11.4-Mev and 12.6-Mev excitation energy. For comparison the  $Mg^{24}$  levels observed by the  $Na^{23}(p,\gamma)Mg^{24}$  reaction<sup>20</sup> are indicated at the right.

 $Na^{23}+p$  reaction than the  $Ne^{20}+\alpha$  scattering. This result is not surprising since the scattering of the even parity spin zero particles can only involve  $Mg^{24}$  states whose parity is even or odd according as l and hence Jis even or odd. Furthermore isobaric spin one (T=1)levels of  $Mg^{24}$  are forbidden for the alpha scattering but permitted for the  $Na^{23}+p$  reactions. The first T=1state of  $Mg^{24}$  should occur at  $\sim 9.4$ -Mev<sup>21</sup> excitation.

<sup>&</sup>lt;sup>21</sup>  $(Na^{24}-Mg^{24})+(Coulomb energy difference Mg^{24}-Na^{24}) - (n-H) = 5.33+4.65-0.78 = 9.4 MeV.$ 

The density of the T=1 states of Mg<sup>24</sup> should then correspond to that of the excited states of Na<sup>24</sup> which have been experimentally studied by Sperduto and Buechner.<sup>22</sup> Since for these T=1 levels there may be possible shifts of several hundred kilovolts between neighboring isobars, it is not possible to identify which of the Na<sup>23</sup>+p resonances correspond to the Na<sup>24</sup> states.

However, all the Mg<sup>24</sup> levels seen by alpha scattering should be accessible also to the Na<sup>23</sup>+p reaction although the cross section may be low. The first three alpha resonances which overlap Burling's proton data do indeed match within experimental uncertainties the  $(p,\gamma)$  resonances. Such is not the case for the three highest-energy alpha resonances. Perhaps more efficient gamma detectors or observation of the Na<sup>23</sup> $(p,\alpha)$ Ne<sup>20</sup> yield function would permit the detection of these states of Mg<sup>24</sup>.

For each alpha resonance the ratio of  $\gamma_{\lambda}^2$  to the Wigner limit is also given in Table I. If the compound state can be described in terms of an alpha particle in

a potential, then the reduced width should be in the order of the Wigner limit.

In the last column of Table I is given the ratio of  $\gamma_{\lambda}^2$  to the average level spacing, D, of states of the same angular momentum and parity. This ratio is significant in relation to Teichman and Wigner's second sum rule<sup>4</sup> which states that on the average  $\gamma_{\lambda}^2 \approx 3D/2\pi K'$ , where K' is the average wave number of the particle in the nucleus. For the range of excitation of Mg<sup>24</sup> covered in the present experiment it is reasonable to treat K' as a constant and hence  $\gamma_{\lambda}^2/D$  measures the fluctuation of individual  $\gamma^{2's}$  from the average given by the sum rule. The observed extreme fluctuation is about a factor of ten. Most values of the ratio lie within a factor of two of  $\gamma^2/D=0.4\times10^{-13}$  cm. This fluctuation is perhaps smaller than would be anticipated from the discussion of Teichman and Wigner.<sup>4</sup>

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<sup>&</sup>lt;sup>22</sup> A Sperduto and W. W. Buechner, Phys. Rev. 88, 574 (1953).

FIG. 4. Oscilloscope picture of the proportional counter pulses at a bombarding energy of 3.923 Mev and  $\theta_{lab}=90^{\circ}$ . The group of large pulses results from elastically scattered alpha particles while the smaller pulses are believed to be caused by protons from the Ne<sup>20</sup>( $\alpha, p$ )Na<sup>23</sup> reaction.



