PHYSICAL REVIEW VOLUME 93, NUMBER 4 FEBRUARY 15, 1954

Energy Levels in N^{14} from the Scattering of Protons by C^{13}†

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(Received November 2, 1953)

The differential cross section for the elastic scattering of protons by C^{13} has been determined in the energy range from 0.45 to 1.60 Mev at angles of 50, 90, 120, 140, and 160 degrees in the center-of-mass system. Marked anomalies were found in the scattering at 0.55, 1.16, 1.47, and 1.55 Mev corresponding to excited states in N^{14} at 8.06, 8.62, 8.90, and 8.98 Mev. The spins and parities of these states have been determined from a preliminary analysis of the data which also indicates an effect due to the broad resonance at 1.25 Mev in $C^{13}(p,\gamma)N^{14}$ corresponding to an excited state in N^{14} at 8.70 Mev. The assignment for the 8.06-Mev level is $J=1^-$, for the 8.62-Mev level $J=0^+$ and for the broad 8.70 Mev level $J=0^-$. The probable assignment for the 8.90-Mev level is $J=3^-$ and for the 8.98-Mev level $J=1^+$.

The elastic scattering of protons by C^{12} was also measured from 300 kev to 550 kev at angles of every ten degrees in the center-of-mass system from 30 to 160 degrees.

I. INTRODUCTION

FIVE levels were reported by Seagrave[1] in the nucleus N^{14} from a study of the excitation curve of the reaction $C^{13}(p,\gamma)N^{14}$. The spins and parities of these states were not completely determined in this experiment nor in the study of the radiative cascade transitions made by Woodbury, Day, and Tollestrup.[2] In order to obtain more information, we have measured the differential cross section for the $C^{13}(p,p)C^{13}$ at several different scattering angles.

Before measuring the $C^{13}(p,p)C^{13}$ differential cross section we measured the $C^{12}(p,p)C^{12}$ differential cross section. In a previous article Jackson and Galonsky[3] reported the results of a partial wave analysis of the differential cross section for the $C^{12}(p,p)C^{12}$ obtained by Goldhaber and Williamson.[4] Their analysis led to values of the resonance energies and widths that differed somewhat from the proton capture data.[1,5,6] In addition, the experimental and calculated scattering cross section could not be brought into agreement below 1 Mev. We have measured the differential cross section for $C^{12}(p,p)C^{12}$ at thirteen different angles from 300 to 600 kev to investigate these discrepancies at the low energy. Jackson *et al.*[7] have more recently remeasured the $C^{12}(p,p)C^{12}$ differential cross section and analyzed[8] their new data; this new measurement has resolved their previous discrepancies.

This paper describes the experiments on the elastic scattering of protons by C^{13} and C^{12} and presents the data obtained. It also discusses the probable assignments of the levels in N^{14} from a preliminary analysis of the data. A more complete theoretical analysis of the $C^{13}(p,p)C^{13}$ data will be presented in a forthcoming paper.

The 2-Mv electrostatic accelerator recently reconstructed at the Kellogg Radiation Laboratory provided a steady source of protons which is maintained homogeneous to better than 0.05 percent by an 80 degree electrostatic analyzer of 1-meter radius and 1-millimeter entrance and exit slits. The target was placed at the "object" position of the 180-degree, double-focusing, magnetic, proton spectrometer described by Snyder *et al.*,[9] It has been remounted so as to allow a continuously variable scattering angle from 0–160 degrees with the incident beam.

The magnetic field of the spectrometer was measured by a null reading magnetometer similar to that described previously.[10] It consisted of a coil carrying a current, measured by a Leeds and Northrup potentiometer, suspended in the magnetic field. The restoring torque which just balanced the torque produced by the field acting on the coil was produced by a quartz fiber. The indicator was a beam of light reflected from a mirror on the coil to a 920 phototube. The signal from the phototube was converted into 60-cycle ac, the phase of which depended on which half of the phototube received the most light and the amplitude depended on the difference of the illumination of the two halves of the phototube. This signal was amplified and then fed into an amplidyne through a phase detector. The field of the spectrometer generator was controlled by the output of the amplidyne. Thus any error of the field was automatically corrected. It was possible to reproduce the magnetic field measurements to an accuracy of better than 0.1 percent over extended periods of time and to regulate the magnetic field to about 0.02 percent for short periods.

A scintillation counter placed at the "image" position of the magnetic spectrometer was used to count the scattered protons. It consisted of a 931-A photomulti-

† Assisted by the joint program of the U. S. Office of Naval Research and U. S. Atomic Energy Commission.

[1] J. D. Seagrave, Phys. Rev. 85, 197 (1952).
[2] Woodbury, Day, and Tollestrup, Phys. Rev. 92, 1199 (1953).
[3] H. L. Jackson and A. I. Galonsky, Phys. Rev. 84, 401 (1951).
[4] G. Goldhaber and R. M. Williamson, Phys. Rev. 82, 495 (1951).
[5] W. A. Fowler and C. C. Lauritsen, Phys. Rev. 76, 314 (1949).
[6] D. M. Van Patter, Phys. Rev. 76, 1264 (1949).
[7] Jackson, Galonsky, Eppling, Hill, Goldberg, and Cameron, Phys. Rev. 89, 365 (1953).
[8] H. L. Jackson and A. I. Galonsky, Phys. Rev. 89, 370 (1953).
[9] Snyder, Rubin, Fowler, and Lauritsen, Rev. Sci. Instr. 21, 852 (1950).
[10] C. C. Lauritsen and T. Lauritsen, Rev. Sci. Instr. 19, 916 (1948).

plier tube and a scintillating screen prepared by dusting zinc sulfide powder on the end of a Lucite cylinder.

II. EXPERIMENTAL PROCEDURE

The targets used were made by a method developed by Seagrave[1] by cracking C^{13} enriched methyl iodide on a tantalum strip heated to a bright orange. This method was used for making thin targets for the $C^{13}(p,\gamma)N^{14}$ experiments. The targets used in the scattering experiments were made by cracking a thick layer of normal carbon from methane on the heated tantalum strip in order to provide a light element backing, then cracking the C^{13} enriched carbon on top of the normal carbon. This was done in order to conserve the relatively expensive C^{13} enriched methyl iodide. The methyl iodide enriched to 61 percent C^{13} was obtained from Eastman Kodak Company. When the tantalum strip was cooled the carbon layer usually formed a large blister which was then removed from the strip as a foil. One can obtain foils 1 cm$\times$3 cm or larger in this manner. The carbon foils thus obtained were mounted on a copper target blank.

The first targets used for the scattering experiments on C^{12} were made by holding a copper target blank in the flame of benzene until a thick layer of soot was deposited. No difference could be detected in the scattering from the two kinds of targets.

A profile or momentum spectrum curve of the protons scattered by each target was made at a selected energy not near any resonance before using the target for a yield curve to check it for contaminations on the surface.

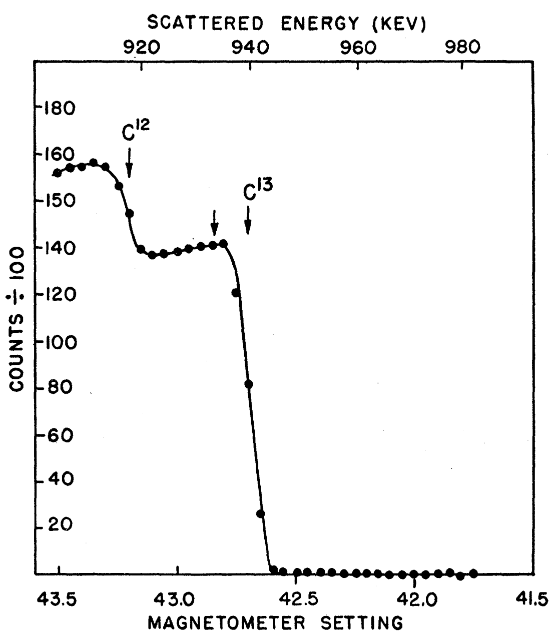

FIG. 1. A profile or $H\rho$ plot for a C^{13} enriched target at a scattering angle of 160 degrees in the center-of-mass system and the proton bombarding voltage of 1272 kev. The arrow indicates the point used for a yield curve.

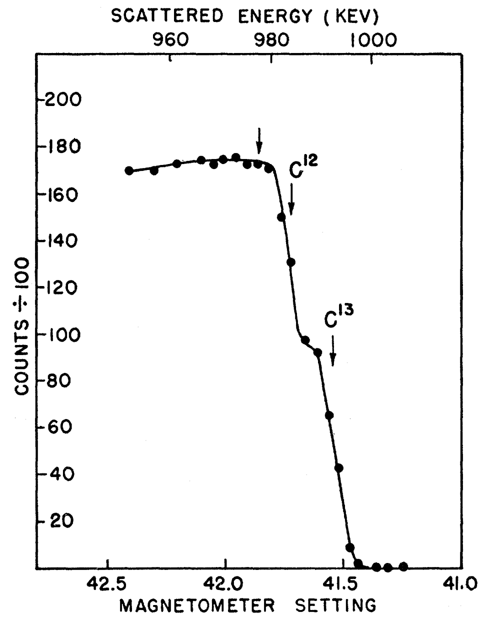

FIG. 2. Profile for a C^{13} enriched target at a scattering angle of 90 degrees in the center-of-mass system and a proton energy of 1150 kev. The separation of the steps from the two isotopes of carbon approaches the resolution of the spectrometer for the slit width employed.

A profile is a curve of the number of counts plotted against the magnetometer setting (inversely proportional to momentum) with the bombarding energy and scattering angle held constant. Figure 1 shows a typical profile at a scattering angle of 160 degrees in the center-of-mass system for $C^{13}(p,p)$. At this angle the protons scattered by the two isotopes differ enough because of the different recoil energy to be easily separated by the spectrometer. Each point of the yield curves at the large scattering angles was taken with the spectrometer field set so as to measure the full number of counts from C^{13} but not to count the protons scattered from C^{12}. Each point represents the scattering by a thin lamina in the thick target which is determined by the energy interval accepted by the spectrometer and the energy loss of the incident and scattered particles in this lamina.

Figure 2 shows another profile at 90 degrees. At this angle the separation of the protons scattered from the two isotopes approaches the resolution of the spectrometer for the slit width employed to give sufficient counts and, hence, the protons scattered from the two isotopes of carbon cannot be separated for the scattering angles of less than 90 degrees. Since the C^{12} and C^{13} scattered protons could not be separated at the small scattering angles the number of counts had to be corrected for the C^{12} present. Both a normal carbon target and C^{13} enriched target were placed on the target backing. The target backing could be raised or lowered so that the two foils could be alternately placed in the proton beam. Each point of the yield curves for

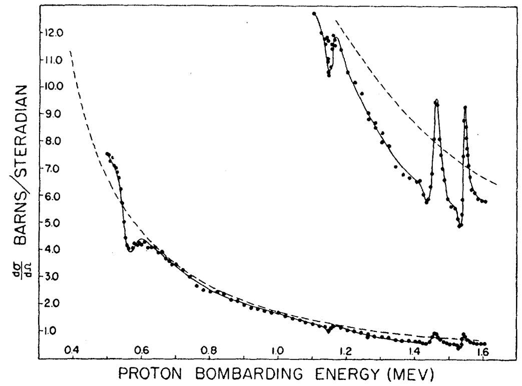

FIG. 3. Differential cross section for the elastic scattering of protons by C^{13} at a scattering angle of 50 degrees in the center-of-mass system. The dotted line is the Rutherford cross section. The solid line is the experimental curve. The upper curve is scaled by a factor of 10.

$C^{13}(p,p)$ was obtained by subtraction using the formula $N=1.65N_1-0.65N_2$, where N is the number of protons that would be scattered from a pure C^{13} target, N_1 is the number of protons scattered from the C^{13} enriched target (61 percent C^{13}+39 percent C^{12}) and N_2 is the number of protons scattered from a normal carbon target (1 percent C^{13}+99 percent C^{12}).

III. EXPERIMENTAL RESULTS

In the region covered by the yield curves (0.45–1.60 Mev) four anomalies were found. These correspond to the 0.556- and 1.16-Mev resonances observed by Seagrave[1] in the $C^{13}(p,\gamma)N^{14}$ reaction and the 1.46- and 1.55-Mev resonances observed by Seagrave but not definitely attributed to the $C^{13}(p,\gamma)N^{14}$ reaction because of the very low gamma-ray yield. These scattering experiments definitely show that excited states corresponding to these resonance energies occur in N^{14}. The very broad resonance at 1.25 Mev in the $C^{13}(p,\gamma)N^{14}$ reaction was not observed as a pronounced anomaly in the scattering measurements but was indicated in the theoretical analysis of the yield curve. Figures 3–8 show the results obtained.

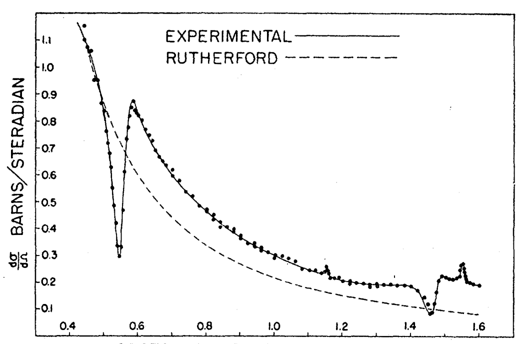

FIG. 4. Differential cross section for the elastic scattering of protons by C^{13} at a scattering angle of 90 degrees in the center-of-mass system.

An attempt was made to study the scattering at the 1.7-Mev resonance, but the resonance is so narrow that it could not be resolved. The resonance is about 2 kev wide, and the over-all resolution of the experimental procedure is about 3–4 kev at this energy.

The differential cross section for $C^{12}(p,p)C^{12}$ was measured from 300 to 600 kev at thirteen different angles. Figures 9–13 show the results obtained. The

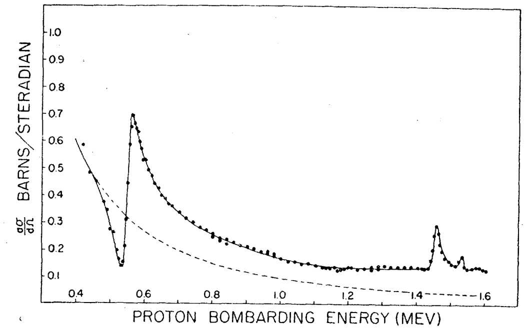

FIG. 5. Differential cross section for the elastic scattering of protons by C^{13} at a scattering angle of 120 degrees in the center-of-mass system. Note that the 1.16-Mev resonance has no effect at this angle.

data for $C^{12}(p,p)C^{12}$ are presented as the ratio of the observed cross section divided by the Rutherford cross section.

The differential cross section for the elastic scattering of protons by C^{13} and C^{12} was calculated using the formulas of Brown *et al.*[11] and Snyder *et al.*[9] The stopping cross section of carbon used for the low energies (400–600 kev) was that measured by Reynolds *et al.*[12] For

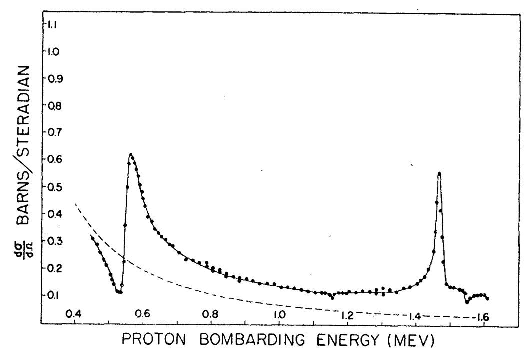

FIG. 6. Differential cross section for the elastic scattering of protons by C^{13} at a scattering angle of 140 degrees in the center-of-mass system.

the high energies (500–1700), the stopping cross section was calculated using Bethe's formula[13] and Segrè's value for the average ionization potential of carbon,

[11] Brown, Snyder, Fowler, and Lauritsen, Phys. Rev. **82**, 159 (1951).

[12] Reynolds, Whaling, Wenzel, and Dunbar, Phys. Rev. **92**, 742 (1953).

[13] M. Stanley Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 264 (1937).

$I=74.4$ ev.[14] The stopping cross sections agree to less than one percent in the overlapping region but are lower than the stopping cross section calculated by Hirshfelder and Magee[15] by about 6 percent.

The efficiency of the counter was calculated by scattering protons from copper and assuming that the scattering from copper obeys the Rutherford scattering law. The stopping cross section of copper used is that measured by Whaling and Wenzel.[16]

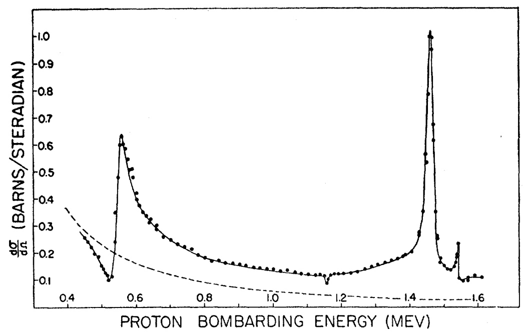

FIG. 7. Differential cross section for the elastic scattering of protons by C^{13} at the scattering angle of 160 degrees in the center-of-mass system.

At least 10 000 counts were taken at each point so that the statistical uncertainty given by the coefficient of variation $\pm 1/\sqrt{N}$ is 1 percent or less except at the smaller angles where the C^{12} scattering was subtracted, in which case the statistical uncertainty is about 2 or 3 percent. Systematic errors are believed to be less than one percent. The probable error in the absolute magnitude of the various solid angles is assumed to be about 3 percent[17] whereas the relative error in the solid angles is less than one percent. The error in the current integrator is about 1 percent. The uncertainty in the composition of the target is 2 percent. The reproducibility of results, which is affected by errors in settings and target smoothness, straggling, etc., was in most cases within 1 percent at the large scattering angles, 2–3 percent at 90 degrees, and 4–5 percent at 50 degrees. The error in the stopping cross section of carbon is assumed to be about 3 percent. This gives an uncertainty in the absolute magnitude of a little less than 5 percent for the large scattering angles and 8 percent for 50 degrees.

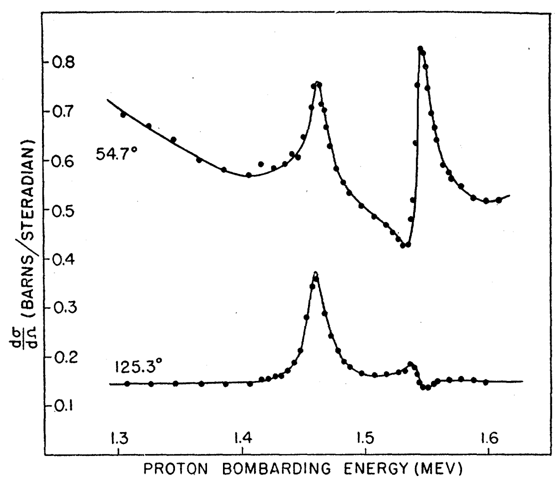

FIG. 8. Differential cross section for the elastic scattering of protons by C^{13} at the angles where $P_2(\cos\theta)=0$. Note that the 1.47-Mev resonance does not show any interference while the 1.55-Mev resonance does.

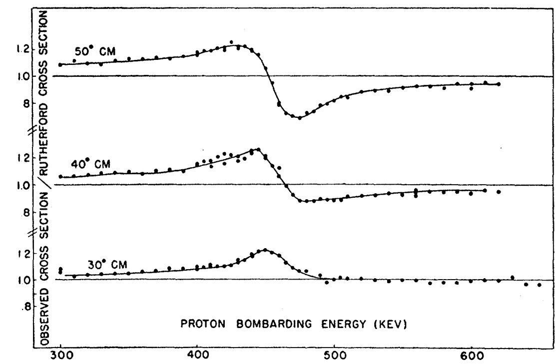

FIG. 9. The elastic scattering of protons by C^{12} at 30, 40, and 50 degrees in the center-of-mass system.

IV. DISCUSSION

A quantitative comparison of the values E_r and Γ with those derived from the proton-capture data awaits a detailed theoretical fit of the scattering data. However, the parities and in some cases the spins of the states can be determined from the scattering data without a detailed theoretical analysis. The states formed by partial waves of odd orbital angular momentum cannot show interference, i.e., a decrease, at 90 degrees, whereas those formed by even angular mo-

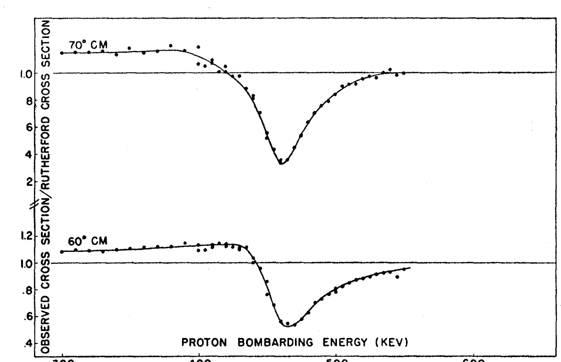

FIG. 10. The elastic scattering of protons by C^{12} at 60 and 70 degrees in the center-of-mass system.

[14] J. Mather and E. Segrè, Phys. Rev. **84**, 191 (1951).
[15] J. Hirshfelder and J. Magee, Phys. Rev. **73**, 207 (1948).
[16] W. Whaling and W. A. Wenzel (private communication).

[17] The efficiency of the counter is included in the effective solid angle; hence, this error is the assumed error in the stopping cross section for copper.

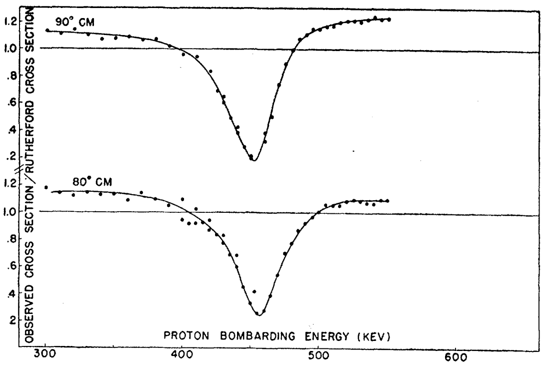

FIG. 11. The elastic scattering of protons by C^{12} at 80 and 90 degrees in the center-of-mass system.

mentum will usually show interference. Hence, if the parity of the target nucleus is known the parity can usually be determined from a study of the scattering at 90 degrees.

The dimensionless reduced width, $\theta^2=\gamma^2(\hbar^2/2Ma)^{-1}$, where γ^2 is the reduced level width of Wigner and Eisenbud,[18] has been calculated by Woodbury[19] employing the Coulomb tables of Block *et al.*,[20] with an interaction radius of $1.41(13^{\frac{1}{3}}+1)\times 10^{-13}$ cm. The quantity $1/\theta^2$ may be interpreted as the number of nuclear transversals of the incident particle in the compound system. Wigner's criterion[21] places an upper limit on θ^2 of 3, and with this criterion one is able to place an upper limit on the possible l value of the incident particle. See Table I.

The capture gamma-ray work of Seagrave[1] and the radiative cascade measurements of Woodbury, Day, and Tollestrup[2] show that the 8.06-Mev level in N^{14} formed by protons of 0.56 Mev is formed by s-wave protons. The scattering agrees with this finding. However, they were unable to assign a definite J value to this state. The proton has a spin of $\frac{1}{2}^+$ and C^{13} has a spin of $\frac{1}{2}^-$, thus the state can be either $J=0^-$ or 1^-. The theoretical scattering formula reduces to a relatively simple form when only s-wave phase shifts are taken into account, *viz.*,

$$d\sigma/d\omega=\tfrac{3}{4}|f_1|^2+\tfrac{1}{4}|f_0|^2,$$

where

$$f_1=(1/k)[-\tfrac{1}{2}\eta(\csc^2\tfrac{1}{2}\theta)\exp(i\eta\ln\csc^2\tfrac{1}{2}\theta)+\sin\delta_0^1\exp(i\delta_0^1)],$$

$$f_0=(1/k)[-\tfrac{1}{2}\eta(\csc^2\tfrac{1}{2}\theta)\exp(i\eta\ln\csc^2\tfrac{1}{2}\theta)+\sin\delta_0^0\exp(i\delta_0^0)].$$

δ_0^0 and δ_0^1 are the s-wave phase shifts for channel spin 0 and 1, respectively; $k=1/\lambda\!\!\!^{-}=\mu v/\hbar$ and $\eta=zZe^2/\hbar v$. The maximum and minimum of this resonance are tabulated along with the theoretical maximum and minimum for $J=0$ and 1 in Table II. It was assumed in calculating the theoretical maximum and minimum that all of the nonresonance phase shifts are zero. This shows that the best fit is with $J=1^-$ for this level in N^{14}.

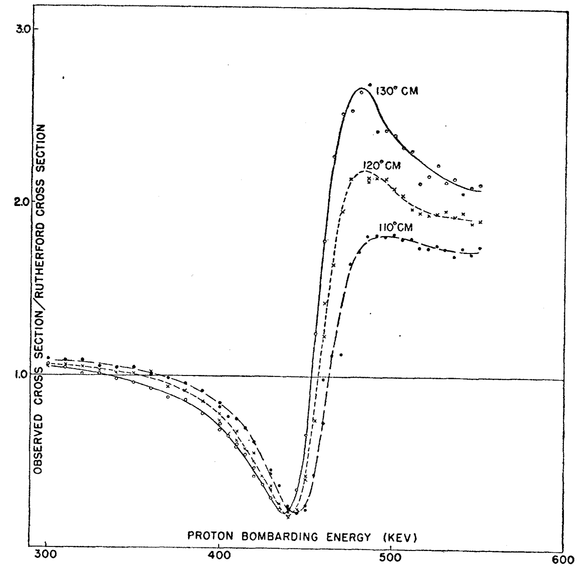

FIG. 12. The elastic scattering of protons by C^{12} at 110, 120, and 130 degrees in the center-of-mass system.

TABLE I. The dimensionless reduced width $\theta^2=\gamma^2(\hbar^2/2Ma)^{-1}$ of five of the resonances in $C^{13}(P,\gamma)N^{14}$.

E_R (Mev)			θ^2		
	s wave	p wave	d wave	f wave	g wave
0.55	0.4	13			
1.16		0.04	0.3	∞	
1.25	0.7	4.5			
1.47		0.05	0.5	∞	
1.55		0.016	0.1	4	

TABLE II. Table of the theoretical and experimental maximum and minimum cross section for the 0.56-Mev resonance. The best fit is $J=1^-$.

Scattering angle		Theoretical cross section (barns/steradian) $J=1^-$	$J=0^-$	Experimental cross section (barns/steradian)
50	Max	4.85	4.76	4.3
	Min	3.47	4.63	4.0
90	Max	0.77	0.66	0.87
	Min	0.25	0.57	0.30
120	Max	0.63	0.40	0.70
	Min	0.12	0.26	0.14
140	Max	0.61	0.35	0.62
	Min	0.10	0.20	0.11
160	Max	0.59	0.31	0.64
	Min	0.08	0.16	0.10

[18] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[19] H. H. Woodbury, Ph.D. thesis, California Institute of Technology, 1953 (unpublished).
[20] Block, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).
[21] E. P. Wigner, Am. J. Phys. **17**, 99 (1949).

The broad level in N^{14} at 8.70 Mev is also formed by s-wave protons at 1.25 Mev. This level does not show up in our measurements explicitly. This would indicate that if there is a resonance here the state must be $J=0^-$ because the effect is so small. Mr. Gerald Speisman has calculated the s-wave phase shifts from the data in the neighborhood of this resonance and has found that the $J=0$ s-wave phase shift appears indeed to be passing through a resonance. This state therefore has been assigned $J=0^-$.

The data for the 1.16-Mev resonance show a symmetrical peak at 90 degrees. This indicates that it is formed by protons with odd angular momentum. Wigner's criterion[21] eliminates the possibility that the state is formed by protons of angular momentum 3 or larger. Thus, the state is probably formed by p-wave protons. In a preliminary analysis of the data, Professor R. F. Christy found agreement with the assignment of $J=0^+$ but not for 1^+ or 2^+. The 8.06-Mev level in N^{14} has been assigned $J=0^+$.

Because of the dip in the scattering at 90 degrees, the 8.90-Mev level in N^{14} is formed by either s, d, or g-wave protons at 1.47 Mev. Wigner's criterion eliminates the possibility of g-wave and complexity of the angular distribution of the scattering rules out s-wave. The scattering at 54.7 degrees and 125.3 degrees [the angles where $P_2(\cos\theta)=0$] show symmetrical peaks which indicates that the state is formed by d-wave protons (see Fig. 8). The maximum of the resonance is best fitted with the assignment $J=3^-$.

The scattering near the 1.55-Mev resonance has a symmetrical peak at 90 degrees which would indicate formation by p- or f-wave protons. The parity is therefore probably even. The f wave is excluded by the Wigner criterion. The peak of the scattering could not be fitted with the assignments of $J=0^+$ or 2^+. It cannot be formed by d-wave protons because the scattering at 54.7 and 125.3 degrees shows interference. By a process of elimination, this state has been assigned $J=1^+$. A perfect fit to simple theory cannot be expected in view of nonresonance potential scattering.

TABLE III. Parameters for the 460-kev resonance in $C^{12}(p,\gamma)$.

Investigator:	Milne (present paper)	Jackson and Galonsky[a]	Jackson and Galonsky[b]	Fowler and Lauritsen[c]	Seagrave[d]
Reaction:	$C^{12}(p,p)$	$C^{12}(p,p)$	$C^{12}(p,p)$	$C^{12}(p,\gamma)$	$C^{12}(p,\gamma)$
E_p (kev)[e]	462	470	461	456	450
(kev)[e]	35	36	34	35	36
E_r (Mev)	2.370	2.379	2.369		
(kev)	32	33	31	32	32.5

[a] See reference 3.
[b] See reference 8.
[c] See reference 5.
[d] See reference 1.
[e] Laboratory system.

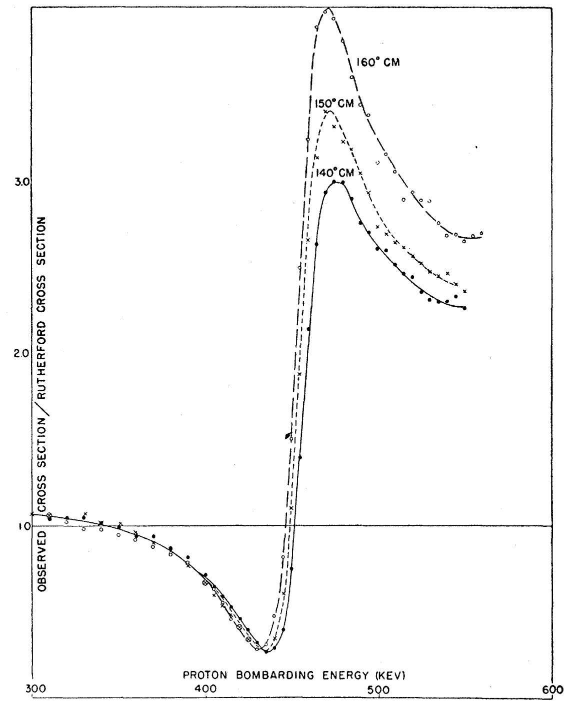

FIG. 13. The elastic scattering of protons by C^{12} at 140, 150, and 160 degrees in the center-of-mass system.

A phase-shift analysis has been performed on the C^{12} scattering data. The experimental cross section was consistently 6–8 percent higher than the calculated theoretical cross section. This was attributed to systematic errors in the stopping cross section. The resonance energy and width obtained from this analysis is tabulated in Table III along with previous values.

I wish to thank Dr. W. D. Warters for many hours of help in taking the data. I am also grateful to Professor R. F. Christy and Mr. Gerald Speisman for discussions of the theoretical aspect of this problem and to Dr. W. A. Fowler and Dr. J. D. Seagrave for suggesting this problem.