Energy Levels in Lu^{176} and Hf^{176}†

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The energies of the three gamma rays in cascade with the β^- emission of Lu^{176} have been determined to be 306, 203, and 89 kev. Peak intensity and conversion x-rays suggest the assignment of all transitions as $E2$. The energies support the theory of Bohr and Mottelson. The total K x-ray intensity shows no excess to be ascribed to a K-capture branch setting a limit of $K/\beta^-<0.1$. The half-life of Lu^{176} is $(2.15\pm0.10)\times10^{10}$ years.

EARLIER experimental work on the nuclides Lu^{176} and Hf^{176} has indicated the decay scheme shown in Fig. 1.[1,2] The question of the existence of an orbital capture branch could not be definitely settled, although it was not prominent. The great interest in this nuclide derives not only from its own high spin and moment, but from the fact that its decay makes accessible Hf^{176} in states of high spin. The theory of Bohr and Mottelson[3] can now be compared with experiment for several levels.

Through the kindness of Dr. Frank H. Spedding we have obtained a 4.974-gram sample of highly purified Lu_2O_3. This sample was used for measurements on our low-level scintillation spectrometer. A typical spectrum is shown in Fig. 2. The gamma-ray energies are found to be 306, 203, and 89 kev by comparison with a series of known radiations, chief weight being placed on the 279-kev transition in Hg^{203}. The 122-kev peak is due to backscattering.

In order to convert the peak areas to absolute intensities, corrections were made for the following factors: (1) Geometry. The source was spread over a rectangular area of 26.4 cm^2 to minimize self-absorption. Its geometry was determined by comparison of the 306- and 203-kev photo-peak areas in the geometry used to those at a distance of 10.1 cm from the crystal face, where the geometry could be calculated easily. Scattered gamma rays from the shield do not affect this result. The geometry was 5.4 percent. (2) Self-absorption. This was corrected for by interpolation from the tables of Davisson and Evans.[4] Absorption by the aluminum can was also corrected for in this way. (3) Stopping power of the crystal and fraction of pulses appearing in the "photo-peak." These were obtained from the data of Novey[5] on a crystal of the same dimensions; in the case of the latter effect these were checked by our own measurements. The agreement was satisfactory. (4) Escape of the iodine K x-ray. This factor is of importance for the 55-kev peak. Novey's value[5] was used.

The values obtained for the total intensity of each radiation in the sample appear in Table I. The first column shows the actual intensities. The second shows gamma-ray intensities corrected for K conversion assuming all are $E2$ transitions. The tables of Rose *et al.*[6] were used for the more energetic gammas, while the experimental value of McGowan[7] was used for the 89-kev transition. The intensity of this peak is the least accurate; values obtained using Bohr and Mottelson's value for α_{total} are given in parentheses. L and M conversion has been neglected except for the 89-kev line; this may cause an appreciable error for the 203-kev line.

The calculated intensities for the K x-ray assume a fluorescent yield of 0.92 in this region. X-rays of hafnium appear due to internal conversion, while those of lutetium appear due to self-absorption in the sample,

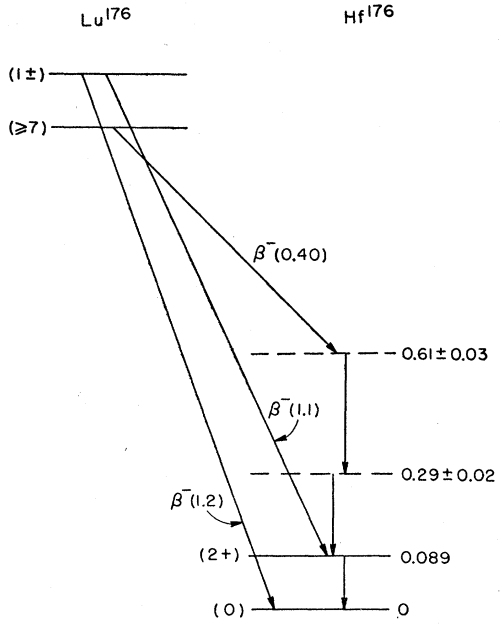

FIG. 1. Decay scheme of Lu^{176}—Hf^{176} (previous work).

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[1] G. Scharff-Goldhaber, as reported in M. Goldhaber and R. D. Hill, Revs. Modern Phys. **24**, 179 (1952).

[2] J. R. Arnold and T. Sugihara, Phys. Rev. **90**, 332 (1953).

[3] A. Bohr and B. R. Mottelson, Phys. Rev. **89**, 316 (1953); Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, 16 (1953).

[4] C. M. Davisson and R. Evans, Revs. Modern Phys. **24**, 79 (1952).

[5] T. B. Novey, Phys. Rev. **89**, 672 (1953).

[6] M. Rose *et al.*, Phys. Rev. **83**, 79 (1951).

[7] F. K. McGowan, Phys. Rev. **87**, 542 (1952).

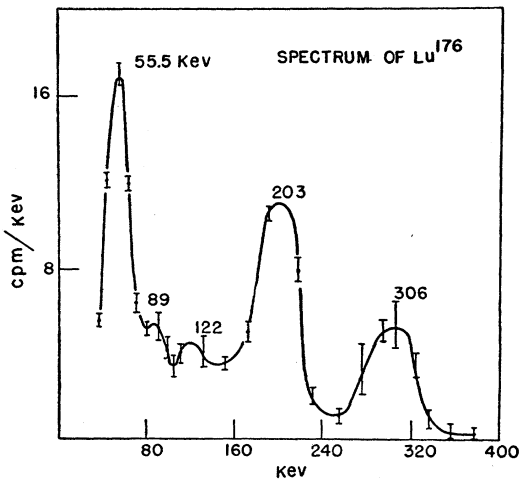

FIG. 2. Example of spectrum of Lu^{176}.

which occurs largely by photoeffect. This results in a "build-up" of K x-rays in a sample of finite thickness. We calculated the "build-up" intensity by assuming a uniform source of infinite extent and finite depth. For exponential absorption the fraction absorbed is given by

$$f_{\text{abs}}=(\tfrac{1}{2}x)[(2x-3)+(x+3)e^{-x}-x^2\text{Ei}(-x)], \quad (1)$$

where x is the source thickness in terms of the mean absorption length. If we now assume that the absorption occurs uniformly through the source (which is not a bad assumption), make a small correction for edge effects, and from the tables of Davisson and Evans obtain the fraction of the absorption due to photo-effect, we get the values given in the last column of Table I (a correction factor of 0.8 has been used for the fraction of the photo-effect due to K electrons). The calculation is of limited accuracy, but it suffices to show that the x-rays observed can be well accounted for without assuming a K-capture branch. Our experimental intensity for the 89-kev transition would allow $K/\beta^-\approx 0.06$; Bohr and Mottelson's value gives $K/\beta^-\approx 0.01$. Both are within the experimental error. A safe limit would be $K/\beta^-<0.1$.

Table II shows a comparison of the energies of the three transitions with the theory of Bohr and Mottelson.[3] The agreement is seen to be very good, with a tendency to a lower value of B for the high energies, which is also in accord with theory.[8] The next higher transition is estimated as 420 kev, which is to be compared with the value of 400 kev found by Suttle[9] for the energy of the β^- transition. The drop in the value of B and the experimental uncertainties leave open the possibility that there may be a small contribution of a singly forbidden β^- transition of very low energy. No gamma ray of corresponding energy has been seen, however. Hf^{176}, then, becomes one of the best cases yet recorded in favor of the model of Bohr and Mottelson.

TABLE I. Absolute intensities in disintegrations per minute per mg Lu_2O_3.

Energy (kev)	Electromagnetic Radiation	Corrected for K conversion	K x-rays expected from conversion	K x-rays expected from "build-up"
306	3.68	3.92	0.22	0.16
203	3.32	3.88	0.52	0.30
89	0.62	1.41	0.72	0.16
89[a]	(0.78)	(1.76)	(0.90)	(0.20)
K x-ray	2.35		1.46	0.62
			(1.64)	(0.66)

[a] The numbers in parentheses are the values obtained by assuming $\alpha_{\text{total}}=4$.

The half-life of Lu^{176} has been determined from the data given above, assuming an isotopic abundance of 2.6 percent in natural lutetium. The value obtained is $(2.15\pm 0.1)\times 10^{10}$ years. This value is probably independent of the presence of a minor orbital capture branch, which would give rise to Yb^{176}, if the theory is to be believed, with about the same series of gamma transitions.

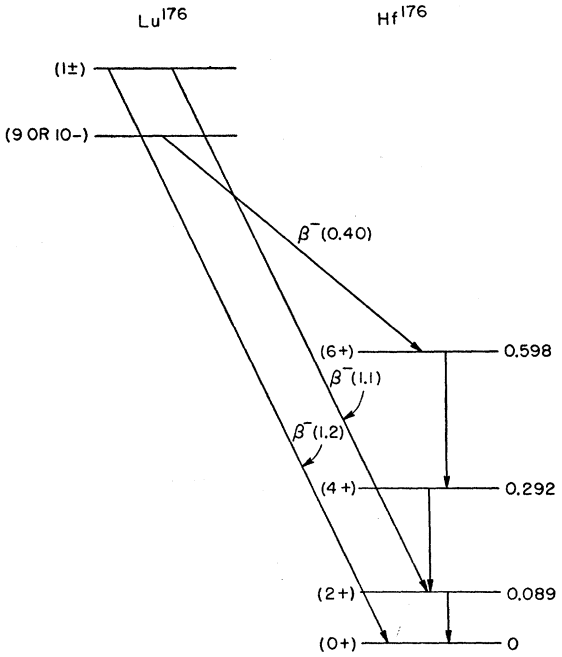

FIG. 3. Decay scheme of $Lu^{176}-Hf^{176}$ (present work).

It appears that this half-life is short enough for geochemical usefulness. Despite the rarity of lutetium, the fact that Hf^{176} is a 5 percent species and the great difference in geochemistry of the two elements bring this nuclide into the range of modern isotope-dilution techniques.

The decay scheme as presently established is shown in Fig. 3. The assignment of a spin to the ground state

[8] A. Bohr and B. R. Mottleson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, 16 (1953), see p. 91.

[9] A. D. Suttle, Ph.D. thesis, University of Chicago, 1952 (unpublished).

TABLE II. Energies of Hf^{176} transitions.

Energy (kev)	I_{upper}	$4I-2$	B (kev)
89	2	6	14.8
203	4	14	14.5
306	6	22	13.9
(420)[a]	(8)	(30)	(14)

[a] Estimated value.

of Lu^{176} is based on Klinkenberg's[10] theoretical statement that the parity is odd, and the ft value of the β^- decay indicating a third-forbidden transition.[9]

One may infer the possible presence of other odd-odd nuclei of high spin in this region. Such species, if their half-lives were too short for occurrence in nature, would be difficult to excite by ordinary means. Even first-forbidden transitions would be of rather long life, with correspondingly low intensity. Thus a number of the activities now known in this region may be isomeric states.

[10] P. F. A. Klinkenberg, Revs. Modern Phys. 24, 63 (1952).

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Microwave Spectrum of O_2†

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Twenty-four lines of the microwave spectrum of oxygen molecule have been carefully measured. The theoretical origin of this spectrum is reexamined and the effect of centrifugal distortion is taken into account. The agreement between theory and experiment is satisfactory. It is pointed out that this spectrum is not suitable to determine the velocity of light.

I. INTRODUCTION

THE microwave spectrum of oxygen is due to its $^3\Sigma$ ground state. The rotational levels are split into triplets by the interaction of the spins of the unpaired electrons and the end-over-end rotation. Magnetic dipole transitions within these triplets give rise to 25 observable lines centered near 60 000 Mc/sec and one line at approximately double the center frequency.

An expression for the energy levels of this so-called ρ-type triplet was first given by Kramers,[1] who considered the spin-spin interaction, and showed it to be equivalent to a coupling of the total electron spin S and the figure axis of the molecule proportional to $3\cos^2\theta-1$, where θ is the angle between S and the figure axis.

Hebb[2] considered the interaction of a component of electronic angular momentum, perpendicular to the figure axis and precessing about it, and the electron spin. This interaction was found to have exactly the same form as that considered by Kramers.

Schlapp[3] recalculated the energy levels including the magnetic interaction $\mu\mathbf{K}\cdot\mathbf{S}$, of the electron spin and the end-over-end rotation, where $\mathbf{K}$ is the angular momentum of the end-over-end rotation and μ is the coupling constant. His formulas for the frequencies are

$$\nu_+(K)=(W_{J=K}-W_{J=K+1})/h=-(2K+3)B+\lambda-(K+1)\mu+[(2K+3)^2B^2+\lambda^2-2\lambda B]^{\frac{1}{2}}, \quad (1a)$$

$$\nu_-(K)=(W_{J=K}-W_{J=K-1})/h=(2K-1)B+\lambda+K\mu-[(2K-1)^2B^2+\lambda^2-2\lambda B]^{\frac{1}{2}}, \quad (1b)$$

where B is the usual rotational constant and λ is the coupling constant of the Kramers' interaction. Using these formulas he could explain the infrared data obtained by Dieke and Babcock.[4]

The unresolved microwave transitions near 60 000 Mc/sec were first observed by Beringer[5] and then by Strandberg, Meng, and Ingersoll.[6] The low-pressure, resolved lines were observed and measured by Burk-

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[1] H. A. Kramers, Z. Physik 53, 422 (1929).

[2] M. H. Hebb, Phys. Rev. 49, 610 (1936).

[3] R. Schlapp, Phys. Rev. 51, 342 (1937).

[4] G. H. Dieke and H. D. Babcock, Proc. Natl. Acad. Sci. U. S. 13, 670 (1927).

[5] R. Beringer, Phys. Rev. 70, 53 (1946).

[6] Strandberg, Meng, and Ingersoll, Phys. Rev. 75, 1524 (1949).