for a region located a few mm from the entrance window the maximum will, in fact, occur at around 0°C. (At such a pressure the fluorescent beam decreases in intensity by more than 25 percent in the first 5 mm of its path.[4]) In general, more strongly absorbed components should exhibit intensity maxima at lower pressures. Irrespective of the direction from which one observes the resonance, in order to draw any definite conclusions one has to make large corrections for the decay of the beam, shape of the exciting line, angle of observation, imprisonment of the radiation, distance from the observation window (reabsorption), and so forth.[5] As a result, no reasonably accurate results for the relative intensities can be expected without using a highly refined technique. It is for this reason that direct measurements of the absorption are superior to an indirect method consisting of a determination of the intensity of resonance fluorescence.

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[2] A. C. G. Mitchell and M. W. Zemansky, *Resonance Radiation and Excited Atoms* (Cambridge University Press, Cambridge, 1934), see pp. 224 and 235–6.
[3] A. v. Malinowski, Ann. Physik 44, 935 (1914), Fig. 1 and pp. 951–2.
[4] Absorption coefficient data for 2537A are given, for instance, in M. W. Zemansky, Phys. Rev. 36, 219 (1930).
[5] A very good discussion of these corrections can be found in P. Kunze, Ann. Physik 85, 1013 (1928).

Internal Compton Effect in Ba^{137}

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(Received December 16, 1953)

CONTINUOUS gamma rays accompanying internal conversion were recently detected[1] for the first time. The process bears the same relationship to the Compton effect as internal pair production does to pair production, and will be referred to as the internal Compton effect. The ratio of the total number of gamma rays between 50 and 200 kev to the number of internally converted electrons was found to be in crude agreement with the ratio predicted by the only theory available, the semiclassical calculations of Wang Chang and Falkoff,[2] but the angular distribution differed markedly from the theoretical prediction. The disagreement is not surprising, for the theory neglects the recoil of the electron upon emission of the real gamma ray, an approximation which is not completely justifiable for the electron and gamma-ray momenta involved. The theory also ignores the mechanism by which the electron is ejected and is, therefore, independent of the multipole character of the virtual gamma ray and of the initial spatial distribution of the electron. Finally, it ignores the effect of the Coulomb field on the electron in the intermediate and final state.

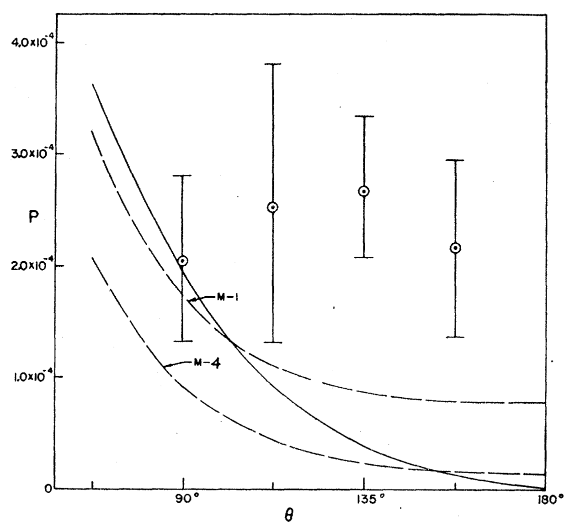

FIG. 1. Angular correlation between direction of conversion electron from Ba^{137m} and of the gamma ray of the continuous radiation. Circled points represent the experimental measurements of Brown and Stump; the solid curve is the value predicted on the basis of semiclassical theory. The dotted curves represent the values predicted for the actual case, an $M4$ transition, and, for comparison, for an $M1$ transition. P is the probability of emission of a gamma ray of the continuous radiation with an energy between 50 and 200 kev into a unit solid angle at an angle of $\theta°$ with respect to the electron.

All but the last defect are eliminated by a quantum calculation using the Born approximation. This latter approximation is still quite a serious one for Ba^{137}; in fact, the Born approximation calculation of the internal-conversion probability, which forms one of the two parts of the calculation, is only 0.4 of the true value. It would seem, though, that the ratio of continuous gamma rays to internally converted electrons should be more reliable than either calculation separately, since quite similar approximations are involved. Nevertheless, as seen from Fig. 1, the quantum calculations do not represent an improvement. The error is not due to the neglect of L shell conversion and it is highly unlikely that it is due to the neglect of $E5$ conversion. The source of the error must then be the neglect of the Coulomb field. It should be noted that in the experimental results the beta-decay bremsstrahlung background was subtracted off on the basis of a calculation which also ignored the effect of the Coulomb field.

The quantum calculations, which should be valid for small charge and large energy, have been performed for an arbitrary magnetic multipole. It is found that the ratio of continuous gamma rays to internally converted electrons is independent of the nuclear matrix element and *decreases* as the charge or the multipole increases. The transition rate for continuous gammas, however, increases as Z increases. The details of these calculations will be submitted for publication shortly.

[1] H. B. Brown and R. Stump, Phys. Rev. 90, 1061 (1953).
[2] C. S. Wang Chang and D. L. Falkoff, Phys. Rev. 76, 365 (1949).

Kinematic Criterion for Meson Production in Fundamental Particle Collisions*

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(Received December 10, 1953)

WHEN one or several mesons are made in a nucleon-nucleon or pion-nucleon collision at cosmotron energies,[1] the angle of recoil of the nucleons in the laboratory system cannot exceed a maximum value $\bar{\theta}_m$ which depends on the energy of the incident nucleon or pion and on the number of mesons produced. As an example, θ_m is 68° for a collision of a 2.2-Bev neutron with a proton in which one pion is made. This result can be used to establish that a particle which goes off at an angle greater than θ_m in an inelastic n-p collision cannot be a nucleon and must, therefore, be a pion (or a heavy meson). Such a criterion is useful in the analysis of cloud-chamber pictures.

In order to obtain $\bar{\theta}_m$, the maximum possible velocity of a nucleon in the center-of-mass system (c.m.s.) must be calculated. In order that the nucleon have maximum velocity, it is necessary that all of the other particles move in a direction opposite to the nucleon. The masses, momenta, velocities, and total energies of the particles in the c.m.s. will be denoted by m_i, p_i, v_i, and E_i, respectively; the subscript $i=1$ pertains to the nucleon considered; the other particles are labeled by $i=2, \cdots k$. In view of conservation of energy and momentum, v_1 is given by

$$v_1 = c^2 \sum_{i=2}^{k} p_i / (E - \sum_{i=2}^{k} E_i), \qquad (1)$$

where $E \equiv \Sigma_{i=1}{}^k E_i$ is the total energy in the c.m.s. v_1 is to be

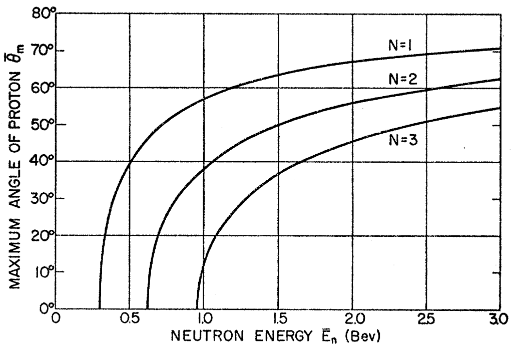

FIG. 1. Maximum angle $\bar{\theta}_m$ of recoil proton in n-p collision with production of N pions as a function of kinetic energy $\bar{E}_n$ of incident neutron in laboratory system.

maximized subject to the condition that

$$\sum_{i=1}^{k} E_i = \text{constant}.$$

This problem is equivalent to that of finding the extremum of

$$K \equiv v_1 + \lambda \sum_{i=1}^{k} E_i, \tag{2}$$

where λ is a Lagrangian multiplier. K can be written

$$K = c^2 \sum_{i=2}^{k} p_i / (E - \sum_{i=2}^{k} E_i) + \lambda c \{ [m_1^2 c^2 + (\sum_{i=2}^{k} p_i)^2]^{\frac{1}{2}} + \sum_{i=2}^{k} (m_i^2 c^2 + p_i^2)^{\frac{1}{2}} \}. \tag{3}$$

The extremum of K with respect to variations of $p_2, p_3, \cdots p_k$ is obtained from the conditions

$$\partial K / \partial p_j = 0. \quad (j = 2, 3, \cdots k) \tag{4}$$

In view of $\partial E_j / \partial p_j = v_j$, Eqs. (4) become

$$\frac{\partial K}{\partial p_j} = c^2 / (E - \sum_{i=2}^{k} E_i) + c^2 \sum_{i=2}^{k} p_i v_j / (E - \sum_{i=2}^{k} E_i)^2 + \lambda [c^2 \sum_{i=2}^{k} p_i / (E - \sum_{i=2}^{k} E_i) + v_j] = 0. \quad (j = 2, 3, \cdots k) \tag{5}$$

In order that the $k-1$ equations (5) for λ be simultaneously satisfied, these equations must be identical, giving

$$v_2 = v_3 = \cdots = v_k. \tag{6}$$

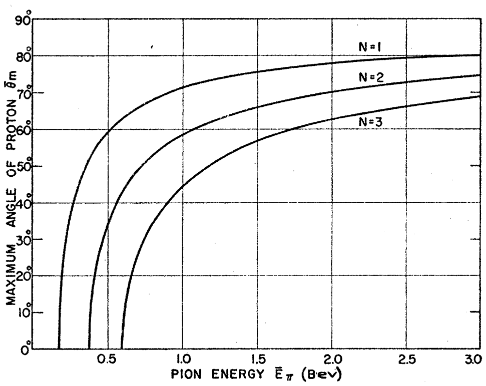

FIG. 2. Maximum angle $\bar{\theta}_m$ of recoil proton in pion-nucleon collision with production of N pions as a function of kinetic energy $\bar{E}_\pi$ of incident pion in laboratory system.

This result implies that particles 2, 3, $\cdots k$ move like a single particle of mass

$$m' = \sum_{i=2}^{k} m_i.$$

The momentum of m' is p_1, so that

$$(c^2 p_1^2 + m_1^2 c^4)^{\frac{1}{2}} + [c^2 p_1^2 + (\sum_{i=2}^{k} m_i)^2 c^4]^{\frac{1}{2}} = E. \tag{7}$$

Equation (7) can be solved for p_1, and hence for v_1, with the result

$$v_1 = c \left(\frac{A}{4E^2 m_1^2 + A} \right)^{\frac{1}{2}}, \tag{8}$$

where

$$A \equiv [(E + \sum_{i=2}^{k} m_i c^2)^2 - m_1^2 c^4][(E - \sum_{i=2}^{k} m_i c^2)^2 - m_1^2 c^4]. \tag{9}$$

The angle $\bar{\theta}$ of the nucleon in the laboratory system is given by

$$\tan\bar{\theta} = \frac{v_1 (1 - V^2/c^2)^{\frac{1}{2}} \sin\theta}{V + v_1 \cos\theta}, \tag{10}$$

where V is the velocity of the c.m.s. with respect to the laboratory, and θ is the angle between the directions of the nucleon and the incident particle in the c.m.s. The maximum value of $\bar{\theta} (\equiv \bar{\theta}_m)$ is obtained from

$$d \tan\bar{\theta} / d\theta = 0, \tag{11}$$

which gives $\cos\theta = -v_1/V$. Upon inserting this result in Eq. (10), one finds

$$\tan\bar{\theta}_m = v_1 (1 - V^2/c^2)^{\frac{1}{2}} / (V^2 - v_1^2)^{\frac{1}{2}}. \tag{12}$$

Equations (8) and (12) determine $\bar{\theta}_m$. Figure 1 shows $\bar{\theta}_m$ for production of $N = 1$, 2, and 3 pions in n-p collisions as a function of the kinetic energy $\bar{E}_n$ of the incident neutron in the laboratory system. Figure 2 shows $\bar{\theta}_m$ for production of $N = 1$, 2, and 3 pions in pion-nucleon collisions as a function of the kinetic energy $\bar{E}_\pi$ of the incident pion.

We note that Eqs. (8), (9), and (12) are general, and can be used for any particle m_1 when the total energy E and the masses $m_1, m_2, \cdots m_k$ are known.[2] These equations can be applied to find the maximum angle of a particle which results from the decay in flight of a meson or a V particle.

A further application of Eqs. (8) and (9) concerns the maximum possible momentum $\bar{p}_1$ which a particle can have in the laboratory system as a function of the laboratory angle $\bar{\theta}_1$. Upon assuming a value of m_1, v_1 can be calculated, and $\bar{p}_1$ is obtained from v_1 and V. If the observed momentum exceeds $\bar{p}_1$, this fact can be used to aid in identifying the particle.

I would like to thank Dr. R. P. Shutt and Dr. H. S. Snyder for helpful discussions.

* Work done under the auspices of the U. S. Atomic Energy Commission.
[1] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. 91, 758 (1953).
[2] Equations (8) and (9) are unchanged if an arbitrary number of γ rays or neutrinos accompany $m_1, m_2, \cdots m_k$. For it may be readily proved that the modification of Eq. (7) due to such messless particles yields a value of $\bar{p}_1$ which is greatest when the total energy of the messless particles is zero.

Phase-Shift Analysis of High-Energy Nucleon-Nucleon Scattering*

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(Received December 7, 1953)

THE 240-Mev proton-proton scattering data[1] have been analyzed in terms of s-wave and p-wave anomalies.[2] If the s-wave phase shift ${}^1K_0$ is assumed to be ${}^1K_0 = 31.3°$,[3] then, neglecting Coulomb interaction, the combinations of p-phase shifts